

From ILS to Hybrid ILS ... and other extensions

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Outline

- ▶ Introduction to ILS
- ▶ Applications of ILS
- ▶ Hybrid ILS and other Extensions
 - Hybrid with other metaheuristics
 - SimILS
 - Two-stage Optimization using ILS
 - MathILS



Outline

▶ Applications:

- Market Basket Analysis
- BonArea
- Supply Chain Design for ecommerce
- SEAT/Volkswagen
- Zara (INDITEX & OESIA)



Grup Alimentari Guissona

ZARA

SEAT

Local optimization algorithms

▶ Local search

1. Get a initial solution x (current solution). Use a constructive heuristic.
2. Search the neighborhood. While there is an untested neighbor of x :
 - * 2.1 Let x' be an untested neighbor of x ;
 - * 2.2 If $c(x') < c(x)$ set $x = x'$; (x' is the new current solution)
3. Return x (local optimal solution).



Local optimization algorithms

▶ Design of a local optimization algorithm:

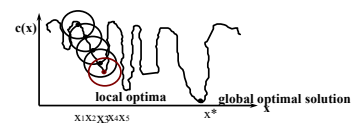
- Obtain an **initial solution**
 - * Heuristic
 - * Random solution
- Define the **neighborhood**
 - * Specific for each problem
- How to **search the neighborhood**
 - * Complete search
 - * First improvement



Local optimization algorithms

▶ Comments

- The search stops at the first local optimum solution with respect to the neighborhood N .
- The final solution highly depends on the initial solution and on the neighborhood.
- No way back out of unattractive local optima...



Multi-Start



- Iterative improvement or hill-descending
 - 1. Get a initial random solution x .
 - 2. Run an **local optimization** (output x)
 - 3. If $\text{cost}(x) < \text{cost}(x_{\text{best}})$ set $x_{\text{best}} = x$;
 - 4. If the stop criteria is not verified, go back to step 1.
 - 5. Output the best solution found.

- * Comments
 - Successive repetition of local improvement.
 - Easy to implement.
 - Random solutions may be very bad.

ILS 7

Iterated Local Search



- A Local Search Method...
 - Single chain on search
- Search on the space of local optimal solutions
- Combines local optimization with a big transition/large step/perturbation.
 - Perturbation should not be easily undone by the local search
 - Most important aspect of the ILS
- Able to make large changes at any stage of the algorithm.

ILS 8

Iterated Local Search



- Get an **initial solution** x ;
 - * Heuristic method or a random solution.
 - * **local optimization** method
- For a certain number of iterations:
 - **Perturbation Step**
 - * method that makes a large modification based in optimization and on the structure of the solution x , resulting in x' .
 - Small-steps
 - * **local optimization** method, initial solution x' ; final solution x'' .
 - **Perform an accept/reject test**
 - * accept all solutions, accept with a certain probability or accept only if it is a better solution.
 - * If x'' is accepted, then $x = x''$.
- Return the best solution found.

ILS 9

Iterated Local Search



- A simple implementation...
 - **Generate Initial Solution**
 - * Greedy Heuristic
 - **Local Search Method**
 - * First improvement local search
 - * Definition of neighborhood
 - **Perturbation Method**
 - * One move of a high level neighborhood
 - **Acceptance Criteria**
 - * Accept if a better solution is found

Often leads to very good performance

Only requires few lines of additional code

ILS 10

Iterated Local Search



- | | |
|--|---|
| <ul style="list-style-type: none"> ► Generate Initial Solution <ul style="list-style-type: none"> ▪ Randomized Greedy Heuristic ▪ Random solution ► Acceptance Criteria <ul style="list-style-type: none"> ▪ Better ▪ Random Walk ▪ Simulated Annealing type ▪ Restart | <ul style="list-style-type: none"> ► Local Search Method <ul style="list-style-type: none"> ▪ Local search ▪ Tabu search ► Perturbation <ul style="list-style-type: none"> ▪ Higher level of neighborhood ▪ Strength of the perturbation <ul style="list-style-type: none"> * Big/small ▪ Adaptive memory ▪ Modify input data ▪ Optimized perturbation |
|--|---|

ILS 11

Iterated Local Search



- Improving ILS
 - **Relationship between local search and perturbation.**
 - * Perturbation must lead to a new region of the solution space that cannot be reached by a local search method.
 - * Perturbation should not be easily undo by the local search.
 - Perturbation can incorporate problem-specific information.
 - * As for example optimization methods
 - * Destruction and construction approach
 - A good perturbation transforms one excellent solution into a excellent starting point to a local search.
 - Local search method must be fast.
 - Complexity must be added progressively and in a modular way.

