Limited budgets and equality of opportunity in health

Guillem Lopez-Casasnovas
guillem.lopez@upf.edu
Center for Research in Health and Economics (CRES)
Department of Economics - Universitat Pompeu Fabra
Ramon Trias Fargas, 25-27 - 08005 Barcelona, Spain

Héctor Pifarré i Arolas[1]
pifarreiarolas@demogr.mpg.de
Center for Research in Health and Economics (CRES)
Max Planck Institue for Demographic Research
Konrad-Zuse-Strasse 1 - 18057 Rostock, Germany

Frederic Udina
frederic.udina@upf.edu
Department of Economics - Universitat Pompeu Fabra
Ramon Trias Fargas, 25-27 - 08005 Barcelona, Spain

Keywords: equality of opportunity; Health; limited budget; capacity to benefit

Classification codes:
I140 Health and Inequality
I180 Health: Government Policy; Regulation; Public Health
D630 Equity, Justice, Inequality, and Other Normative Criteria and Measurement

Acknowledgements: The authors thank Carmen Herrero, Humberto Llavador, Adeline Lo and Antonio Villar Notario for for advice and support. Manja Janoswki provided excellent research assistance. All errors are our own.

Highlights:

- In applications such as health care, limited resources often prevent the achievement of full equality of opportunity.
- We argue that given a limited budget, society should judge not only between fair and unfair inequalities, but also which disadvantaged groups to target with public compensation.
- Our proposal frames this decision as a trade-off between compensating the worst-off groups and groups with a higher capacity to benefit.

[1] Corresponding author: Email: pifarreiarolas@demogr.mpg.de; Phone: +49 (0)381 2081-171.

Limited budgets and equality of opportunity in health

October 22, 2015

Abstract

Roemer's theory of equality of opportunity (EOp) makes a crucial distinction between fair and unfair inequalities, based on the assumption that groups experiencing unfair forms of inequality should be compensated. We argue that when there are insufficient resources to fully compensate groups or individuals for existing unfair inequalities — a very real concern when applying EOp to actual policies — it is equally important to decide which of the groups facing illegitimate inequalities should receive public compensation. In our work we focus on inequalities arising from uneven access to resources, both public and private. In the case of healthcare, these kinds of inequalities can be driven by both untargeted public provisions, such as universal healthcare systems, and by private spending on health. To guide public choice in these important situations, we propose a novel framework that extends social deliberation to cover the application of equality of opportunity.

1 Introduction

Since its publication in 1998, Roemer's theory of equality of opportunity (EOp) has gained ground in numerous applications. A research area that is thus far underexplored is the theory's application to too-real contexts in which social justice is unattainable due to resource constraints. That is, in many scenarios (and perhaps in most scenarios) inequalities that are already present in the population cannot be overcome to achieve EOp with the resources currently at hand. When it is not possible to achieve EOp, we argue that, in addition to identifying unfair sources of inequality, it is also crucial to decide which groups

to target with public compensation policies. We offer a framework to guide public choice in these situations.

In our scenario an EOp planner must make the following decision: given a number of groups of individuals facing unfair inequalities and a dearth of resources to compensate all individuals, the planner must decide which groups should be targeted first. Roemer suggests a Rawlsian approach: i.e., compensating the groups of individuals who are worst off. We argue that this choice is not inherent to the ethical criterion of EOp. Crucially, a critique of Roemer's stance is that the resulting allocation does not account for the effectiveness of the policy in the reduction of *unfair inequalities*. Moreover, our work shows that accepting Roemer's formulation of the principle of EOp does not require that the principle is implemented according to Rawlsian principles. We contribute to the theory of EOp by first showing that a separation of the ethical criterion from the method chosen to allocate the resources is possible while maintaining the core of the EOp criterion. We propose an allocation method whereby the social decision is broadened vis-à-vis the selection of targeted groups under an explicit trade-off between efficiency and equity[1].

To contextualize our argument we briefly review Roemer's proposal. In his book (Roemer, 1998), Roemer formalizes external circumstances and effort, individual responsibility, and public compensation, in a way that recovers the concept of individual responsibility. According to his formalization, individuals' outcomes can be attributed to both effort and circumstances. Roemer posited that effort is the only legitimate source of differences, thereby implying that compensation should be provided for inequalities rooted in circumstances[2]. The major strength of Roemer's work is that it provides a framework a society can use to address inequality, while retaining the responsibility for supplying most of the normative content. Many authors have taken advantage of the flexibility of the framework and have applied Roemer's strategies for EOp in different fields. The framework has, for example, been applied to education (Peragine and Serlenga, 2008 and 2009; Bratti, 2008; Calo-Blanco and Villar Notario, 2009) and development aid (Llavador and Roemer, 2001). More recently, the evaluation of health inequalities has been a prominent and very promising application of

[1] Equality of opportunities has been studied using to different definitions in a number of works. For a comparison between Roemer's and Van der gaer's approach see Ooghe et al.(2007).

[2] Lefranc et al. (2009) argue that luck should be considered a third fundamental component in the decomposition. In this work, however, we restrict ourselves to the original framework by Roemer.

the framework (Fleuerbaey and Shockhaert, 2009; see Schokkaert, 2015; and Wagstaff and Kanbur, 2015 for a recent debate on the topic) [3].

Returning to Roemer's original contribution, the normative choice open to society is the identification of tolerable sources of inequality; in Roemer's terminology, this is the distinction between circumstances and effort. The framework offers a very large degree of flexibility, since it is possible to include, within Roemer's theoretical construction, ethical criteria that may be seen as being at opposite extremes of conceptions of justice. We revert to utilitarianism when we judge individual outcomes as being determined solely by effort, and embrace Rawlsian ideals when we judge outcomes as being dependent on circumstances only.

