A Model of the Twin Ds: Optimal Default and Devaluation*

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August 10, 2015

Abstract

Defaults are typically accompanied by large devaluations. This paper characterizes jointly optimal default and exchange-rate policy in an economy with limited enforcement of debt contracts and downward nominal wage rigidity. Under optimal policy, default occurs during contractions and is accompanied by large devaluations. The latter inflate away real wages thereby reducing involuntary unemployment. By contrast, under fixed exchange rates, optimal default takes place in the context of involuntary unemployment. Fixed-exchange-rate economies are shown to have stronger default incentives and therefore can support less external debt in the long run than economies with optimally floating rates. *(JEL E43, E52, F31, F34, F38, F41)*

Keywords: Sovereign Default, Exchange Rates, Optimal Monetary Policy, Capital Controls, Downward Nominal Wage Rigidity, Currency Pegs.

*Schmitt-Grohé and Uribe thank the National Science Foundation for research support. We thank for comments Javier Bianchi, Robert Kollmann and seminar participants at the University of Bonn, Columbia University, Harvard University, Seoul National University, the European Central Bank, the Philadelphia Fed, MIT, the University of Pennsylvania, LSE, the Board of Governors, the CIREQ-ENSAI workshop, the Cornell PSU conference, and the IFM October 2014 Program meeting of the NBER. The views expressed herein are those of the authors and should not be interpreted as reflecting the views of the Federal Reserve System.

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1 Introduction

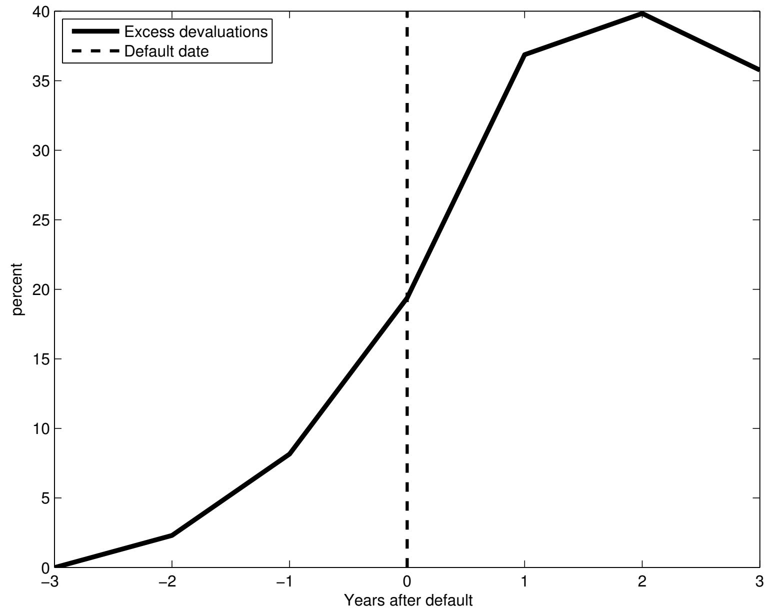
There exists a strong empirical link between sovereign default and devaluation. Using data for 58 countries over the period 1970 to 1999, Reinhart (2002) estimates that the unconditional probability of a large devaluation in any 24-month period is 17 percent. At the same time, she estimates that conditional on the 24-month period containing a default event, the probability of a large devaluation increases to 84 percent. Reinhart refers to this phenomenon as the Twin Ds.

Figure 1 provides further evidence of the Twin Ds phenomenon. It displays the median excess depreciation of the nominal exchange rate around 116 sovereign defaults that occurred in 70 countries over the period 1975 to 2013. It shows that typically in a window encompassing three years prior and after a default event, the exchange rate depreciates 35-40 percent more than in the unconditional median window of the same width. A feature of the Twin Ds uncovered by figure 1, is the deceleration in the rate of devaluation that takes place shortly after default. The Twin Ds has to do more with a change in the level of the nominal exchange rate than with a switch to a higher rate of depreciation. This observation is of particular interest because it helps discriminate among possible explanations of the Twin Ds phenomenon.

The Twin Ds phenomenon suggests some connection between the decision to default and the decision to devalue. In this paper, this connection is created by combining lack of commitment to repay sovereign debt with downward nominal wage rigidity. When the government chooses both default and devaluation optimally, the typical default episode is shown to occur after a string of increasingly negative output shocks. In the run-up to default, consumption experiences a severe contraction putting downward pressure on the demand for labor. Absent any intervention by the central bank, downward nominal wage rigidity would prevent real wages from adjusting downwardly and involuntary unemployment would emerge. To avoid this scenario, the optimal policy calls for a devaluation of the domestic currency, which reduces the real value of wages. In a calibrated version of the model, the minimum devaluation necessary to implement the optimal allocation at the time of default exceeds 35 percent. Thus, the benevolent government's desire to preserve employment during a severe external crisis gives rise endogenously to the Twin Ds, the joint occurrence of large devaluations and sovereign default.