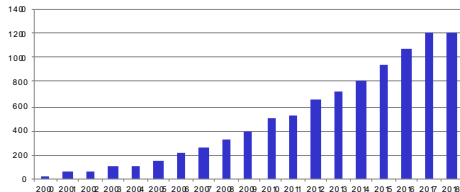
ILS 12

Iterated Local Search

- ▶ Google Scholar's number of publications

“Iterated Local Search”

- About 10,000 publications



ILS 13

Iterated Local Search

- ▶ ILS applied to Complex and large-scale real problems

Accuracy

- Complex problems.
- Large scale problems.

Speed

- Fast answer.
- Analysis of several scenarios.

Flexibility

- Fast changes.
- Different constraints in different areas.

Simplicity

- Need of fast implementation

ILS 14

Traveling Salesman Problem

- ▶ Traveling Salesman Problem

- Given a number of cities and the costs (distances) of traveling from any city to any other city...
- What is the least-cost round-trip route that visits each city exactly once and then returns to the starting city?
- <http://www.math.uwaterloo.ca/tsp/>

ILS 15

Traveling Salesman Problem

- ▶ Generate Initial Solution

- Constructive Heuristic: nearest neighbor, insertion heuristic

- ▶ Local Search Method

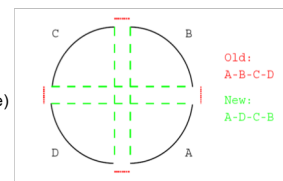
- 2-opt/3-opt Neighborhood

- ▶ Perturbation Method

- One 4-opt move (double-bridge)

- ▶ Acceptance Criteria

- Accept only if the best solution improved



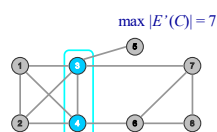
ILS 16

Maximum Cut-Clique Problem

- ▶ Given a clique C , its edge neighborhood (cut-clique) is defined by the set of edges $E'(C) = \{(i,j) \in E : i \in C \text{ and } j \in V \setminus C\}$, and $|E'(C)|$ is its size. Denote $N(i) = \{j \in V : (i,j) \in E\}$.

- ▶ Maximum Cut-Clique

- Maximum edge neighborhood clique



ILS 17

ILS for Cut-Clique Problem

- ▶ Perturbation

- Select randomly one node
- Build the clique with all nodes in the previous clique and fully connected to this node
Set $C \leftarrow [C \cap N(i)] \cup \{i\}$;
Set $U \leftarrow \emptyset$ and $C' \leftarrow C$;

- ▶ Local Optimization

- Add, Swap and Aspiration moves
- R-ILS (random version)
- D-ILS (deterministic version)

ILS 18

Maximum Cut-Clique Problem

► Traditional approach

- Solve with Integer Linear Programming Software
- Branch-and-Bound / Branch-and-cut general exact algorithm
- Obtain the Optimal Solution / Lower Bound
- CPLEX Optimizer

ILS 20

Computational results:

Intel Core i7-2600 with 3.40GHz and 8GB RAM; using CPLEX 11.2

The ---- symbol means that CPLEX was not able to read and preprocess the model in one hour

time in seconds

Instance	V	E	d	MC problem		MCC Problem		
				$\Phi(G)$	N(C)	C	N(C)	time
d1-RTN	2418	9317	0.0032	10	195	8	1273	605.11
d3-RTN	4755	26943	0.0024	18	1097	----	----	----
d7-RTN	6511	44615	0.0021	18	1576	----	----	----
d15-RTN	7965	62136	0.0020	18	1979	----	----	----
d30-RTN	10101	91803	0.0018	21	13099	----	----	----
d66-RTN	13308	148035	0.0017	----	----	----	----	----
c-fat200-1	200	1534	0.077	12	72	9	81	0.05
c-fat200-2	200	3235	0.163	24	264	17	306	0.09
c-fat200-5	200	8473	0.426	58	1682	44	1892	0.05
c-fat500-1	500	4459	0.036	14	98	11	110	0.76
c-fat500-2	500	9139	0.073	26	338	19	380	0.80
c-fat500-5	500	23191	0.186	64	2048	48	2304	0.83
c-fat500-10	500	46627	0.374	126	7938	94	8930	0.58

