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From ILS to Hybrid ILS ... and other extensions

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ILS 1

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Outline

- ▶ Introduction to ILS
- ▶ Applications of ILS
- ▶ Hybrid ILS and other Extensions
 - Hybrid with other metaheuristics
 - SimILS
 - Two-stage Optimization using ILS
 - MathILS



ILS 2

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Outline

- ▶ Applications:
 - Market Basket Analysis
 - BonArea
 - Supply Chain Design for ecommerce
 - SEAT/Volkswagen
 - Zara (INDITEX & OESIA)



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Local optimization algorithms



- ▶ Local search
 1. Get a initial solution x (current solution). Use a constructive heuristic.
 2. Search the neighborhood. While there is an untested neighbor of x :
 - * 2.1 Let x' be an untested neighbor of x ;
 - * 2.2 If $c(x') < c(x)$ set $x=x'$; (x' is the new current solution)
 3. Return x (local optimal solution).

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Local optimization algorithms



- ▶ Design of a local optimization algorithm:
 - Obtain an **initial solution**
 - * Heuristic
 - * Random solution
 - Define the **neighborhood**
 - * Specific for each problem
 - How to **search the neighborhood**
 - * Complete search
 - * First improvement

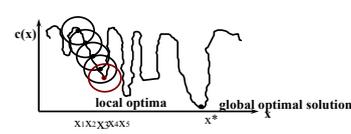
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Local optimization algorithms



- ▶ Comments
 - The search stops at the first local optimum solution with respect to the neighborhood N .
 - The final solution highly depends on the initial solution and on the neighborhood.
 - No way back out of unattractive local optima...



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Multi-Start

- ▶ Iterative improvement or hill-descending
 - 1. Get a initial random solution x .
 - 2. Run an **local optimization** (output x)
 - 3. If $\text{cost}(x) < \text{cost}(x_{\text{best}})$ set $x_{\text{best}} = x$;
 - 4. If the stop criteria is not verified, go back to step 1.
 - 5. Output the best solution found.
- * Comments
 - Successive repetition of local improvement.
 - Easy to implement.
 - Random solutions may be very bad.

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Iterated Local Search

- ▶ A Local Search Method...
 - Single chain on search
- ▶ Search on the space of local optimal solutions
- ▶ Combines local optimization with a big transition/large step/perturbation.
 - Perturbation should not be easily undone by the local search
 - Most important aspect of the ILS
- ▶ Able to make large changes at any stage of the algorithm.

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Iterated Local Search

- ▶ Get an **initial solution** x ;
 - * Heuristic method or a random solution.
 - * **local optimization** method
- ▶ For a certain number of iterations:
 - **Perturbation Step**
 - * method that makes a large modification based in optimization and on the structure of the solution x , resulting in x' .
 - Small-steps
 - * **local optimization** method, initial solution x' ; final solution x'' .
 - **Perform an accept/reject test**
 - * accept all solutions, accept with a certain probability or accept only if it is a better solution.
 - * If x'' is accepted, then $x = x''$.
- ▶ Return the best solution found.

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Iterated Local Search

- ▶ A simple implementation...
 - **Generate Initial Solution**
 - * Greedy Heuristic
 - **Local Search Method**
 - * First improvement local search
 - * Definition of neighborhood
 - **Perturbation Method**
 - * One move of a high level neighborhood
 - **Acceptance Criteria**
 - * Accept if a better solution is found

Often leads to very good performance

Only requires few lines of additional code

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Iterated Local Search

<ul style="list-style-type: none"> ▶ Generate Initial Solution <ul style="list-style-type: none"> ▪ Randomized Greedy Heuristic ▪ Random solution ▶ Acceptance Criteria <ul style="list-style-type: none"> ▪ Better ▪ Random Walk ▪ Simulated Annealing type ▪ Restart 	<ul style="list-style-type: none"> ▶ Local Search Method <ul style="list-style-type: none"> ▪ Local search ▪ Tabu search ▶ Perturbation <ul style="list-style-type: none"> ▪ Higher level of neighborhood ▪ Strength of the perturbation <ul style="list-style-type: none"> * Big/small ▪ Adaptive memory ▪ Modify input data ▪ Optimized perturbation
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Iterated Local Search

- ▶ Improving ILS
 - **Relationship between local search and perturbation.**
 - * Perturbation must lead to a new region of the solution space that cannot be reached by a local search method.
 - * Perturbation should not be easily undo by the local search.
 - Perturbation can incorporate problem-specific information.
 - * As for example optimization methods
 - * Destruction and construction approach
 - A good perturbation transforms one excellent solution into a excellent starting point to a local search.
 - Local search method must be fast.
 - Complexity must be added progressively and in a modular way.