A natural question is what are the predicted equilibrium dynamics around default when the central bank is unwilling or unable to apply the optimal devaluation policy. This question is relevant in light of the fact that a number of debt crises have taken place under fixed exchange rates. Prominent examples are the default events in Greece and Cyprus in 2012

Figure 1: Excess Devaluation Around Default, 1975-2013



Note. The solid line displays the median of the cumulative devaluation rate between years -3 and t, for $t = -3, \ldots, 3$, conditional on default in year 0 minus the unconditional median of the cumulative devaluation rate between years -3 and t. Countries with less than 30 consecutive years of exchange rate data were excluded, resulting in 116 default episodes over the period 1975 and 2013 in 70 countries. Data Sources: Default dates, Uribe and Schmitt-Grohé (2015). Exchange rates, World Development Indicators, code: `PA.NUS.FCRF`.

and 2013, respectively. Motivated by this question, we characterize the optimal default policy under a currency peg. In this case, the central bank loses its ability to counteract the inefficiencies associated with downward nominal wage rigidity during periods of depressed aggregate demand. As a consequence, the contraction around default is accompanied by involuntary unemployment, which in the calibrated version of the economy reaches 20 percent of the labor force.

Further, the model predicts that in the long run, economies with fixed exchange rates can support less external debt than economies in which the exchange rate floats optimally. The reason is that ex-ante the fixed-exchange-rate economy has stronger incentives to default than the economy with optimal exchange-rate policy. This is so because in the fixed-exchange rate economy the resources that are freed up by default have the additional benefit of contributing to moderate the unemployment problem. Interestingly, ex-post the probability of default is not predicted to be higher in the fixed exchange-rate economy, because the lower ex-post level of debt reduces the gains from default.

Unlike most of the related literature on sovereign default, our starting point is a decentralized economy. Individual households can borrow or lend in international financial markets and are subject a tax on debt. In addition, households and firms interact in competitive factor and product markets in which prices are set in nominal terms and nominal wages are downwardly rigid. The government chooses optimally the paths of three policy instruments, the nominal exchange rate, the debt tax, and the decision to default on the country's net foreign debt obligations.

The paper establishes two decentralization results that unfold twice the social planner real setup in which most models of default à la Eaton-Gersovitz are cast. The first unfolding allows households to make optimal consumption and savings decisions but maintains the assumption of a real economy. The second unfolding goes one step further and considers an environment in which all transactions are performed in nominal prices and wages are downwardly rigid. This second decentralization result shows that real economies in the tradition of Eaton and Gersovitz can be interpreted as the centralized version of models with downward nominal wage rigidity.

The present paper is related to several strands of literature. An important body of work focuses on the fiscal consequences of devaluations, emphasizing either flow or stock effects. Models of balance-of-payment crises à la Krugman (1979) focus on increases in the rate of devaluation as a way to generate seignorage revenue flows when a government suffering from structural fiscal deficits is forced to abandon an unsustainable currency peg. This explanation has been used to understand the defaults of the early 1980s in Latin America, which were followed by a decade of high inflation. Under this hypothesis, the nominal exchange rate continues to grow at higher rates after the default. However, the typical pattern of default and devaluation is one in which high rates of devaluation stop within a year after default. This is reflected by the post-default flattening of the exchange rate path shown in figure 1. A literature that goes back to Calvo (1988) views devaluation as an implicit default on (the stock of) domestic-currency denominated government debt. Recent developments along this line include Aguiar et al. (2013), Corsetti and Dedola (2014), Da Rocha (2013), Du and Schreger (2015), and Sunder-Plassmann (2013). This channel is not open in the model studied in the present paper because debt is assumed to be denominated in foreign currency. This assumption is motivated by the empirical literature on the original sin, which documents that virtually all of the debt issued by emerging countries is denominated in foreign currency (see, for example, Eichengreen, Hausmann, and Panizza, 2005).

The real side of the model developed in this paper builds on recent contributions to the theory of sovereign default in the tradition of Eaton and Gersovitz, especially, Aguiar and Gopinath (2006), Arellano (2008), Hatchondo, Martinez, and Sapriza (2010), Chatterjee and Eyigungor (2012), and Mendoza and Yue (2012). This literature has made significant progress in identifying features of the default model that help deliver realistic predictions for the average and cyclical behavior of key variables of the model, such as the level of external debt and the country interest rate premium. We contribute to this literature by establishing that the social planner allocation in models of the Eaton-Gersovitz family can be decentralized by means of a debt tax. And we extend this literature by merging it with the literature on optimal exchange-rate policy (e.g., Galí and Monacelli, 2005; Kollmann, 2002; and Schmitt-Grohé and Uribe, 2015). Moussa (2013) builds a framework similar to the present one to study the role of debt denomination. Kriwoluzky, Müller, and Wolf (2014) study an environment in which default takes the form of a re-denomination of debt from foreign to domestic currency. Finally, Yun (2014) presents a model in which sovereign default causes the monetary authority to loose commitment to stable exchange-rate policy.