ILS 21

Computational results of the ILS:

Intel Core i7-2600 with 3.40GHz and 8GB RAM

100 runs

time in seconds

Instance	V	E	d	MCC Problem		
				C	N(C)	time
d1-RTN	2418	9317	0.0032	8	1273	0.1762
d3-RTN	4755	26943	0.0024	12	3526	0.4743
d7-RTN	6511	44615	0.0021	15	5656	0.6777
d15-RTN	7965	62136	0.0020	16	7772	0.8757
d30-RTN	10101	91803	0.0018	21	13099	1.1317
d66-RTN	13308	148035	0.0017	28	22379	1.4081
c-fat200-1	200	1534	0.077	9	81	0.1385
c-fat200-2	200	3235	0.163	17	306	0.0866
c-fat200-5	200	8473	0.426	44	1892	0.0664
c-fat500-1	500	4459	0.036	11	110	0.5451
c-fat500-2	500	9139	0.073	19	380	0.3595
c-fat500-5	500	23191	0.186	48	2304	0.2381
c-fat500-10	500	46627	0.374	94	8930	0.2111

ILS 22

Market Basket Analysis

- The main objective is to analyze large dataset of store transactions
- Obtain relevant insights to do a better planning of the Marketing strategies and operations.
 - Product placement
 - Optimal product-line offering
 - Personalized marketing campaigns
 - Product promotions

ILS 23

Market Basket Analysis

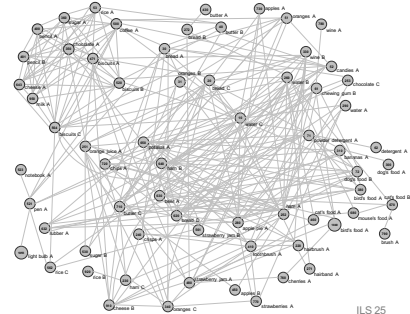
- Raeder and Chawla (2011) say "... no techniques currently available in the literature sufficiently addresses the problem of finding meaningful relationships in a large transaction databases."
- Dataset
 - a household panel database for the British ice cream market.
 - 691 different varieties of products available in the British market.