In this work, we regard Roemer's formalization of EOp as two separate contributions. First, Roemer defines an ethical criterion based on an interpretation of equality of opportunity: any two individuals exerting the same effort should attain the same level of advantage. Second, Roemer offers an allocation method such that a given budget meets EOp. In a context of scarce resources in which social justice as defined by EOp might not be achievable, the allocation method allows for a stage at which the groups who are to be compensated are identified. Thus, an important dimension of social deliberation is opened. We propose a different allocation that accounts for the capacity of recipients of public compensation to benefit. The rationale behind our proposal is that, in the application of EOp to health inequalities, society might also be concerned about the degree to which the health of certain groups may be expected to improve when given health care resources. We favor an allocation method based on a tradeoff between compensating the individuals who are worst off, and those who would benefit the most. Hence, our proposal effectively extends society's choice to cover all aspects of the application of the theory of EOp.

Our work is related to the literature on the so-called bankruptcy problems, which proposes solutions to the allocation of a divisible good[4] among agents when the total amount of the good is insufficient to cover all their demands. The problem addressed in the bankruptcy solution literature (for an extensive survey of the literature, see Thompson, 2003) closely resembles our own. Our particular proposal for implementing EOp is to the proposal discussed by Herrero and

[3] See Williams and Cookson (2000) for a survey of other normative theories of health inequality

[4] The literature tends to focus on divisible goods; nevertheless, for an exploration of the problem under indivisible goods see Herrero and Martinez (2008)

Villar Notario (1994). The authors presented a method for the allocation of a public budget to different objectives when the available funds are insufficient to completely satisfy all the objectives. In their work, the authors studied the properties of a number of sharing rules inspired by both the bankruptcy and axiomatic negotiations literatures. An important distinction between our work and the rules suggested in the bankruptcy literature (and in Herrero and Villar Notario, 1994) is that we introduce concerns about the recipients' capacities to benefit from transfers.

The work is organized as follows. In the first section we formally review the framework for EOp presented in Roemer (1998). In the second section we justify our allocation strategy and our priority setting method within the health care framework. Preliminary, we impose further assumptions on the health state functions and the effort decisions. We continue by distinguishing between impediments to achieving a fair health status distribution that stem from budgetary restrictions and those that stem from the complexities of the compensation policy. For the sake of clarity, we tackle these two types of impediments separately. To convey the basic intuition, we use a simplified case that shows how budgetary limitations might preclude the social planner from achieving EOp in health regardless of the allocation method. We then show that simple policies can achieve EOp in very specific settings only. Since these situations are not the focus of our work, we offer a modified social objective which allows us to set aside the concerns regarding the complexity of the policies. We then present the derivation of our allocation method and discuss how the choice of different metrics allows us to cover the full spectrum of social decisions regarding which groups should be targeted for intervention. A final section concludes.

2 The formalization of EOp

In this section we briefly introduce the basic elements of the theory of EOp. The problem, in short, is the following. Consider a set of individuals who can achieve a certain health status (or any other relevant dimension of welfare). Suppose the health status is a function of the amount of health care consumed by individuals, the effort they exert (individually), and their circumstances. Our goal is to decide how given public budget should be distributed across individuals to achieve a health status distribution which meets the ethical criteria of EOp.

The basis of EOp's normative structure lies precisely in the distinction between circumstances and effort, which we elaborate in this section. Define *circumstances* to be that which society judges to be beyond the responsibility of the individual. *Effort* is constituted by all of the actions that society judges as being within the responsibility of the individual. It follows from the characterization of circumstances, that we may create a classification of individuals by types. A *type* is a subset of individuals who share the same circumstances that are relevant to the attainment of health. Examples of these circumstances include individuals' genetic predisposition to illnesses, their education, and their capacity to benefit from treatment.

We denote $\top = \{1, \ldots, T\}$ as the set of T types into which we divide the population. The relationship between resources, effort and health is given by the health status function. The *health status* of an individual is a function of the effort and resources allocated, indexed by the type.

We denote $u^t(x, e)$ as the health status function for type t, where x are the resources and e is the effort[5]. As we stated above, society must choose an allocation of health care spending that may be dependent on effort and on type. While we know that the aim of the policy is to attain EOp, let it be for now any rule that satisfies the following definition:

Definition 1. (Policy) A policy is a T-tuple of functions that specify, for each type, the resources devoted as a function of effort. We denote it $\phi = \left(\phi^1, \ldots, \phi^T\right)$ and call each function ϕ^t an allocation rule. Then, $\phi^t(e)$ is the amount of resources a type t individual receives if she exerts effort e.

In this work, however, we exclude policies that are not constant on the type; that is, policies that are not a function of effort. It is then reasonable to believe, in turn, that the effort exerted by individuals is dependent on the policy. Given a policy, the individuals of a given type t generate a distribution of effort, given by a cumulative probability function $F^t_{\phi^t}$; $F^t_{\phi^t}(e)$ is the cumulated probability up to and including e. In some cases we may assume that effort has a discrete probability distribution, while in others we may assume a continuous distribution with a convex support, say an interval (infinite or not) with a density function $f^t_{\phi^t}$.

We now specify the notation for the budget constraint. Let ω be the per

[5] Indexing by type is equivalent to including another variable or vector of variables that includes type characteristics, i.e., $u(C, x, e)$, where C is the vector of individual characteristics that constitute a type (see Roemer, 2002 for a complete exposition of this notation).

capita disposable resources. The amount of (per capita) resources assigned to type t is

$$\omega^t = \sum_e \phi^t(e) P(e) \text{ or } \omega^t = \int_R \phi^t(e) \, dF^t_{\phi^t}(e)$$

depending on whether effort is a discrete or a continuous variable. Denote α^t the proportion of type t in the population. Then the global constraint is

$$\omega = \sum_t \alpha^t \omega^t. \tag{1}$$

Let $\rho^t = \omega^t/\omega$ be the per capita share of the resource for type t. Then $\sum_t \rho^t = 1$.