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Iterated Local Search

- ▶ Google Scholar's number of publications
- ▶ "Iterated Local Search"
 - About 10,000 publications

Year	Number of Publications
2010	100
2011	150
2012	200
2013	250
2014	300
2015	400
2016	500
2017	600
2018	800
2019	1200

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Iterated Local Search

- ▶ ILS applied to Complex and large-scale real problems

Accuracy	<ul style="list-style-type: none"> • Complex problems. • Large scale problems.
Speed	<ul style="list-style-type: none"> • Fast answer. • Analysis of several scenarios.
Flexibility	<ul style="list-style-type: none"> • Fast changes. • Different constraints in different areas.
Simplicity	<ul style="list-style-type: none"> • Need of fast implementation

ILS 14

Traveling Salesman Problem

- ▶ Traveling Salesman Problem
 - Given a number of cities and the costs (distances) of traveling from any city to any other city...
 - What is the least-cost round-trip route that visits each city exactly once and then returns to the starting city?
 - <http://www.math.uwaterloo.ca/tsp/>

ILS 15

Traveling Salesman Problem

- ▶ Generate Initial Solution
 - Constructive Heuristic: nearest neighbor, insertion heuristic
- ▶ Local Search Method
 - 2-opt/3-opt Neighborhood
- ▶ Perturbation Method
 - One 4-opt move (double-bridge)
- ▶ Acceptance Criteria
 - Accept only if the best solution improved

ILS 16

Maximum Cut-Clique Problem

- ▶ Given a clique C , its edge neighborhood (cut-clique) is defined by the set of edges $E'(C) = \{(i,j) \in E : i \in C \text{ and } j \in V \setminus C\}$, and $|E'(C)|$ is its size. Denote $N(i) = \{j \in V : (i,j) \in E\}$.
- ▶ Maximum Cut-Clique
 - Maximum edge neighborhood clique

$\max |E'(C)| = 7$

ILS 17

ILS for Cut-Clique Problem

- ▶ Perturbation
 - Select randomly one node
 - Build the clique with all nodes in the previous clique and fully connected to this node
 - Set $C \leftarrow [C \cap N(i)] \cup \{i\}$;
 - Set $U \leftarrow \emptyset$ and $C' \leftarrow C$;
- ▶ Local Optimization
 - Add, Swap and Aspiration moves
 - R-ILS (random version)
 - D-ILS (deterministic version)

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Maximum Cut-Clique Problem

- ▶ Traditional approach
 - Solve with Integer Linear Programming Software
 - Branch-and-Bound / Branch-and-cut general exact algorithm
 - Obtain the Optimal Solution / Lower Bound
 - CPLEX Optimizer

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Computational results:

Intel Core i7-2600 with 3.40GHz and 8GB RAM; using CPLEX 11.2

The ---- symbol means that CPLEX was not able to read and preprocess the model in one hour

time in seconds

Instance	V	E	d	MC problem		MCC Problem		
				$\phi(G)$	N(C)	C	N(C)	time
d1-RTN	2418	9317	0.0032	10	195	8	1273	605.11
d3-RTN	4755	26943	0.0024	18	1097	----	----	----
d7-RTN	6511	44615	0.0021	18	1576	----	----	----
d15-RTN	7965	62136	0.0020	18	1979	----	----	----
d30-RTN	10101	91803	0.0018	21	13099	----	----	----
d66-RTN	13308	148035	0.0017	----	----	----	----	----
c-fat200-1	200	1534	0.077	12	72	9	81	0.05
c-fat200-2	200	3235	0.163	24	264	17	306	0.09
c-fat200-5	200	8473	0.426	58	1682	44	1892	0.05
c-fat500-1	500	4459	0.036	14	98	11	110	0.76
c-fat500-2	500	9139	0.073	26	338	19	380	0.80
c-fat500-5	500	23191	0.186	64	2048	48	2304	0.83
c-fat500-10	500	46627	0.374	126	7938	94	8930	0.58