ILS 24

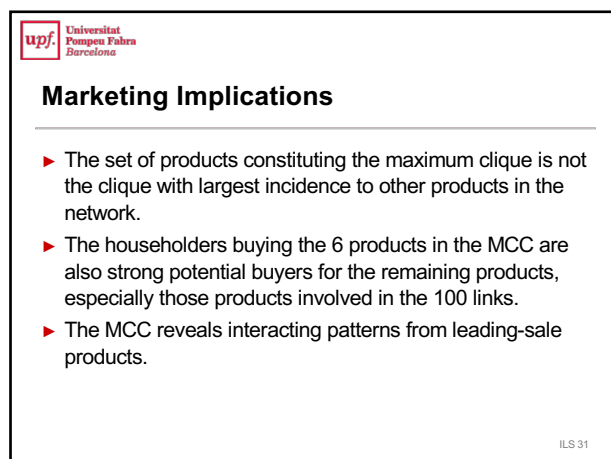
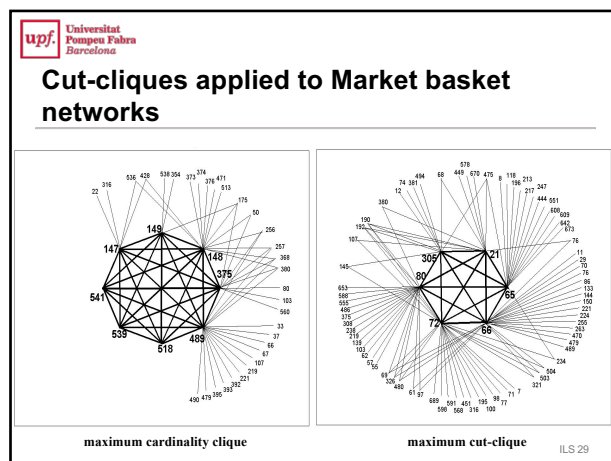
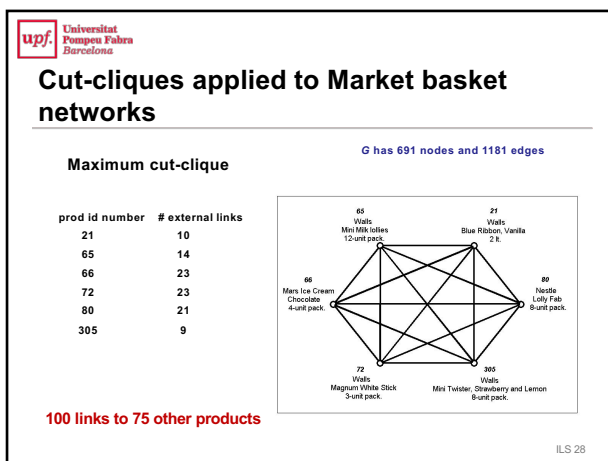
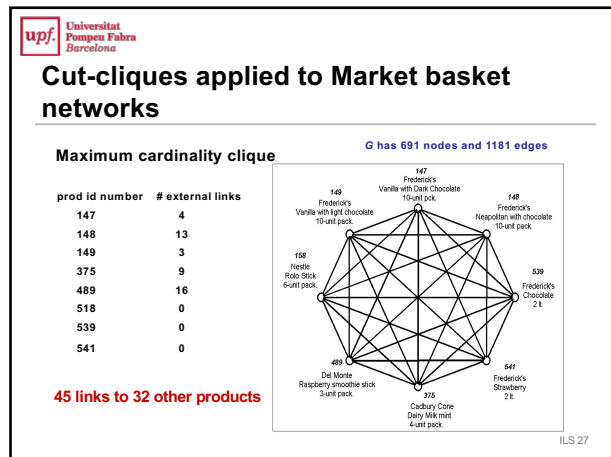
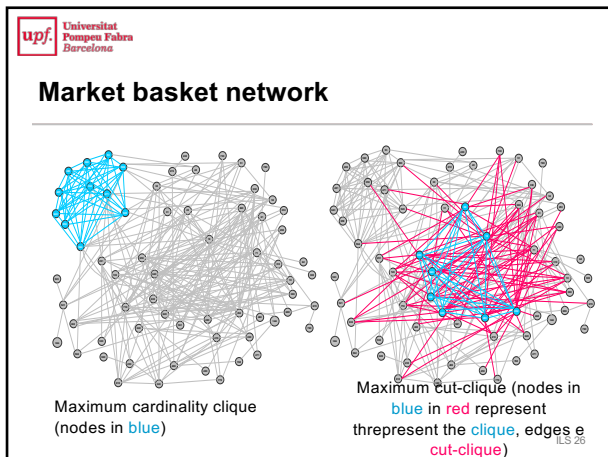
Market basket network

nodes:
products in a store

edges:
represent pairs of products (i, j) bought together by a customer on a given purchase visit to the store

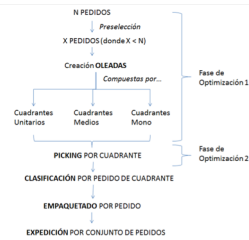
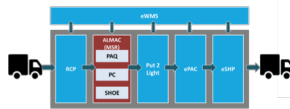


ILS 25



Warehouse Zara

- ▶ Optimization of the picking of the online orders
- ▶ This warehouse prepares 30,000 to 500,000 orders a day.