The remainder of the paper is organized as follows. Section 2 presents the model and derives the competitive equilibrium. Section 3 derives the key decentralization results and characterizes analytically the equilibrium under optimal default and devaluation policy. Section 4 analyzes quantitatively the typical default episode under the optimal policy in the context of a calibrated version of the model. Section 5 characterizes analytically and quantitatively the equilibrium dynamics under a currency peg. Section 6 extends the model to allow for long-maturity debt and incomplete exchange-rate pass-through. Section 7 concludes.

2 The Model

The theoretical framework embeds imperfect enforcement of international debt contracts à la Eaton and Gersovitz (1981) into the small open economy model with downward nominal wage rigidity of Schmitt-Grohé and Uribe (2015). We begin by describing the economic decision problem of households, firms, and the government interacting in a decentralized economic environment.

2.1 Households

The economy is populated by a large number of identical households with preferences described by the utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where c_t denotes consumption. The period utility function U is assumed to be strictly increasing and strictly concave and the parameter β, denoting the subjective discount factor, resides in the interval $(0, 1)$. The symbol $\mathbb{E}_t$ denotes the mathematical expectations operator conditional upon information available in period t. The consumption good is a composite of tradable consumption, c_t^T, and nontradable consumption, c_t^N. The aggregation technology is of the form

$$c_t = A(c_t^T, c_t^N), \tag{2}$$

where A is an increasing, concave, and linearly homogeneous function.

Households have access to a one-period, state noncontingent bond, which is assumed to be denominated in tradables.[1] We let d_{t+1} denote the level of debt assumed in period t and due in period $t+1$ and q_t^d its price. The sequential budget constraint of the household is given by

$$P_t^T c_t^T + P_t^N c_t^N + P_t^T d_t = P_t^T \tilde{y}_t^T + W_t h_t + (1-\tau_t^d) P_t^T q_t^d d_{t+1} + F_t + \Phi_t, \tag{3}$$

where P_t^T denotes the nominal price of tradable goods, P_t^N the nominal price of nontradable goods, $\tilde{y}_t^T$ the household's endowment of traded goods, W_t the nominal wage rate, h_t hours worked, τ_t^d a tax on debt, F_t a lump-sum transfer received from the government, and Φ_t nominal profits from the ownership of firms. Households are assumed to be subject to a debt limit that prevents them from engaging in Ponzi schemes.

The variable $\tilde{y}_t^T$ is stochastic and is taken as given by the household. Households supply inelastically $\bar{h}$ hours to the labor market each period, but may not be able to sell all of them, which gives rise to the constraint

$$h_t \leq \bar{h}. \tag{4}$$

Households take h_t as exogenously given.

Households choose contingent plans $\{c_t, c_t^T, c_t^N, d_{t+1}\}$ to maximize (1) subject to (2)-(4) and the no-Ponzi-game debt limit, taking as given P_t^T, P_t^N, W_t, h_t, Φ_t, q_t^d, τ_t^d, F_t, and $\tilde{y}_t^T$. Letting $p_t \equiv P_t^N/P_t^T$ denote the relative price of nontradables in terms of tradables, the optimality conditions associated with this problem are (2)-(4), the no-Ponzi-game debt limit, and

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t, \tag{5}$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N),$$

$$(1-\tau_t^d) q_t^d \lambda_t = \beta \mathbb{E}_t \lambda_{t+1},$$

where λ_t/P_t^T denotes the Lagrange multiplier associated with (3).

[1] In section 6.1, we show that the key results of the paper are robust to allowing for long-term debt.

2.2 Firms

Nontraded output, denoted y_t^N, is produced by perfectly competitive firms. Each firm operates a production technology of the form

$$y_t^N = F(h_t). \tag{6}$$

The function F is assumed to be strictly increasing and strictly concave. Firms choose the amount of labor input to maximize profits, given by

$$\Phi_t = P_t^N F(h_t) - W_t h_t. \tag{7}$$

The optimality condition associated with this problem is $P_t^N F'(h_t) = W_t$. Dividing both sides by P_t^T yields

$$p_t F'(h_t) = w_t,$$

where $w_t \equiv W_t / P_t^T$ denotes the real wage in terms of tradables.