A key contribution of Roemer is his formalization of a method for making a just comparison of the effort exerted by different types. Given that the effort distribution is influenced by the circumstances, how can we *fairly* compare the amount of effort exerted by the different types? A possible solution is to draw a distinction between the *level* and the *degree* of effort as formalized in this definition.

Definition 2. (Effort level, effort degree and indirect health status) For $\pi \in (0,1)$, let $e^t(\pi, \phi^t)$ be the *level of effort* exerted by an individual of the type t in the π^{th} quantile of effort of the type . We call π a *degree of effort*. These levels and degrees are characterized by the equations

$$\pi = \int_0^{e^t(\pi,\phi^t)} dF^t_{\phi^t}, \qquad t \in \top$$

for the case of continuous effort distribution, and similarly for the discrete case.

The *indirect health status function* gives the health status of an individual of type t who receives the resources determined by the policy ϕ and exerts the π degree of effort of the type distribution of effort, and is defined by

$$v^t(\pi, \phi^t) = u^t\left(\phi^t\left(e^t(\pi, \phi^t)\right), e^t(\pi, \phi^t)\right)$$

Central to the concept of equality of opportunity is the assumption that

we should be comparing individuals according to their degree of effort and not according to their level, which is influenced by their circumstances. We formalize this ethical criterion in the following definition.

Definition 3. (Social criteria and strong social justice) The ethical criterion of *equality of opportunity* (EOp) states that when any two individuals, independent of type, exert the same *degree* of effort (i.e., individuals who are in the same position in their respective type distributions), they must achieve the same health status. That is,

$$\forall i, j \in \top,\ \forall \pi \in (0,1)\,, \quad v^i\left(\pi, \phi^i\right) = v^j\left(\pi, \phi^j\right) \tag{2}$$

When we refer to *strong social justice* according to a given criterion, we mean the state in which the advantages of all of the individuals satisfy the requirements of the chosen ethical criterion; in this case, EOp.

Having defined the fair distribution of health status, the remaining question is how we can design policies that achieve that social objective. Roemer proceeded in the following manner. In his presentation, he referred to EOp as described in definition 3 *(2)*. However, when implementing EOP — i.e., the choice of policy — Roemer defined the policy that resulted from his proposed methodology as an EOp policy regardless of whether it actually achieved EOp as described in definition 3. It could be argued that Roemer made EOp a criterion that is a subsidiary of the allocation method, as what he actually treated as EOp was the outcome of his rule for choosing the policy.

Even if we accept Roemer's proposed method, which is presented in the remainder of the section, the method does not necessarily follow directly from the theory of EOp. In other words, a society that accepts the normative criterion of definition 3 does not need to support the ethical choices inherent to his proposed method. This is where our contribution lies. While we accept EOp as a criterion, we propose a different methodology to guide the choice of policies. When we refer to EOp, we are considering the criterion in definition 3. This allows for the possibility that EOp is not actually achieved, which is otherwise ruled out by the nature of Roemer's procedure. If we proceed while maintaining this separation between *ethical criterion* and *method,* we can still discuss Roemer's rule for a choice of policy as a particular method. Recall that we are seeking a distribution of resources that leads to the achievement of EOp. Ideally, we would meet this criterion for every percentile of effort, whereby all

types accomplish the same level of health. The method proposed by Roemer is the maximization of the minimum health statuses among types for every π of effort.

$$\max_{\phi} \min_{t \in \mathrm{T}} v^t \left(\pi, \phi^t\right)$$

Note that this allocation method is Rawlsian in its conception. It centers the attention on the type who is worst off, for every π of effort. This is the essence of Roemer's implementation and what follows are technical considerations. Given the limitations on the sophistication of the implementation policy, the solution to the program for a given quantile might not correspond with the solution that equalizes health among other quantiles. Therefore, we encounter the problem that we might obtain as many policies as quantiles. The proposed solution consists of assigning a weight equivalent to the population weight for every quantile and solving for this modified problem. Effectively, we assign the same importance to every quantile. In this sense, the solution becomes utilitarianist across quantiles.

$$\max_{\phi} \int_0^1 \min_{t \in \mathrm{T}} v^t \left(\pi, \phi^t\right) \, d\pi \tag{3}$$

The remaining part of the work is devoted to constructing and justifying our proposed allocation method.

3 Preliminaries

In this section we explore why EOp might not be achieved and why this is particularly relevant in applications to health. We begin with the justification of the relevance of our proposal. Suppose first that EOp cannot be achieved, while disregarding the reason. The main concern that motivates our contribution is that the Rawlsian approach to the choice of policy proposed in Roemer's original contribution is particularly ill-suited for health applications. This is because it excludes any consideration of the capacity to benefit from health care of the individuals to be treated who face unfair inequalities. In the extreme case in which the type of individuals who are the worst off cannot improve with medical treatment, using *(3)* would still lead to allocating the entirety of the budget to individuals of this type. This criticism has been applied to all of

the methods based on Rawlsian inspiration; interestingly, this precise point was raised by Harsanyi (1975) in his critique of Rawls' maxmin principle. The following example, drawn from Harsanyi's text, illustrates our critique.

As a first example, consider a society consisting of one doctor and two patients, where both patients are critically ill with pneumonia. Their only chance for recovery is via antibiotic treatment, but the amount of treatment available is sufficient for treatment of only one patient. Of these two patients, individual A is an otherwise healthy person (apart from his present attack of pneumonia). On the other hand, individual B is a terminal cancer victim whose life would be prolonged by merely several months, given treatment of the antibiotic. Which patient should be given the antibiotic? According to the difference principle, it should be given to the cancer victim, who is obviously the less fortunate of the two patients.