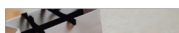
ILS 32

Warehouse Zara

- ▶ **Input data**
 - Online order to be prepared
 - Due date of the orders
 - Detail information of each SKU of the orders
 - Location of each SKU of the orders in the warehouse
 - Distance matrix of each location in the warehouse
- ▶ **Decision Variables**
 - Orders to be prepared in the next preparation (Oleada)
 - Orders associated with each group
 - Routing of the picking of each group

ILS 33

Warehouse Zara

- ## ► Constraints
- Number of orders by group (size of the preparation area)
 - Number of groups by preparation phase (oleada)
 - Number of employees
 - Number of SKUs by group
 - Due date at packing area
 - Other special constraints...
- 
- A photograph showing a white Zara shopping bag with a black strap and a small white card, both resting on a light-colored wooden surface. The Zara logo is visible on both items.



ILS 34

Warehouse Zara

- ▶ **Algorithm**
 - Clustering, Assignment, Routing Algorithm (CARA)
 - Greedy Randomized Adaptive Search Procedure (GRASP)
 - Iterated Local Search (ILS)
- ▶ **Results**
 - Run the algorithm for the small instance with 500 orders, 2 area-stocks, 200 positions, and more than 400 items.
 - Be able to prepare more 25% order per hour!

ILS 35

Hybrid ILS and Extensions

- ▶ Hybrid with other metaheuristics
- ▶ SimILS
- ▶ MathILS



Word cloud: Blum et al. (2011) Hybrid metaheuristics in combinatorial optimization: A survey. Applied Soft Computing 11:4135-4151

ILS 36

ILS... SimILS... MathILS...

- ▶ ILS represents one of the most efficient yet easy-to-implement frameworks for solving combinatorial optimization problems.
- ▶ **Easy to implement and adapt.**
- ▶ Most real-life problems are **complex** and filled with **uncertainty**.
- ▶ By integrating simulation inside the local search process, SimILS framework extends the virtues of ILS to stochastic COPs as well.
- ▶ The same by using exact methods, lower bounds etc...

ILS 37

Hybrid ILS with other metaheuristics

- ▶ Use the ILS structure with other metaheuristics
- ▶ **Local Optimization Phase**
 - Tabu Search
 - VNS
 - Simulated Annealing
 - Variable Neighborhood Search
 -
- ▶ **Perturbation Phase**
 - Large neighborhood change

ILS 38

Distribution problem

- ▶ **Extended Vehicle Routing Problem**
 - Heterogeneous fleet (7 different truck capacities)
 - Time windows in the stores
 - Constraints of assigning some trucks to some stores.
 - Maximum driving hours
 - Multitrip for some vehicles
 - Sales constraints
- ▶ **Minimize operative costs**

ILS 39

Distribution problem

- ▶ **GILS-VND Algorithm**
 - GILS-VND, combina un Iterated Local Search (ILS), Greedy Randomized Adaptive Search Procedure (GRASP), y Variable Neighborhood Descent (VND).
 - ILS Structure
 - * Initial solution: **GRASP**
 - * Local improvement: **Variable Neighborhood Descent**
 - * Perturbation: **Refine using random neighborhood**
 - Coelho V.N., Grasas A., Ramalhinho H., Coelho I.M., Souza M.J.F. (2016), An ILS-based Algorithm to Solve a Large-scale Real Heterogeneous Fleet VRP with Multi-trips and Docking Constraints, European Journal of Operational Research 250 (2): 367–376.

ILS 40

Distribution problem

- ▶ **Computational results**

Statistical results for the new set of instances: GILS-VND vs. Company .

Instance	Company	GILS-VND			
		Best	Average	Std. dev.	Gap (%)
K	32,472	30,507	30,692	70	-5.48
L	18,997	16,327	16,427	42	-13.53
M	20,266	18,017	18,146	43	-10.46
N	51,609	45,523	45,813	100	-11.23
O	45,764	40,846	40,995	66	-10.42