2.3 Downward Nominal Wage Rigidity

We model downward nominal wage rigidity by imposing a lower bound on the growth rate of nominal wages of the form

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0. \tag{8}$$

The parameter γ governs the degree of downward nominal wage rigidity. The higher is γ, the more downwardly rigid are nominal wages. The decision to model nominal rigidity as downward nominal wage rigidity is empirically motivated. Schmitt-Grohé and Uribe (2015) document that downward nominal wage rigidity is pervasive in emerging-market economies. For example, during the 1998-2001 crisis in Argentina, nominal hourly wages remained flat (actually they increased from 7.87 pesos in 1998 to 8.14 pesos in 2001) in spite of the fact that subemployment (the sum of involuntary unemployment and involuntary part -time employment) increased by 10 percentage points and that the nominal exchange rate was fixed at one dollar per peso. This evidence suggests the presence of downward nominal wage rigidity. The period following the collapse of the Argentine currency convertibility (i.e., post December 2001) features sizable increases in nominal hourly wages. This suggests that nominal wages are upwardly flexible. This evidence favors a formulation in which wage rigidity is one sided as opposed of two-sided. Consumer prices in Argentina do not appear to be downwardly rigid to the same degree as nominal wages. Over the period 1998 to 2001, nominal consumer prices fell by about 1 percent per year. Taken together, this evidence suggests that in Argentina around the 2001 crisis, wages were more downwardly rigid than were product prices. The empirical relevance of downward nominal wage rigidity extends to the periphery of Europe around the Great Contraction of 2008. Schmitt-Grohé and Uribe (2015) show that between 2008 and 2011 nominal hourly wages in 13 peripheral European countries increased on average by 2 percent per year, despite the fact that unemployment increased massively and that all countries were either on the Euro or pegging to the Euro. In the boom period that preceded the crisis (2000 to 2007) nominal hourly wages increased on average by more than 7 percent per year. Again, this suggests a formulation in which nominal wages are downwardly rigid.

The presence of downwardly rigid nominal wages implies that the labor market will in general not clear. Instead, involuntary unemployment, given by $\bar{h} - h_t$, will be a regular feature of this economy. We assume that wages and employment satisfy the slackness condition

$$(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0. \tag{9}$$

This condition states that periods of unemployment ($h_t < \bar{h}$) must be accompanied by a binding wage constraint. It also states that when the wage constraint is not binding ($W_t > \gamma W_{t-1}$), the economy must be in full employment ($h_t = \bar{h}$).

2.4 The Government

At the beginning of each period, the country can be either in good or bad financial standing in international financial markets. Let the variable I_t be an indicator function that takes the value 1 if the country is in good financial standing and chooses to honor its debt and 0 otherwise. If the economy starts period t in good financial standing ($I_{t-1} = 1$), the government can choose to default on the country's external debt obligations or to honor them. If the government chooses to default, then the country enters immediately into bad standing and $I_t = 0$. Default is defined as the full repudiation of external debt. While in bad standing, the country is excluded from international credit markets, that is, it cannot borrow or lend from the rest of the world. Formally,

$$(1 - I_t)d_{t+1} = 0. \tag{10}$$

Following Arellano (2008), we assume that bad financial standing lasts for a random number of periods. Specifically, if the country is in bad standing in period t, it will remain in bad standing in period $t + 1$ with probability $1 - \theta$ and will regain good standing with probability θ. When the country regains access to financial markets, it starts with zero external obligations.

We assume that the government rebates the proceeds from the debt tax in a lump-sum fashion to households. In periods in which the country is in bad standing ($I_t = 0$), the government confiscates any payments of households to foreign lenders and returns the proceeds to households in a lump-sum fashion. The resulting sequential budget constraint of the government is

$$f_t = \tau_t^d q_t^d d_{t+1} + (1 - I_t) d_t, \tag{11}$$

where $f_t \equiv F_t / P_t^T$ denotes lump-sum transfers expressed in terms of tradables.[2]

2.5 Foreign Lenders

Foreign lenders are assumed to be risk neutral. Let q_t denote the price of debt charged by foreign lenders to domestic borrowers during periods of good financial standing, and let r^* be a parameter denoting the foreign lenders' opportunity cost of funds. Then, q_t must satisfy the condition that the expected return of lending to the domestic country equal the opportunity cost of funds. Formally,

$$\frac{\text{Prob}\{I_{t+1} = 1 | I_t = 1\}}{q_t} = 1 + r^*. \tag{12}$$

This expression can be equivalently written as

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0.$$

2.6 Competitive Equilibrium

In equilibrium, the market for nontraded goods must clear at all times. That is, the condition

$$c_t^N = y_t^N \tag{13}$$

must hold for all t.

We assume that each period the economy receives an exogenous and stochastic endowment equal to y_t^T per household. This is the sole source of aggregate fluctuations in the present model. Movements in y_t^T can be interpreted either as shocks to the physical availability of tradable goods or as shocks to the country's terms of trade.

[2]It can be shown that the equilibrium dynamics are identical if one replaces the lump-sum transfer f_t with a proportional tax on any combination of the three sources of household income, $w_t h_t$, $\tilde{y}_t^T$, and Φ_t / P_t^T.