It is important to note that while one might reject *(3)*, this does not invalidate EOp as an ethical principle since *(3)* is not the unique program that implements EOp. Recognizing this fact, we propose abandoning the pure Rawlsian program and instead designing policies based on a more flexible framework. While society might be concerned with the well-being of the types of individuals who are the worst off, the capacity to benefit should be a key component of any policy that is adopted. Our contribution is to allow society to choose the particular weight given to the two potentially conflicting objectives in a program that seeks to achieve EOp when implementing policy. However, before we describe our proposal in detail, we will clarify why we think it is reasonable to assume that EOp cannot be achieved in the majority of applications.

In several contexts and most definitely in the case of health care policy, individuals start with certain levels of advantage (health endowments or health gained by income, for example) that are not distributed based on ethical considerations. The origin of these initial differences can be traced to a variety of factors. In the case of health individuals might have different behavioral or biological characteristics, such as different health-related habits, lelvels of access to private health insurance, or even genetics. Without disregarding the importance of such factors, in this work, we emphasize the role of health care spending in creating initial differences in health status. Regardless of the share of the health budget to the pursuit of EOp, there is bound to be a large fraction of total spending that is allocated without regard to the ethical principle of EOp. There might, for example, be a high degree of elasticity of private spending on health, as the opportunity costs for prevention may be lower for

certain higher income groups, and these groups may choose to spend money on expensive healthy food, free time for exercise, and health education. Public spending, perhaps with a universalist flavor, might also contribute to these initial differences. Regardless of the source of these initial differences in health status accross types, it is entirely possible and even reasonable to think that it might not be feasible to fully compensate individuals for those unfair inequalities with the budget allocated to EOp policies. It is in such context that the choice of the program implemented to achieve EOp becomes relevant. As an aside, we should not that another reason why a strong social justice (according to EOp) might not be feasible is technical in nature. As we explained in the previous section, there could be a conflict between policies designed to achieve EOp. Conflicts are more likely to arise if policies are limited to be constant in effort. We illustrate this point later on for policies that only depend on the type. In this section, we explore separately both of the reasons why a EOp distribution might not be feasible. We then show under what circumstances we can achieve a weakened version of EOp before turning to our proposal.

3.1 Assumptions

Throughout the remainder of the work, we make the following assumptions regarding the effort decisions, the advantage functions, and the behavior of individuals.

1. Achievement functions $u^t(x, e)$ are defined for all nonnegative values of their arguments, unbounded for any fixed positive value of any of its arguments, and twice differentiable with continuity, i.e. C^2 functions. The first partial derivatives are strictly positive and the second derivatives with respect to the same argument twice are strictly negative.

2. Both resources and effort are necessary and sufficient for obtaining a positive achievement: $u^t(x, e) = 0$ if and only if $x = 0$ or $e = 0$.

3. The assignment policy is determined by Roemer's method, so it is the optimal solution of *(3)*when all of the resources are allocated. Furthermore, we restrict ourselves to constant policies.

4. Public compensation does not offset private effort. Formally, if $\phi_a^t > \phi_b^t$, then $F^t_{\phi_a^t} \geq F^t_{\phi_b^t}$ (first order stochastic dominance).

We clarify here the extent to which our assumptions are restrictive. Our first assumption is mainly technical, as it states that the health status function is well-behaved. However, we also assume that there is no upper bound to how much health can be ameliorated with health care. That is, we allow for different capacities to benefit, but we assume that even though some individuals may have an arbitrarily small capacity to translate health care spending into a better health status, it is always possible for them to improve their health. The second assumption states that health status requires a positive amount of resources, but we do not require that the resources are fully determined by the social planner. Resources may originate from other public interventions (not aimed at achieving EOp), privately by individuals ("other resources"), or as allocated by the social planner. Further along, we do impose a restrictive assumption whereby we require that "other resources" are unaffected by the social planner to obtain EOp. This simplifying assumption precludes important considerations such as the crowding-out of private resources.

We assume constant policies in our exercise. While our message would remain mostly unchanged if we allowed for more complicated policies, we choose to restrict our analysis to compensation policies that depend on the type only. We have chosen this approach because it simplifies the presentation of our proposal, and it allows us to present our results under the realistic assumption that the policy space is highly constrained. This could be because of the feasibility constraints related to polic implementation or because of some political economy reasons. Finally, our most restrictive assumption is intended to prevent a situation in which agents reduce their efforts to the extent that the gains garnered from the allocation of further resources are fully offset[6]. Some of our results rely on this asumption to ensure that the optimization problem is well defined. Our claim is, however, that our proposal could be applied, even if there is some "crowding out", as long as there is an arbitrarily large public budget such that social justice can be attained. That is, achieving some form of social justice (we elaborate on this later on) is feasible. We think it would be far-fetched to assume otherwise, since such assumption would lend support to the rather extreme expectation that individuals would simply undo any public compensation effort, regardless of its magnitude, by reducing their effort.

[6]See Calsamiglia (2009) for an in-depth treatment of the issue.

3.2 Insufficient budget

We now look at a context in which the budget is insufficient. In order to focus solely on this issue, we present a limited case in which we rule out the possibility of conflicts in the policies required by each quantile by assuming a single level of effort for each type (although not necessarily the same level across types). That is, the distribution of effort is characterized by a single effort for each type, with the clear implication being that EOp is achieved by completely equalizing advantages across types. This is the simplest scenario, as the achievement of EOp is limited by the availability of public funds. We want to emphasize that we use this simplistic scenario strictly for presentation purposes, as our proposal does not rely on single efforts by types of distributions. In fact, this is an assump-tion we abandon later on. Having established the aim of our exercise, suppose now that all the resources available for individuals are publicly provided and distributed according to the ethical objective of EOp. Then, as we state formally in proposition 1, strong social justice is always achieved in our context.