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Computational results of the ILS:

Intel Core i7-2600 with 3.40GHz and 8GB RAM

100 runs

time in seconds

Instance	V	E	d	MCC Problem		
				C	N(C)	time
d1-RTN	2418	9317	0.0032	8	1273	0.1762
d3-RTN	4755	26943	0.0024	12	3526	0.4743
d7-RTN	6511	44615	0.0021	15	5656	0.6777
d15-RTN	7965	62136	0.0020	16	7772	0.8757
d30-RTN	10101	91803	0.0018	21	13099	1.1317
d66-RTN	13308	148035	0.0017	28	22379	1.4081
c-fat200-1	200	1534	0.077	9	81	0.1385
c-fat200-2	200	3235	0.163	17	306	0.0866
c-fat200-5	200	8473	0.426	44	1892	0.0664
c-fat500-1	500	4459	0.036	11	110	0.5451
c-fat500-2	500	9139	0.073	19	380	0.3595
c-fat500-5	500	23191	0.186	48	2304	0.2381
c-fat500-10	500	46627	0.374	94	8930	0.2111

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Market Basket Analysis

- ▶ The main objective is to analyze large dataset of store transactions
- ▶ Obtain relevant insights to do a better planning of the Marketing strategies and operations.
 - Product placement
 - Optimal product-line offering
 - Personalized marketing campaigns
 - Product promotions

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Market Basket Analysis

- ▶ Raeder and Chawla (2011) say "... no techniques currently available in the literature sufficiently addresses the problem of finding meaningful relationships in a large transaction databases."
- ▶ Dataset
 - a household panel database for the British ice cream market.
 - 691 different varieties of products available in the British market.

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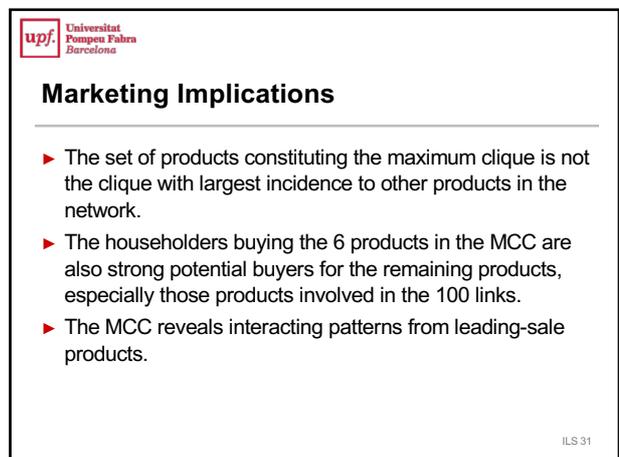
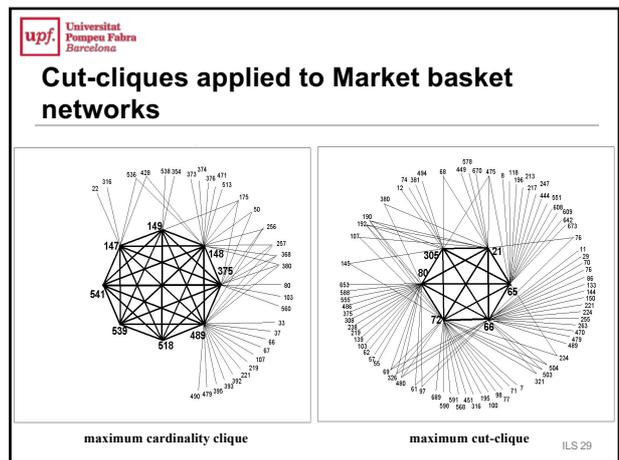
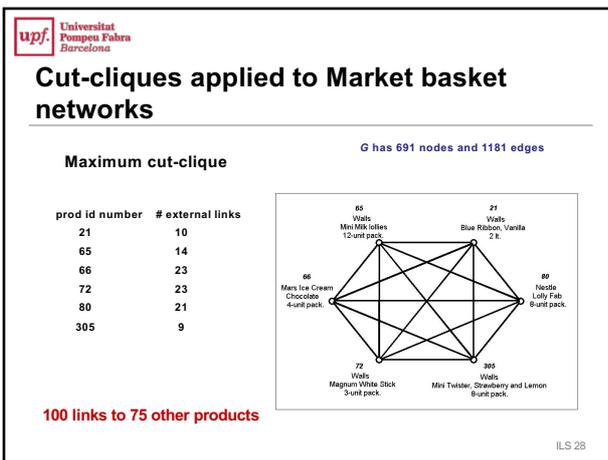
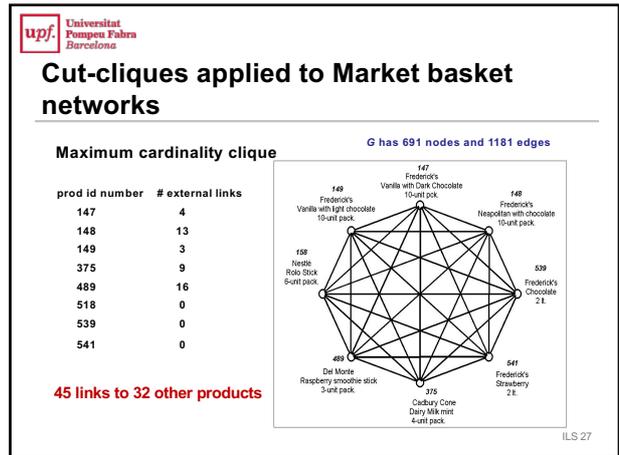
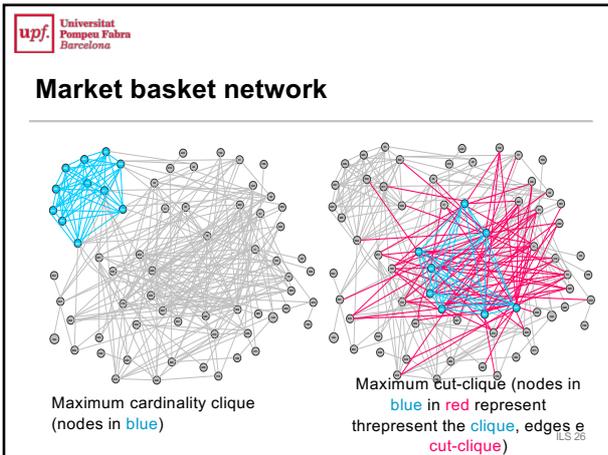
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Market basket network