As in much of the literature on sovereign default, we assume that if the country is in bad financial standing ($I_t = 0$), it suffers an output loss, which we denote by $L(y_t^T)$. The function $L(\cdot)$ is assumed to be nonnegative and nondecreasing. Thus, the endowment received by the household, $\tilde{y}_t^T$, is given by

$$\tilde{y}_t^T = \begin{cases} y_t^T & \text{if } I_t = 1 \\ y_t^T - L(y_t^T) & \text{otherwise} \end{cases}. \tag{14}$$

As explained in much of the related literature, the introduction of an output loss during financial autarky improves the model's predictions along two dimensions. First, it allows the model to support more debt, as it raises the cost of default. Second, it discourages default in periods of relatively high output.

We assume that $\ln y_t^T$ obeys the law of motion

$$\ln y_t^T = \rho \ln y_{t-1}^T + \mu_t, \tag{15}$$

where μ_t is an i.i.d. innovation with mean 0 and variance σ_μ^2, and $|\rho| \in [0, 1)$ is a parameter.

In any period t in which the country is in good financial standing, the domestic price of debt, q_t^d, must equal the price of debt offered by foreign lenders, q_t, that is,

$$I_t(q_t^d - q_t) = 0. \tag{16}$$

In periods in which the country is in bad standing d_{t+1} is nil. It follows that in these periods the value of τ_t^d is immaterial. Therefore, without loss of generality, we set $\tau_t^d = 0$ when $I_t = 0$, that is,

$$(1 - I_t)\tau_t^d = 0. \tag{17}$$

Combining (3), (6), (7), (10), (11), (13), (14), and (16) yields the market-clearing condition for traded goods,

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t[q_t d_{t+1} - d_t].$$

We assume that the law of one price holds for tradables.[3] Specifically, letting P_t^{T*} denote the foreign currency price of tradables and $\mathcal{E}_t$ the nominal exchange rate defined as the domestic-currency price of one unit of foreign currency (so that the domestic currency

[3] Section 6.2 extends the model to allow for imperfect exchange-rate pass through.

depreciates when $\mathcal{E}_t$ increases), the law of one price implies that

$$P_t^T = P_t^{T*}\mathcal{E}_t.$$

We further assume that the foreign-currency price of tradables is constant and normalized to unity, $P_t^{T*} = 1$. Thus, we have that the nominal price of tradables equals the nominal exchange rate,

$$P_t^T = \mathcal{E}_t.$$

Finally, let

$$\epsilon_t \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$$

denote the gross devaluation rate of the domestic currency. We are now ready to define a competitive equilibrium.

Definition 1 (Competitive Equilibrium) *A competitive equilibrium is a set of stochastic processes* $\{c_t^T,\ h_t,\ w_t,\ d_{t+1},\ \lambda_t,\ q_t,\ q_t^d\}$ *satisfying*

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t[q_t d_{t+1} - d_t], \tag{18}$$

$$(1 - I_t)d_{t+1} = 0, \tag{19}$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \tag{20}$$

$$(1 - \tau_t^d)q_t^d \lambda_t = \beta \mathbb{E}_t \lambda_{t+1}, \tag{21}$$

$$I_t(q_t^d - q_t) = 0, \tag{22}$$

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{23}$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{24}$$

$$h_t \leq \bar{h}, \tag{25}$$

$$(h_t - \bar{h})\left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t}\right) = 0, \tag{26}$$

$$I_t\left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*}\right] = 0, \tag{27}$$

given processes $\{y_t^T, \epsilon_t, \tau_t^d, I_t\}$ *and initial conditions* w_{-1} *and* d_0.

3 Equilibrium Under Optimal Policy

This section characterizes the optimal default, devaluation, and debt tax policies. When the government can choose freely ϵ_t and τ_t^d, the competitive equilibrium can be written in a more compact form, as stated in the following proposition.

Proposition 1 (Competitive Equilibrium When ϵ_t and τ_t^d Are Unrestricted) *When the government can choose ϵ_t and τ_t^d freely, stochastic processes $\{c_t^T, h_t, d_{t+1}, q_t\}$ can be supported as a competitive equilibrium if and only if they satisfy the subset of equilibrium conditions (18), (19), (25), and (27), given processes $\{y_t^T, I_t\}$ and the initial condition d_0.*

The key step in establishing this proposition is to show that if processes $\{c_t^T, h_t, d_{t+1}, q_t\}$ satisfy conditions (18), (19), (25), and (27), then they also satisfy the remaining conditions defining a competitive equilibrium, namely, conditions (20)-(24) and (26). To show this, pick λ_t to satisfy (20). When I_t equals 1, set q_t^d to satisfy (22) and set τ_t^d to satisfy (21). When I_t equals 0, set $\tau_t^d = 0$ (recall convention (17)) and set q_t^d to satisfy (21). Set w_t to satisfy (23). Set ϵ_t to satisfy (24) with equality. This implies that the slackness condition (26) is also satisfied. This establishes proposition 1.