Proposition 1. *Let each type have a unique nonzero effort level. Then, under assumptions 1, 2, 3 and 4, strong social justice is achieved.*

Proof. Under a single effort level for a given type, functions v^t are now simply

$$v^t\left(\phi^t\right) = u^t\left(\phi^t, e^t\left(\phi^t\right)\right)$$

where $e^t(\phi^t)$ denotes the effort level applied by all individuals of type t as a response to receiving ϕ^t resources. By assumption 4, v^t as a function of its single argument is monotonic and strictly increasing with continuity. It can be shown that, if two types have different advantages, it is possible to reassign resources to reduce the difference. To reassign: choose i, j such that $v^i(\phi^i)$ is the minimum among all types and $v^j(\phi^j)$ is the maximum among types. Since functions v^t are continuous and increasing, there exists $\delta > 0$ such that

$$v^i\left(\phi^i\right) < v^i\left(\phi^i + \delta\alpha^i\right) < v^j\left(\phi^j - \delta\alpha^j\right) < v^j\left(\phi^j\right).$$

Then we can adjust ϕ^i by $\phi^i + \delta\alpha^i$ and ϕ^j by $\phi^j - \delta\alpha^j$ to obtain a new policy that still satisfies the global constraint (1). It may be that several types share the same minimum given by $v^j\left(\phi^j\right)$. To achieve a policy that improves Roemer's criterion, reassignment may require repeating the application. □

The achievement of EOp is threatened by the existence of initial differences, not by the availability of budgetary resources per se. After all, if the types did not have initial levels of health, it would always be possible to equalize them at the bottom by assigning zero health for all of the types. Proposition 1 formalizes the idea that in the absence of initial levels of health, EOp is achieved regardless of the budget. Assume now that the types have some initial health status independent of the policy selected. We consider a case in which this is due to some arbitrary initial assignment of resources that was made without necessarily respecting any justice requirements. In this case, public funds might have been distributed in a previous stage according to some other criterion (for instance, utilitarianism), as privately provided resources (for instance, provisions by the family), or a combination of the two. Regardless of the reason why, we assume that all of the types hold some initial amount of resources, denoted by x_0^t, and that this does not depend on the posterior public resources allocated. Then, in our context, whether EOp can be achieved depends strictly on the size of the public budget.

Proposition 2. *Let each type have a unique nonzero effort level. Then, under assumptions 1 and 2 and a given* $x_0^t,\ \forall t \in \mathrm{T}$ *(where* $\exists\, i,\, j:\ x_0^i > 0,\ x_0^j = 0$ *), there exist budgets* ω_S *and* ω_L *such that:*

(i) For $\omega < \omega_S$*, equality of health statuses among types is unattainable.*

(ii) For $\omega \geq \omega_L$*, strong social justice is achieved.*

(iii) If strong social justice has been achieved, any $\triangle\omega > 0$ *is distributed such that:*

$$\forall i, j \in \mathrm{T},\ \frac{\partial u^i}{\partial x}\frac{d\phi^i}{d\omega} + \frac{\partial u^i}{\partial e}\frac{de^i}{d\phi^i}\frac{d\phi^i}{d\omega} = \frac{\partial u^j}{\partial x}\frac{d\phi^j}{d\omega} + \frac{\partial u^j}{\partial e}\frac{de^j}{d\phi^j}\frac{d\phi^j}{d\omega}$$

.

Proof. If (i) were false, letting ω go to zero will give us equality of health statuses among $u^t\left(x_0^t + 0, e^t\right)$ with some x_0^t positive and some zero; this is impossible given assumption 2. To prove (ii), apply proposition 1 with health status functions $u_*^t\left(x, e\right) = u\left(x_0^t + x, e\right) - u\left(x_0^t, e\right)$. We have a budget ω_* with equality of health statuses that we may assume to be the minimal one. Then, take $\omega_L = \omega_* + \sum_t x_0^t$. (iii) follows by differentiating with respect to ω (using the chain rule) the equality $u^i\left(\phi^i, e^i\left(\phi^i\right)\right) = u^j\left(\phi^j, e^j\left(\phi^j\right)\right)$ (see proof of prop. 1) where now $\phi^t = \phi^t\left(\omega\right)$. □

We can extract a number of valuable insights from proposition 2 that hold

in more complex settings. First, the amount of public funds available matters for the equality of health statuses. Second, once a state of strong social justice is reached, further resources are allocated so that every type obtains a share. The size of the portion depends on the ability of the types to transform resources into health. This last point is important, and we build our contribution around it. Intuitively, if all of the types receive some share of a hypothetical marginal increment of resources, we should be *as close as possible* to social justice. Otherwise, the additional funds would be channeled to those types who are experiencing unfair inequalities. Having characterized the issue of the scarcity of resources to the extent that is needed for our exercise, we now turn to the problem posed by the limitations on the complexity of the policy.

3.3 Simplicity of the policy

In the presentation of his method, Roemer addresses the possibility of conflicts in the policies necessary to obtain strong social justice for every given quantile. In our context, this is the second main reason why strong social justice might not be achieved. In this part we exclude the initial differences in resources, and focus instead on illustrating the problems that arise when we are restricted to the particular case of constant policies. We have chosen to take this approach because given the limited availability of information for designing compensation schemes, we believe it is useful to design a method that works under the simplest possible policies. We find that very stringent requirements are needed to attain the ethical objective if we restrict ourselves to constant policies when dealing with multiple efforts. Through Example 1, we show that strong social justice is attained only when both types and quantiles of effort have very particular homotheticity properties. We provide the example as an illustration of the degree to which special circumstances are necessary to completely fulfill the desired ethical criterion.