ILS 41

Distribution problem

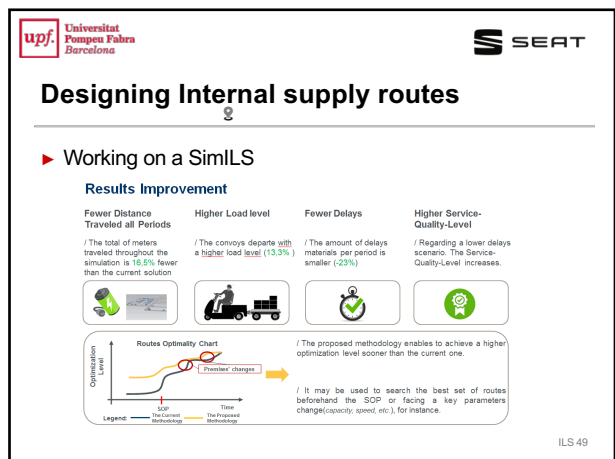
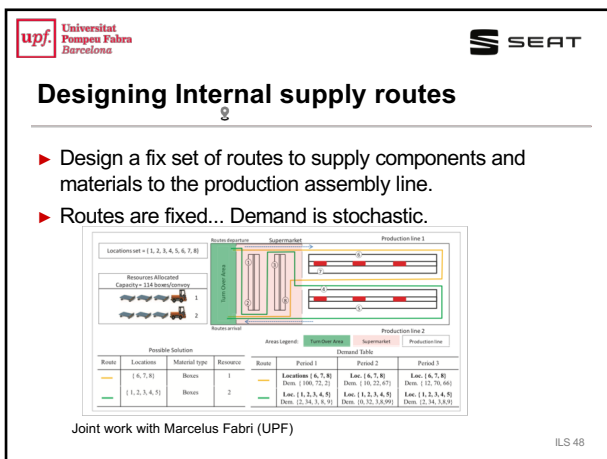
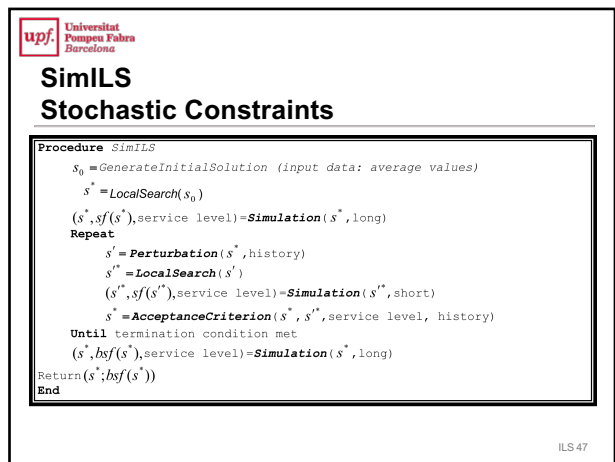
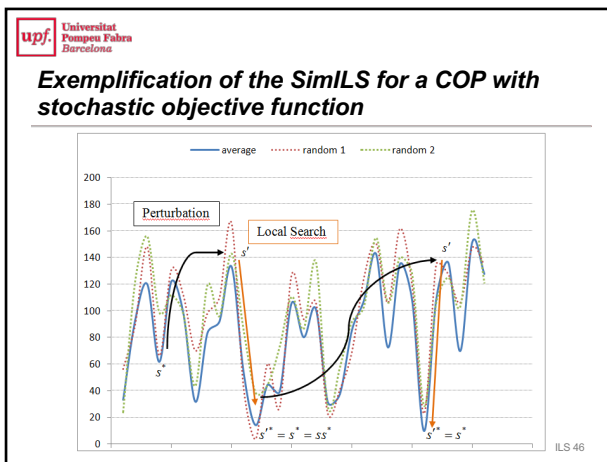
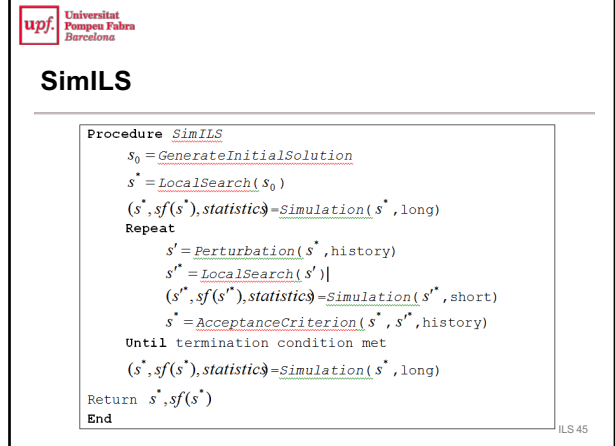
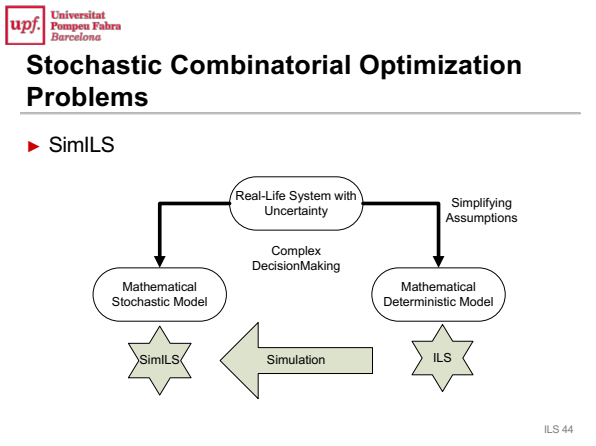
- ▶ **Results**
 - Savings 10% daily with respect to the actual solutions.
 - * A significant daily amount!
 - Savings of 2% compared with Prins' Algorithm with a simpler version of the problem.
 - Smaller number of vehicles need.
 - Better coordination with sales department.