It is noteworthy that the compact set of equilibrium conditions includes neither the lower bound on wages nor the Euler equation of private households for choosing debt. This means that policy can be set to undo the distortions arising from downward nominal wage rigidity and the externality originating in the fact that private agents fail to internalize the effect of their individual borrowing choices on interest rates. Taxes on debt play a similar role in models in which a pecuniary externality arises because borrowers fail to internalize that the value of their collateral depends on their own spending decisions (see Korinek, 2010; Mendoza, 2010; and Bianchi, Boz, and Mendoza, 2012).

The government is assumed to be benevolent. It chooses a default policy I_t to maximize the welfare of the representative household subject to the constraint that the resulting allocation can be supported as a competitive equilibrium. The Eaton-Gersovitz model imposes an additional restriction on the default policy. Namely, that the government has no commitment to honor past promises regarding debt payments or defaults. The lack of commitment opens the door to time inconsistency. For this reason the Eaton-Gersovitz model assumes that the government has the ability to commit to a default policy that makes the default decision in period t an invariant function of the minimum set of aggregate states of the competitive equilibrium of the economy in period t. The states appearing in the conditions of the competitive equilibrium listed in proposition 1 are the endowment, y_t^T, and the stock of net external debt, d_t. Notice that the past real wage, w_{t-1}, does not appear in the compact set of competitive equilibrium conditions. The intuition for why this variable is irrelevant for determining the state of the economy is that, with the policy instruments at its disposal, the government can completely circumvent the distortion created by downward nominal rigidity.[4] Thus, we impose that the default decision in period t be a time invariant function of y_t^T and d_t. We can then define the optimal-policy problem as follows.

Definition 2 (Equilibrium under Optimal Policy) *When ϵ_t and τ_t^d are unrestricted, an equilibrium under optimal policy is a set of processes $\{c_t^T, h_t, d_{t+1}, q_t, I_t\}$ that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t))) \tag{28}$$

subject to

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t[q_t d_{t+1} - d_t], \tag{18}$$

$$(1 - I_t)d_{t+1} = 0, \tag{19}$$

$$h_t \leq \bar{h}, \tag{25}$$

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0 \tag{27}$$

and to the constraint that if $I_{t-1} = 1$, then I_t is an invariant function of y_t^T and d_t and if $I_{t-1} = 0$, then $I_t = 0$ except when reentry to credit markets occurs exogenously. The set of processes must also satisfy the no-Ponzi-game debt limit. The initial values d_0 and I_{-1} are given.

Because I_t depends on d_t and y_t^T, we have that I_{t+1} depends on d_{t+1} and y_{t+1}^T. The expected value of I_{t+1} conditional on information available in period t, $\mathbb{E}_t I_{t+1}$, depends on d_{t+1} and y_t^T. This is because d_{t+1} is determined in period t and because y_t^T is assumed to follow an autoregressive process of order one, which implies that the expected value in period t of any function of y_{t+1}^T depends only on y_t^T. Therefore, by equation (27), in periods t in which the government chooses to honor its debts, the price of debt depends only upon y_t^T and d_{t+1}. Hence we can write equation (27) as

$$I_t \left[q_t - q(y_t^T, d_{t+1}) \right] = 0, \tag{29}$$

where the function $q(\cdot, \cdot)$ is determined in equilibrium.

The solution to the optimal policy problem features full employment at all times. To see this, note that h_t enters only in the objective function (28) and the constraint (25). Clearly,

[4] We will show in section 5 that when the government is not free to choose the path of the devaluation rate, w_{t-1} reappears as a relevant state variable.

because U, A, and F are all strictly increasing, it must be the case that $h_t = \bar{h}$ for all t. This result holds even if the government does not have access to a tax on debt, provided that the intra- and intertemporal elasticities of consumption substitution are equal to each other (which, as argued later in section 4.1, is the case of greatest empirical relevance). The proof of this result is contained in appendix A.4.

Expressing the optimal policy problem of definition 2 in recursive form taking into account that under optimal policy $h_t = \bar{h}$ at all times, it becomes clear that under optimal policy, the equilibrium allocation in the decentralized economy with downward nominal wage rigidity of definition 1 is identical to the equilibrium allocation in the centralized real economy of Eaton and Gersovitz (1981) as presented, for example, in Arellano (2008). This establishes the following proposition:

Proposition 2 (Decentralization) *Real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be interpreted as the centralized version of the decentralized economy with default risk and downward nominal wage rigidity described in definition 1 under optimal devaluation policy and optimal taxation of debt.*

Proof: See appendix A.1.