Example 1. Assume:

(i) There are just two types $t = A, B$ each with a continuum of effort levels, and health status functions of the form $u^t(\varphi, e) = \lambda^t \varphi^{\alpha^t} e^{1-\alpha^t}$.

(ii) There is a policy φ that assigns a constant amount of resources $\varphi^A + \varphi^B = 1$ (total amount of resources per capita is normalized to 1) and achieves strong social justice, that is $\forall \pi \in (0,1)\,,\ v^A\left(\pi, \varphi^A\right) = v^B\left(\pi, \varphi^B\right)$.

Then, the frontier of health statuses is homothetic on the quantiles. If e^t_π

denotes the π quantile in the effort distribution of type t,

$$\forall \phi \in (0,1), \quad \forall \pi, \pi' \in (0,1), \quad \frac{u^A\left(\phi, e_\pi^A\right)}{u^B\left(1-\phi, e_\pi^B\right)} = \frac{u^A\left(\phi, e_{\pi'}^A\right)}{u^B\left(1-\phi, e_{\pi'}^B\right)} \tag{4}$$

Furthermore, this assumption forces the effort quantiles by $e_\pi^A = c_1\left(e_\pi^B\right)^{c_2}$ for some constants $c_1,\ c_2$.

Proof. Simple substitution of u^t into the social justice condition *(4)* gives

$$\left(e_\pi^B\right)^{1-\alpha^B} = \frac{\lambda^A\left(\varphi^A\right)^{\alpha^A}}{\lambda^B\left(\varphi^B\right)^{\alpha^B}}\left(e_\pi^A\right)^{1-\alpha^A}$$

and this gives both results. □

Since we do not wish to restrict our contribution to such special cases, or to relax our restrictions on policies, our strategy is to develop a weaker definition of social justice. In particular, we relax the requirements on the definition of social justice by only accepting differences that arise from the simplicity of the policy.

3.4 Weak social justice

In applications, the feasible level of complexity of policies may be limited. Thus, it might be impossible to achieve a state of strong social justice regardless of the budget. In this section we illustrate how it is still possible to reach a laxer state of social justice even with very simple policies. Recall that proposition 2 establishes that all of the types receive a positive amount of any marginal increase in available public funds once social justice is met. We build our new definition of social justice around this notion by defining a state of *weak social justice* in which all of the types receive a share of any marginal increase in the public budget. This is a state of social justice in which only inequalities derived from the simplicity of the policy are tolerated. The particular situation in which proposition 2 is established coincides with the achievement of strong social justice.When we are dealing with multiple levels of effort, the situation no longer coincides with the achievement of strong social justice. However, it is still possible to find a budget such that by using Roemer's allocation method, all of the types receive a portion of a marginal increase of the public funds. By choosing Roemer's implementation, we relax our definition of social justice in the same way that Roemer did in his contribution. However, as we emphasized

earlier, we depart from Roemer in that we allow only for differences in health that arise from the simplicity of our policy. We maintain that given the exogenous initial differences across types, our relaxed version of social justice might not be achieved. The formalization of this explanation is given in the following definition.

Definition 4. (Sufficient budget and weak social justice): Assume the policy is decided by solving Roemer's program. A sufficient budget is such that, for any budget in excess of the sufficient budget, all of the types would receive a strictly larger assignment under the chosen policy associated with the larger budget. Formally, a total budget ω is *sufficient,* if, for all types t and for all $\bar{\omega} > \omega$, $\phi^t_{\bar{\omega}} > \phi^t_{\omega}$, where ϕ_ω denotes the assignment under the optimal policy in assumption 3. Denote as *weak social justice* the state in which the total budget is at least *sufficient*.

The definition captures the intuition that if every type receives a share of the additional resource pie, it is not possible to move closer to the social justice state. Therefore, a total budget is sufficient if it compensates for initial inequalities among types. We now prove that such a budget exists under fairly general conditions.

Proposition 3. *Under assumptions 1 , 2 , 3 and 4 and a compact support effort distribution for each type (or finite number of values if effort levels are discrete), there exists a (finite) sufficient budget.*

Proof. With these assumptions, functions $v^t(\pi, \phi^t)$ are continuous and non decreasing in both arguments, not bounded on the second argument but bounded for $\pi \in [0,1]$. Function $v(\pi, \Phi) = \min_t v^t(\pi, \phi^t)$ also has these properties. Function $R(\omega) = \max_\Phi \int_0^1 v(\pi, \Phi)\, d\pi$ is increasing and not bounded as ω grows.

If $\Phi = \Phi_\omega$ is the optimal policy for a budget ω, a type t has $\phi^t = 0$ if and only if $v^t(\pi, \phi^t_\omega) > v(\pi, \Phi_\omega)$ for all values of π except possible for some isolated ones. This may occur either because type t has some initial assignment x^t_0 or because the distribution of effort does not contain the zero effort. The only case in which ϕ^t_ω does not increase when ω grows is when $\phi^t_\omega = 0$.

Now we proceed by induction on the number of types T. For $T = 1$ the proposition is clearly true. For T, choose a budget ω_0 large enough to be a sufficient budget for types $t = 1, \ldots, T-1$ while also satisfying $v(\pi, \Phi_{\omega_0}) > v^T(\pi, 0)$ for all π in some interval of positive length. This is also a sufficient budget for all types. □

Having proved its existence, we select a particular *sufficient* budget. Recall that we seek to approach *weak social justice* using our selected policy. However, this new and more lax requirement of social justice could be achieved using a number of budgets. When selecting a budget, our natural choice is the budget that requires the least amount of resources, and that in turn characterizes the health states in a particular *weak social justice* state.