ILS 42

SimILS

- ▶ **Stochastic Combinatorial Optimization Problems**
 - Uncertainty is present – Random Data
 - * Example: Stochastic Demand in Vehicle Routing Problems
 - * Stochastic processing times in scheduling
 - * etc...
 - Strategic Problems
- ▶ Extends ILS to solve Stochastic Models...
- ▶ **SimILS**
 - Simulation + Iterated Local Search

ILS 43



Supply Chain Design for ecommerce

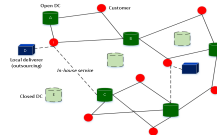
- Supply Chain Design for ecommerce
 - Two-Stage Stochastic Programming Problem
- The goal is to find:
 - the subset of warehouses to be opened;
 - and determine the customer's assignment to the open warehouses
 - ... such that all the demand is served at minimum total cost.
 - Demand is stochastic ...
 - Each customer area must have 2 or 3 warehouses assigned as regular warehouses.



ILS 50

Supply Chain Design for ecommerce

- Two-Stage Optimization Problems
- The problems has two groups of variables interrelated among them ...
 - Strategical decision variables (long term deterministic decision)
 - Operational decision variables (short term decisions), therefore **stochastics** at the moment of the decision process!



ILS 51

Supply Chain Design for ecommerce

- Solving this Two-Stage **Stochastic** Optimization Problem
- **Deterministic Equivalent Model (DEM)**
 - solved by CPLEX.
- **SimILS**
 - Simulation + Iterated Local Search
 - Use simulation to obtain the expected overall cost.
 - Local Search on open/close warehouses
 - Only *promising* solutions are tested in a stochastic environment.
- SimILS results compared favorably with **DEM**, with shorter running times.

ILS 52

Supply Chain Design for ecommerce

- **Deterministic Equivalent Model (DEM)**
 - Stochastic Programming

$$\begin{aligned}
 (\text{SCFLPrp}) \quad \min Z_{\text{stoch}} &= \sum_{j=1}^m f_j y_j + \sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^s \pi_k c_{ijk} x_{ijk} \\
 \text{s.t.} \quad &\sum_{j=1}^m \tau_{ij} \leq R, & \text{for } i = 1, \dots, n; \\
 &\sum_{j=1}^m x_{ijk} = 1, & \text{for } i = 1, \dots, n, \quad k = 1, \dots, s; \\
 &\sum_{i=1}^n d_{ijk} x_{ijk} \leq q_j y_j, & \text{for } j = 1, \dots, m, \quad k = 1, \dots, s; \\
 &x_{ijk} \leq \tau_{ij}, & \text{for } i = 1, \dots, n, \quad j = 1, \dots, m, \quad k = 1, \dots, s; \\
 &y_j \in \{0,1\}, \tau_{ij} \in \{0,1\}, x_{ijk} \geq 0 & \text{for } j = 1, \dots, m; i = 1, \dots, n; k = 1, \dots, s
 \end{aligned}$$