A corollary of this proposition applies to economies without nominal rigidities. Specifically, real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be interpreted as the centralized version of economies with decentralized markets for consumption and borrowing and default risk under optimal taxation of debt. We present the proof of this corollary in appendix A.2. In other words, real models in the Eaton-Gersovitz family can be decentralized by means of a tax on foreign borrowing.

The preceding analysis fully characterizes the real allocation under optimal policy, as we have established that $h_t = \bar{h}$ at all times and that c_t^T and d_{t+1} are determined as in the Eaton-Gersovitz model, whose solution is known. It remains to characterize the exchange-rate policy that supports the optimal real allocation. This step will allow us to ascertain whether the model can capture the empirical regularity that defaults are typically accompanied by nominal devaluations, the Twin Ds phenomenon documented in figure 1. The family of optimal devaluation policies is given by

$$\epsilon_t \geq \gamma \frac{w_{t-1}}{w^f(c_t^T)}, \tag{30}$$

where $w^f(c_t^T)$ denotes the full-employment real wage, defined as

$$w^f(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}). \tag{31}$$

Given the assumed properties of the aggregator function A, the full-employment real wage, $w^f(c_t^T)$, is strictly increasing in the absorption of tradable goods. To see that the family of devaluation policies given in equation (30) can support the optimal allocation, notice that because in the optimal-policy equilibrium $h_t = \bar{h}$ for all t, competitive-equilibrium condition (23) implies that $w_t = w^f(c_t^T)$, for all $t \geq 0$. Combining this expression with (24) yields (30). One can further establish that any devaluation-rate policy from the family (30) uniquely implements the optimal-policy equilibrium. See appendix A.3 for a proof of this claim.

The optimal policy scheme features instrument specialization. Because the optimal devaluation policy ensure that the equilibrium real wage equals the full-employment real wage at all times, exchange-rate policy specializes in undoing the distortions created by nominal rigidities. Recalling from the proof of proposition 1 that τ_t^d is chosen to guarantee satisfaction of the private agent's Euler equation, it follows that tax policy specializes in overcoming the borrowing externality.

4 The Twin Ds

The optimal devaluation policy, given in equation (30), stipulates that the government must devalue in periods in which consumption of tradables experiences a sufficiently large contraction. At the same time we know from the decentralization result of Proposition 2 that under optimal devaluation policy the default decision coincides with the default decision in real models in the Eaton-Gersovitz tradition. In turn, in this family of models default occurs when aggregate demand is depressed. Therefore, the present model has the potential to predict that devaluations and default happen together, that is, that there is a Twin Ds phenomenon. The question remains whether for plausible calibrations of the model, the contraction in aggregate demand at the time of default is associated with large enough declines in the full-employment real wage to warrant a sizable devaluation. This section addresses this question in the context of a quantitative version of the model.

Conducting a quantitative analysis requires specifying an exchange-rate policy. From the family of optimal devaluation policies given in (30), we select the one that stabilizes nominal wages. Specifically, we assume a devaluation rule of the form

$$\epsilon_t = \frac{w_{t-1}}{w^f(c_t^T)}. \tag{32}$$

For $\gamma < 1$, this policy rule clearly belongs to the family of optimal devaluation policies given in (30). The motivation for studying this particular optimal devaluation policy is twofold. First, it ensures no deflation in the long run. This property is appealing because long-run deflation is not observed either in wages or product prices. Second, the selected optimal devaluation policy delivers the smallest devaluation at any given time among all optimal policies that are non deflationary in the long-run. This means that if the selected devaluation policy delivers the Twin Ds phenomenon, then any other nondeflationary optimal devaluation policy will also do so.[5]

4.1 Functional Forms, Calibration, And Computation

We calibrate the model to the Argentine economy. We choose this country for two reasons. First, the Argentine default of 2002 conforms to the Twin Ds phenomenon. Second, the vast majority of quantitative models of default are calibrated to this economy (e.g., Arellano, 2008; Aguiar and Gopinath, 2006; Chatterjee and Eyigungor, 2012; Mendoza and Yue, 2012). The time unit is assumed to be one quarter. Table 1 summarizes the parameterization. We adopt a period utility function of the CRRA type

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

and set $\sigma = 2$ as in much of the related literature. We assume that the aggregator function takes the CES form

$$A(c^T, c^N) = \left[a (c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}.$$

Following Uribe and Schmitt-Grohé (2015), we set $a = 0.26$, and $\xi = 0.5$. We assume that the production technology is of the form

$$y_t^N = h_t^{\alpha},$$

and set $\alpha = 0.75$ as in Uribe and Schmitt-Grohé (2015). We normalize the time endowment $\bar{h}$ at unity. Based on the evidence on downward nominal wage rigidity reported in Schmitt-Grohé and Uribe (2015), we set the parameter γ equal to 0.99, which implies that nominal wages can fall up to 4 percent per year. We also follow these authors in measuring tradable output as the sum of GDP in agriculture, forestry, fishing, mining, and manufacturing in Argentina over the period 1983:Q1 to 2001:Q4. We obtain the cyclical component of this

[5] In the calibrated version of the model studied below, the assumed devaluation rule produces an unconditional standard deviation of the devaluation rate of 29 percent per year. The average standard deviation of the devaluation rate across the 70 countries included in figure 1 is 35 percent.