Definition 5. (Minimum sufficient budget): Given initially allocated resources $x^t \geq 0$, whereby some are positive, the infimum of all socially acceptable budgets is defined as the *minimum sufficient* budget and the corresponding optimal policy is defined as the *minimum sufficient policy* (note that this is also a sufficient budget).

How do we choose a policy when the budget is not suffcient? Roemer's method remains valid; however, it presumes that the priority is compensating the types who are the worst off; a judgment that is not implied by EOp. We propose a method that allows society to decide which types to compensate by taking a stand on the trade-off between the effectiveness of the public intervention and the desire to help the worst off. We denote the choice in allocation resulting from society's stand on the efficiency-equity trade-off involved in a socially selected policy. The next section formalizes this intuition.

4 Our proposal

In the previous section we considered the impediments that prevent us fromachieving full EOp, defined a laxer social justice state and described under which conditions it can be attained. However, we have not considered the key question we wish to address in this work. If the budget is too small to achieve a weak social justice state, how should it be distributed? Or, in other words, what types ought to be prioritized? A natural first proposal for selecting policies within this context is, in the spirit of Herrero and Villar Notario (1994), to choose the policy that minimizes the distance to a weak social justice state. Formally, given an *insufficient budget*, we can obtain a policy from the minimization of the distance to the state of *weak social justice* generated by the *minimum sufficient policy*. In this context, the choice of distance measures implies an ethical stand regarding which groups to prioritize.

Let ω_0 be the minimum sufficient budget for a given set of initially allocated resources $\{x^t\}$, let ϕ_0 be the corresponding minimum sufficient policy , and

let $v^t(\pi, \phi_0)$ be the associated indirect health state functions. We denote by $\mathbf{v}(\pi, \phi_0)$ the vector of all the health statuses of all types for degree π. For a smaller budget ω we say that the *socially-selected* policy is the solution for the program

$$\min_{\phi} \int_0^1 \|\mathbf{v}(\pi, \phi_0) - \mathbf{v}(\pi, \phi)\|_p \, d\pi \tag{5}$$

where $\|.\|_p$ denotes a norm (see below) that measures the distance between two vectors of health statuses, the minimum sufficient policy, $\mathbf{v}(\pi, \phi_0)$, and $\mathbf{v}(\pi, \phi)$ is the vector of achievements under policy ϕ.

It is clear that the choice of distance has profound implications for the resulting policy. Consider a p-norm; then, *(5)* becomes

$$\min_{\phi} \int_0^1 \left(\sum_{t=1}^{t} \left| v^t(\pi, \phi_0) - v^t\left(\pi, \phi^t\right) \right|^p \right)^{\frac{1}{p}} d\pi \tag{6}$$

It is desirable to formulate the problem in this fashion as every metric comprised in the p-norm corresponds to a choice of weights in the trade off between effectiveness of public resources and compensating those who are worse off. For instance, for $p = 1$, *(6)* becomes the minimization of the sum of the differences.

$$\min_{\phi} \int_0^1 \left(\sum_{t=1}^{t} \left| v^t(\pi, \phi_0) - v^t\left(\pi, \phi^t\right) \right| \right) d\pi$$

The choice of this metric is utilitarian, as policies based on this metric would target *the type who could benefit the most*. At the margin, resources are allocated to the type with a larger partial derivative of the health status function with respect to resources. Since in general there is no guarantee that an interior solution is achieved, this could lead to the abandonment of types who are less able to transform resources into health.

On the other extreme, when $p \to \infty$, *(6)* is then:

$$\min_{\phi} \int_0^1 \left(\max_t \left| v^t(\pi, \phi_0) - v^t\left(\pi, \phi^t\right) \right| \right) d\pi$$

The solution allocates resources to the types who are farther away from the minimum sufficient policy advantages, disregarding any considerations related to the effectiveness of the public funds.

While appealing, the program we have sketched above has strong informa-

tion requirements. For the objective function to be well defined it is necessary to determine the minimum sufficient budget, which might not be possible[7]. Precisely one of the strengths of Roemer's initial proposal was the simplicity of its implemention program. To address these concerns, we suggest a modification of the concept of distance minimization in the spirit of Roemer's maxmin principle.

Let $v^{max}(\pi, \phi)$ denote the health state of healthiest type of a given π. Then, conditional on not being in a state of weak social justice, our socially-selected policy is obtained from the minimization of the distance between the healthiest and the least healthy types. That is:

$$\min_{\phi} \int_0^1 \left(\sum_{t=1}^{t} \left| v^{max}(\pi, \phi) - v^t\left(\pi, \phi^t\right) \right|^p \right)^{\frac{1}{p}} d\pi \tag{7}$$

In the simple cases presented in propositions 1 and 2, *(7)* achieves strong social justice assuming there is a sufficient budget. Of course, as we have shown in our exposition, there is no guarantee that this is the case with constant policies and complex effort distributions. The key difference between programs *(7)* and *(6)* is that while *(7)* preserves the ordering across types in the health states that existed before the intervention, *(6)* does not. Given that the differences between the individual types were unfair, the order-preserving property of *(7)* may not be seen as desirable. This is the unfortunate drawback of choosing *(7)* when we do not have the information necessary to carry out the first approach *(6)*.

In this augmented framework in which EOp may not be achievable given the constraints on resources, we show that there are two main choices to be made in pursuing EOp. Given that full equality is probably unachievable, society must decide not only how to distinguish between circumstances and effort but also how to prioritize the distribution of resources across groups who face unfair inequalities. The advantage of our proposal is that while it suggests a course of action from which we can pursue the objective of EOp, namely via the trade-off between effectiveness and compensating those who are most unhealthy, society also retains complete freedom to devide howmuch weight is placed on each objective.