nodes: products in a store

edges: represent pairs of products (i, j) bought together by a customer on a given purchase visit to the store

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Hybrid ILS with other metaheuristics

- ▶ Use the ILS structure with other metaheuristics
- ▶ **Local Optimization Phase**
 - Tabu Search
 - VNS
 - Simulated Annealing
 - Variable Neighborhood Search
 -
- ▶ **Perturbation Phase**
 - Large neighborhood change

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Distribution problem

- ▶ **Extended Vehicle Routing Problem**

 - Heterogeneous fleet (7 different truck capacities)
 - Time windows in the stores
 - Constraints of assigning some trucks to some stores.
 - Maximum driving hours
 - Multitrip for some vehicles
 - Sales constraints
- ▶ **Minimize operative costs**

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Distribution problem

- ▶ **GILS-VND Algorithm**

 - GILS-VND, combina un Iterated Local Search (ILS), Greedy Randomized Adaptive Search Procedure (GRASP), y Variable Neighborhood Descent (VND).
 - ILS Structure
 - * Initial solution: **GRASP**
 - * Local improvement: **Variable Neighborhood Descend**
 - * Perturbation: **Refine using random neighborhood**
 - Coelho V.N., Gragas A., Ramalhinho H., Coelho I.M., Souza M.J.F. (2016), An ILS-based Algorithm to Solve a Large-scale Real Heterogeneous Fleet VRP with Multi-trips and Docking Constraints, European Journal of Operational Research 250 (2): 367–376.

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Distribution problem

- ▶ **Computational results**


Statistical results for the new set of instances: GILS-VND vs. Company .