ILS 53

SimILS for SC Design Problem

```

Procedure SimILS
  s0 = GenerateInitialSolution
  s = LocalSearch(s0)
  (s', sf(s'), statistics) = Simulation(s', long)
  Repeat
    s' = Perturbation(s', statistics)
    s'' = LocalSearch(s')
    (s'', sf(s''), statistics) = Simulation(s'', short)
    s = AcceptanceCriterion(s', s'', history)
  Until termination condition met
  (s', sf(s'), statistics) = Simulation(s', long)
  Return s', sf(s')
End
    
```

(a) open (b) close (c) open-close

Destruction-Reconstruction process

ILS 54

Supply Chain Design for ecommerce

- Some results...
- cap11#,
 - 50 facilities
 - 50 customers
- Capa/b/c#,
 - 100 facilities
 - 1000 customers

Instance	Stochastic Programming			SimILS		
	Z _{stoch}	t (sec)	gap Cplex (%)	Z _{stoch}	t (sec)	gap (%)
cap111	879371.7	1103	0	883339.4	170	0.47
cap112	965382.2	3604	0.01	958047.4	303	-0.76
cap113	1047322.2	3603	0.04	1030653.5	294	-1.59
cap114	1160395.4	3604	0.06	1139246.8	50	-1.82
cap124	975397.6	3603	0.09	949375.9	65	-2.67
capa1	-	-	-	19244150.7	2263	-
capa2	-	-	-	18458612.4	2370	-
capa3	-	-	-	17827599.1	2934	-
capa4	-	-	-	17162132.3	983	-
capb1	-	-	-	13773560.0	809	-
capb2	-	-	-	13378972.7	2645	-
capb3	-	-	-	13238010.0	2086	-
capb4	-	-	-	13084859.8	2380	-
capc1	-	-	-	11704267.9	1363	-
capc2	-	-	-	11571933.3	1812	-
capc3	-	-	-	11340950.9	2830	-
capc4	-	-	-	11335842.8	1928	-

ILS 55

Solution Methods

**Local Search Methods
Metaheuristics**

- Mathematical proved optimal solutions
- Important information on the characteristics and properties of the problem.

- Good solutions for complex and large-scale problems
- Short running times
- Easily adapted

**Integer Programming
Exact Methods**

ILS 56

Math - Iterated Local Search

- Get an initial solution x ;
 - Heuristic method or a random solution.
 - Local optimization method
- For a certain number of iterations:
 - Perturbation Step**
 - Uses an exact method to solve a subproblem or a relaxation of the problem.
 - Small-steps
 - Local optimization method, initial solution x' ; final solution x'' .
 - Perform an accept/reject test
 - accept all solutions, accept with a certain probability or accept only if it is a better solution.
 - If x'' is accepted, then $x = x''$.
- Return the best solution found.

ILS 57

MathILS

- Maybe the first application...
 - Use an exact algorithm to solve a sub problem within a **Iterated Local Search** heuristic for the Job-Shop Scheduling Problem
 - Solving to optimality the one-machine scheduling problem with due dates and delivery times using the Carlier Algorithm.**
 - Laurenço H.R. (1995), Job-Shop Scheduling: computational study of local search and large-step optimization methods. European Journal of Operational Research **83**(2): 347-364. ISSN 0377-2217.
 - PhD Thesis, Lourenço H.R. (1993)

ILS 58

Example of Applications

- Real Applications
 - Maybe the best set of problems to apply Metaheuristics methods...
 - Why?
 - Complex problems with a large number of constraints.
 - Sometimes difficult to model...
 - But, a simplification of the problem is frequently a well-studied optimization problem.
 - Apply **metaheuristics for the real general problem...**
 - And **exact methods for the well-known relaxation problem.**

ILS 59

Metaheuristics

- Which is the best metaheuristic?
 - Begin with a simple method and then turn, if necessary, to a more complicated one or refine the first implementation
 - Small number of parameters
 - Evaluate its performance by:
 - Accuracy
 - Speed
 - Simplicity
 - Flexibility

ILS 60

Conclusions

- Iterated Local Search
 - Simple
 - Easy to implement
 - Robust
 - Highly effective
 - Modularity
- Start simple and add complexity if needed!
- The success of ILS lies in the biased sampling of the set of local optimal.
- More than 6000 publications in google scholar.

Do you want to try to implement an ILS?

ILS 61

Iterated Local Search

► Main References

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