[7]For instance, in the presence of non monotonicities in the health status functions

4.1 Discussion

We have made a number of technical assumptions in this work. One assumption is that of the standard case of a continuous achievement function. This might be considered a limitation of our proposal as many applications require the consideration of categorical data such as health states. This issue has been explored in a recent study that looked at the extensions of the EOp framework categorical data (Herrero and Villar Notario, 2012). Our methodology extends to these situations as well; when dealing with categorical data, our method requires for each category and type the minimization of the distance to the proportion that is in accordance with the weakened definition of EOp.

On a more general note, a critique that is often leveled against EOp is that it does not take into account the traditional tradeoff between efficiency and equity. Our work includes considerations regarding efficiency in the application of EOp, but only as they relate to the attainment of social justice. Hence, we acknowledge the possibility that pursuing the optimal policies using our method could be inefficient from a broader standpoint. Nevertheless, we believe the EOp framework can provide the guidelines for public authorities seeking to design policies focused exclusively on the attainment of equity. This does not preclude the possibility that a society might, on a broader level, still be trying to achieve the traditional balance between efficiency and equity in allocating its resources. In the case of health care, for instance, the relevant policy-makers might devote part of their budget to reducing avoidable mortality, and another part to equitable access. Our proposal addresses the latter objective only.

5 Conclusion

We have argued that the full capacity to decide over the normative content of the theory of equality of opportunity, as formulated by Roemer (1998), requires transferring the decision-maing power over its allocation method. In applying the theory, it is not only relevant for society to establish the legitimacy of inequalities; when the scarcity of resources means that attaining equality of opportunity is impossible, it is also crucial to decide which types should be compensated first. In this work, we outlined the conditions under which this concern matters: namely when public funds are insufficient to compensate for initial differences across types. In this context, Roemer's original proposal advocates for a Rawlsian approach that prioritizes redistribution towards the groups

who are the worst off. Our critique is that this approach fails to consider the capacity of the targeted individuals to benefit from transfers, which is crucial when theory is applied to health care.

We then present a new allocation method based on the intuitive idea of the minimization to an unattainable objective. Through the choice of different metrics, our methodology allows for the inclusion, with varying importance, of the weight attached to the potentially conflicting objectives of compensating the types who are worst of and types who can benefit the most. The main contribution of this work is, therefore, to extend the framework of EOp to allow societal control of the application of equality of opportunity.

References

[1] Calo-Blanco, A., Villar Notario, A., 2009. Education, Utilitarianism, and Equality of Opportunity. Fundación BBVA - Documentos de Trabajo, No. 13.

[2] Calsamiglia, C., 2009. Decentralizing Equality Of Opportunity. International Economic Review. 50 (1), 273-290.

[3] Fleuerbaey, M., Shockhaert, E., 2009. Unfair Inequalities in Health and Health Care. Journal of Health Economics. 28 (1), 73-90.

[4] Harsanyi, J.C., 1975. Can the Maximim Principle Serve as a Basis for Morality? A Critique of John Rawls' Theory. The American Political Science Review. 69 (02), 594-606.

[5] Herrero, C ., Martínez, R., 2008. Egalitarian Rules in Claims Problems with Indivisible Goods. Social Choice and Welfare. 30, 603-617.

[6] Herrero, C., Villar Notario, A., 1994. La Assistencia sanitaria Como un Problema de Asignacion en Presencia de Objectivos Inalcanzables. In: Lopez-Casasnovas, G. (ed.) Analisis economico de la sanidad. Servei catala de salut.

[7] Herrero, C., Villar Notario, A., 2012. An Equity-based Proposal for the Evaluation of Health States. Fundación BBVA - Documentos de Trabajo, No. 15.

[8] Lefranc, A., Pistolesi, N., Trannoy, A., 2009. Equality of opportunity and luck: Definitions and testable conditions, with an application to income in France. Journal of Public Economics. 93 (11-12), 1189-1207.

[9] Llavador, H.G., Roemer, J.E., 2001. An Equal-opportunity Approach to the Allocation of International Aid. Journal of Development Economics. 64 (1), 147-171.

[10] Ooghe, E., Schokkaert, E., Van de gaer, D., 2007. Equality of Opportunity versus Equality of Opportunity Sets. Social Choice and Welfare. 28 (2), 209-230.

[11] Peragine, V., Serlenga, L., 2008. Higher Education and Equality of Opportunity in Italy. In: Bishop, J., Zheng, D.(eds.) Research in Economic Inequality. 16, pp. 67-97.

[12] Roemer, J.E., 2002. Equality of Opportunity: A Progress Report. Social Choice and Welfare. 19 (2), 455-471.

[13] Roemer, J.E., 1998. Equality of Opportunity. Harvard University Press, Cambridge, MA.

[14] Schokkaert, E., 2015. Equality of Opportunity, A Flexible Framework. Health Economics. 24 (10), 1250–1252.

[15] Thompson, W., 2003. Axiomatic and Game-Theoretic Analysis of Bankruptcy and Taxation Problems: A Survey. Mathematical Social Sciences. 45 (3), 249-297.

[16] Wagstaff, A., Kanbur, R., 2015. Inequality of Opportunity: The New Motherhood and Apple Pie?. Health Economics. 24 (10), 1243–1247.

[17] Williams, A., Cookson, R., 2000. Equity in Health. In: Culyer, A.J., Newhouse, J.P. (eds.) Handbook of Health Economics. 1B: Elsevier Science B.V., pp. 1863-1910.