Instance	Company	GILS-VND			
		Best	Average	Std. dev.	Gap (%)
K	32,472	30,507	30,692	70	-5.48
L	18,997	16,327	16,427	42	-13.53
M	20,266	18,017	18,146	43	-10.46
N	51,609	45,523	45,813	100	-11.23
O	45,764	40,846	40,995	66	-10.42

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Distribution problem

- ▶ **Results**

 - Savings 10% daily with respect to the actual solutions.
 - * A significant daily amount!
 - Savings of 2% compared with Prins' Algorithm with a simpler version of the problem.
 - Smaller number of vehicles need.
 - Better coordination with sales department.

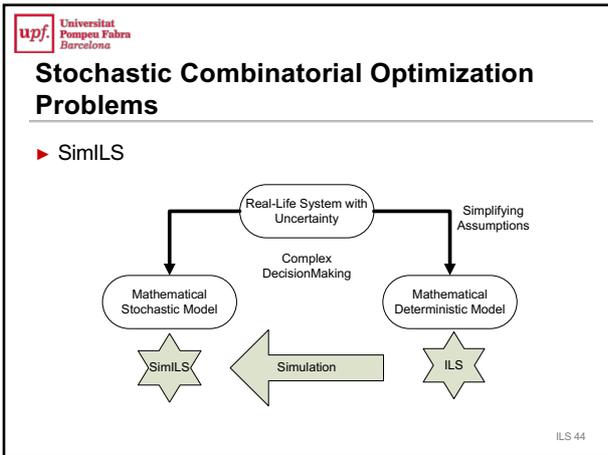
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SimILS

- ▶ **Stochastic Combinatorial Optimization Problems**
 - Uncertainty is present – Random Data
 - * Example: Stochastic Demand in Vehicle Routing Problems
 - * Stochastic processing times in scheduling
 - * etc...
 - Strategic Problems
- ▶ **Extends ILS to solve Stochastic Models...**
- ▶ **SimILS**
 - Simulation + Iterated Local Search

ILS 43

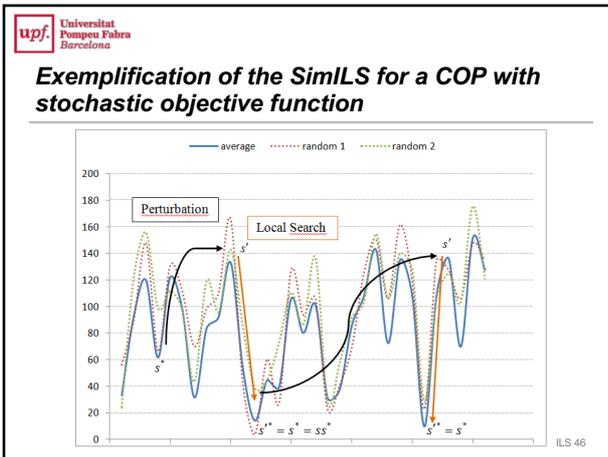


SimILS

```

    Procedure SimILS
      s0 = GenerateInitialSolution
      s* = LocalSearch(s0)
      (s*, sf(s*), statistics) = Simulation(s*, long)
      Repeat
        s' = Perturbation(s*, history)
        s'' = LocalSearch(s')
        (s'', sf(s''), statistics) = Simulation(s'', short)
        s* = AcceptanceCriterion(s*, s'', history)
      Until termination condition met
      (s*, sf(s*), statistics) = Simulation(s*, long)
      Return s*, sf(s*)
    End
  
```

ILS 45



SimILS Stochastic Constraints

```

    Procedure SimILS
      s0 = GenerateInitialSolution (input data: average values)
      s* = LocalSearch(s0)
      (s*, sf(s*), service level) = Simulation(s*, long)
      Repeat
        s' = Perturbation(s*, history)
        s'' = LocalSearch(s')
        (s'', sf(s''), service level) = Simulation(s'', short)
        s* = AcceptanceCriterion(s*, s'', service level, history)
      Until termination condition met
      (s*, bsf(s*), service level) = Simulation(s*, long)
      Return(s*; bsf(s*))
    End
  
```

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Designing Internal supply routes

► Design a fix set of routes to supply components and materials to the production assembly line.

► Routes are fixed... Demand is stochastic.

Route	Locations	Material type	Resource	Period 1	Period 2	Period 3
1	{4, 7, 8}	Boxes	1	Locations {4, 7, 8} Dist: {10, 7, 2}	Loc: {4, 7, 8} Dist: {10, 20, 45}	Loc: {4, 7, 8} Dist: {16, 7, 8}
2	{1, 2, 3, 4, 5}	Boxes	2	Loc: {1, 2, 3, 4, 5} Dist: {2, 10, 3, 5, 9}	Loc: {1, 2, 3, 4, 5} Dist: {10, 12, 3, 8, 9}	Loc: {1, 2, 3, 4, 5} Dist: {12, 16, 3, 8, 9}

Joint work with Marcellus Fabri (UPF)

ILS 48

Designing Internal supply routes

► Working on a SimILS

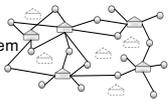
Results Improvement

- Fewer Distance Traveled all Periods**: The total of meters traveled throughout the simulation is 16.6% fewer than the current solution.
- Higher Load level**: The convoys depart with a higher load level (13.3%).
- Fewer Delays**: The amount of delays materials per period is smaller (-23%).
- Higher Service-Quality-Level**: Regarding a lower delays scenario. The Service-Quality-Level increases.

ILS 49

Supply Chain Design for ecommerce

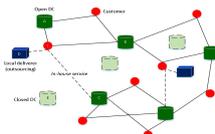
- Supply Chain Design for ecommerce
 - Two-Stage Stochastic Programming Problem
- The goal is to find:
 - the subset of warehouses to be opened;
 - and determine the customer's assignment to the open warehouses
 - ... such that all the demand is served at minimum total cost.
 - Demand is stochastic ...
 - Each customer area must have 2 or 3 warehouses assigned as regular warehouses.



ILS 50

Supply Chain Design for ecommerce

- Two-Stage Optimization Problems
- The problems has two groups of variables interrelated among them ...
 - Strategical decision variables (long term deterministic decision)
 - Operational decision variables (short term decisions), therefore **stochastics** at the moment of the decision process!



ILS 51

Supply Chain Design for ecommerce

- Solving this Two-Stage **Stochastic** Optimization Problem
- Deterministic Equivalent Model (DEM)**
 - solved by CPLEX.
- SimILS**
 - Simulation + Iterated Local Search
 - Use simulation to obtain the expected overall cost.
 - Local Search on open/close warehouses
 - Only *promising* solutions are tested in a stochastic environment.
- SimILS results compared favorably with **DEM**, with shorter running times.

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Supply Chain Design for ecommerce

- Deterministic Equivalent Model (DEM)**
 - Stochastic Programming

$$\begin{aligned}
 (\text{SCFLPrp}) \quad & \min Z_{\text{stoch}} = \sum_{j=1}^m f_j y_j + \sum_{i=1}^n \sum_{k=1}^s \pi_k c_{ij} d_{ik} x_{ijk} \\
 \text{s.t.} \quad & \sum_{j=1}^m \tau_{ij} \leq R, & \text{for } i = 1, \dots, n; \\
 & \sum_{j=1}^m x_{ijk} = 1, & \text{for } i = 1, \dots, n, k = 1, \dots, s; \\
 & \sum_{i=1}^n d_{ik} x_{ijk} \leq q_j y_j, & \text{for } j = 1, \dots, m, k = 1, \dots, s; \\
 & x_{ijk} \leq \tau_{ij}, & \text{for } i = 1, \dots, n, j = 1, \dots, m, k = 1, \dots, s; \\
 & y_j \in \{0,1\}, \tau_{ij} \in \{0,1\}, x_{ijk} \geq 0 & \text{for } j = 1, \dots, m; i = 1, \dots, n; k = 1, \dots, s
 \end{aligned}$$

Open/close Warehouse Regular/not regular Warehouse Demand assignment on scenario k

ILS 53

SimILS for SC Design Problem

```

Procedure SimILS
  s0 = GenerateInitialSolution
  s* = LocalSearch(s0)
  (s*, sf(s*), statistics) = Simulation(s*, long)
  Repeat
    s' = Perturbation(s*, statistics)
    s'' = LocalSearch(s')
    (s'', sf(s''), statistics) = Simulation(s'', short)
    s* = AcceptanceCriterion(s*, s'', history)
  Until termination condition met
  (s*, sf(s*), statistics) = Simulation(s*, long)
  Return s*, sf(s*)
End
    
```

(a) open (b) close (c) open-close

Destruction-Reconstruction process

ILS 54

Supply Chain Design for ecommerce

- Some results...
- cap11#,
 - 50 facilities
 - 50 customers
- Capa/b/c#,
 - 100 facilities
 - 1000 customers

Instance	Stochastic Programming			SimILS		
	Z _{stoch}	t (sec)	gap Cplex (%)	Z _{sim}	t (sec)	gap (%)
cap11	879371.7	1103	0	883339.4	170	0.47
cap12	965382.2	3604	0.01	958047.4	303	-0.76
cap13	1047322.2	3603	0.04	1030653.5	294	-1.59
cap14	1160395.4	3604	0.06	1139246.8	50	-1.82
cap124	975397.6	3603	0.09	949375.9	65	-2.67
capa1	-	-	-	19244150.7	2263	-
capa2	-	-	-	18458612.4	2370	-
capa3	-	-	-	1782599.1	2934	-
capa4	-	-	-	1716232.3	983	-
capb1	-	-	-	13773560.0	809	-
capb2	-	-	-	13378972.7	2645	-
capb3	-	-	-	13238010.0	2086	-
capb4	-	-	-	13084859.8	2380	-
capc1	-	-	-	11704267.9	1363	-
capc2	-	-	-	11571933.3	1812	-
capc3	-	-	-	11340950.9	2850	-
capc4	-	-	-	11335842.8	1928	-

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Solution Methods

**Local Search Methods
Metaheuristics**

- ▶ Good solutions for complex and large-scale problems
- ▶ Short running times
- ▶ Easily adapted

- Mathematical proved optimal solutions
- Important information on the characteristics and properties of the problem.

**Integer Programming
Exact Methods**

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Math - Iterated Local Search

- ▶ Get an initial solution x ;
 - * Heuristic method or a random solution.
 - * local optimization method
- ▶ For a certain number of iterations:
 - **Perturbation Step**
 - * **Uses a exact method to solve a subproblem or a relaxation of the problem.**
 - Small-steps
 - * local optimization method, initial solution x' ; final solution x'' .
 - Perform an accept/reject test
 - * accept all solutions, accept with a certain probability or accept only if it is a better solution.
 - * If x'' is accepted, then $x = x''$.
- ▶ Return the best solution found.

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MathILS

- ▶ Maybe the first application...
 - Use an exact algorithm to solve a sub problem within a **Iterated Local Search** heuristic for the Job-Shop Scheduling Problem
 - **Solving to optimality the one-machine scheduling problem with due dates and delivery times using the Carlier Algorithm.**
 - * Lourenço H.R. (1995), Job-Shop Scheduling: computational study of local search and large-step optimization methods. European Journal of Operational Research **83**(2): 347-364. ISSN 0377-2217.
 - * PhD Thesis, Lourenço H.R. (1993)

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Example of Applications

- ▶ Real Applications
 - Maybe the best set of problems to apply Metaheuristics methods...
 - Why?
 - * Complex problems with a large number of constraints.
 - * Sometimes difficult to model...
 - * But, a simplification of the problem is frequently a well-studied optimization problem.
 - Apply **metaheuristics for the real general problem...**
 - And **exact methods for the well-known relaxation problem.**

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Metaheuristics

- ▶ Which is the best metaheuristic?
 - Begin with a simple method and then turn, if necessary, to a more complicated one or refine the first implementation
 - Small number of parameters
 - Evaluate its performance by:
 - * Accuracy
 - * Speed
 - * Simplicity
 - * Flexibility

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Conclusions

- ▶ Iterated Local Search
 - Simple
 - Easy to implement
 - Robust
 - Highly effective
 - Modularity
- ▶ Start simple and add complexity if needed!
- ▶ The success of ILS lies in the biased sampling of the set of local optimal.
- ▶ More than 6000 publications in google scholar.

Do you want to try to implement an ILS?

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Iterated Local Search

► Main References

- **Lourenço H.R.**, Martin O. and Stützle T. (2010), Iterated Local Search: Framework and Applications. In Handbook of Metaheuristics, 2nd. Edition. Vol.146. M. Gendreau and J.Y. Potvin (eds.), Kluwer Academic Publishers, International Series in Operations Research & Management Science, pp. 363-397.
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