Table 1: Calibration

Parameter	Value	Description
γ	0.99	Degree of downward nominal wage rigidity
σ	2	Inverse of intertemporal elasticity of consumption
y^T	1	Steady-state tradable output
$\bar{h}$	1	Labor endowment
a	0.26	Share of tradables
ξ	0.5	Elasticity of substitution between tradables and nontradables
α	0.75	Labor share in nontraded sector
β	0.85	Quarterly subjective discount factor
r^*	0.01	World interest rate (quarterly)
θ	0.0385	Probability of reentry
δ_1	-0.35	parameter of output loss function
δ_2	0.4403	parameter of output loss function
ρ	0.9317	serial correlation of $\ln y_t^T$
σ_μ	0.037	std. dev. of innovation μ_t
		Discretization of State Space
n_y	200	Number of output grid points (equally spaced in logs)
n_d	200	Number of debt grid points (equally spaced)
n_w	125	Number of wage grid points (equally spaced in logs)
$[\underline{y}^T, \overline{y}^T]$	[0.6523,1.5330]	traded output range
$[\underline{d}, \overline{d}]^{float}$	[0,1.5]	debt range under optimal float
$[\underline{d}, \overline{d}]^{peg}$	[-1,1.25]	debt range under peg
$[\underline{w}, \overline{w}]^{peg}$	[1.25,4.25]	wage range under peg

time series by removing a quadratic trend.[6] The OLS estimate of the AR(1) process (15) yields $\rho = 0.9317$ and $\sigma_\mu = 0.037$. Following Chatterjee and Eyigungor (2012), we set $r^* = 0.01$ per quarter and $\theta = 0.0385$. The latter value implies an average exclusion period of about 6.5 years. Following these authors, we assume that the output loss function takes the form

$$L(y_t^T) = \max\left\{0, \delta_1 y_t^T + \delta_2 (y_t^T)^2\right\}.$$

We set $\delta_1 = -0.35$ and $\delta_2 = 0.4403$. We calibrate β, the subjective discount factor, at 0.85. The latter three parameter values imply that under the optimal policy the average debt to traded GDP ratio in periods of good financial standing is 60 percent per quarter, that the frequency of default is 2.6 times per century, and that the average output loss is 7 percent per year conditional on being in financial autarky. The predicted average frequency of default is in line with the Argentine experience since the late 19th century (see Reinhart et al., 2003). The implied average output loss concurs with the estimate reported by Zarazaga (2012) for the Argentine default of 2001. The implied debt-to-traded-output ratio is in line with existing default models in the Eaton-Gersovitz tradition, but below the debt levels observed in Argentina since the 1970s.

The assumed value of β is low compared to the values used in models without default, but not uncommon in models à la Eaton-Gersovitz (see, for example, Mendoza and Yue, 2012). In section 6.3 we consider values of β of 0.95 and 0.98 and show that the prediction of a Twin Ds phenomenon is robust to these changes. All other things equal, increasing β lowers the predicted default frequency. One way to match the observed default frequency without having to set β at a low value is to incorporate long-maturity debt. We pursue this alternative in section 6.1. The predicted dynamics of the model around default episodes (and in particular the model's predictions regarding the Twin Ds phenomenon) are similar in the model with one-period debt and a low β and in the model with long-maturity debt and a high value of β. The reason why we do not to adopt the long-maturity debt specification as the baseline is that the model with long-maturity debt is computationally more complex, especially in the case of a currency peg, which we analyze in the next section.

We approximate the equilibrium by value function iteration over a discretized state space. We assume 200 grid points for tradable output and 200 points for debt. The transition probability matrix of tradable output is computed using the simulation approach proposed by Schmitt-Grohé and Uribe (2009).

[6]The choice of a quadratic detrending method is motivated by the fact that the log of traded output in Argentina appears to grow faster starting in the 1990s. The results of the paper are robust to removing a log-linear trend.

4.2 Equilibrium Dynamics Around A Typical Default Episode

We wish to numerically characterize the behavior of key macroeconomic indicators around a typical default event. To this end, we simulate the model under optimal policy for 1.1 million quarters and discard the first 0.1 million quarters. We then identify all default episodes. For each default episode we consider a window that begins 12 quarters before the default date and ends 12 quarters after the default date. For each macroeconomic indicator of interest, we compute the median period-by-period across all windows. The date of the default is normalized to 0.

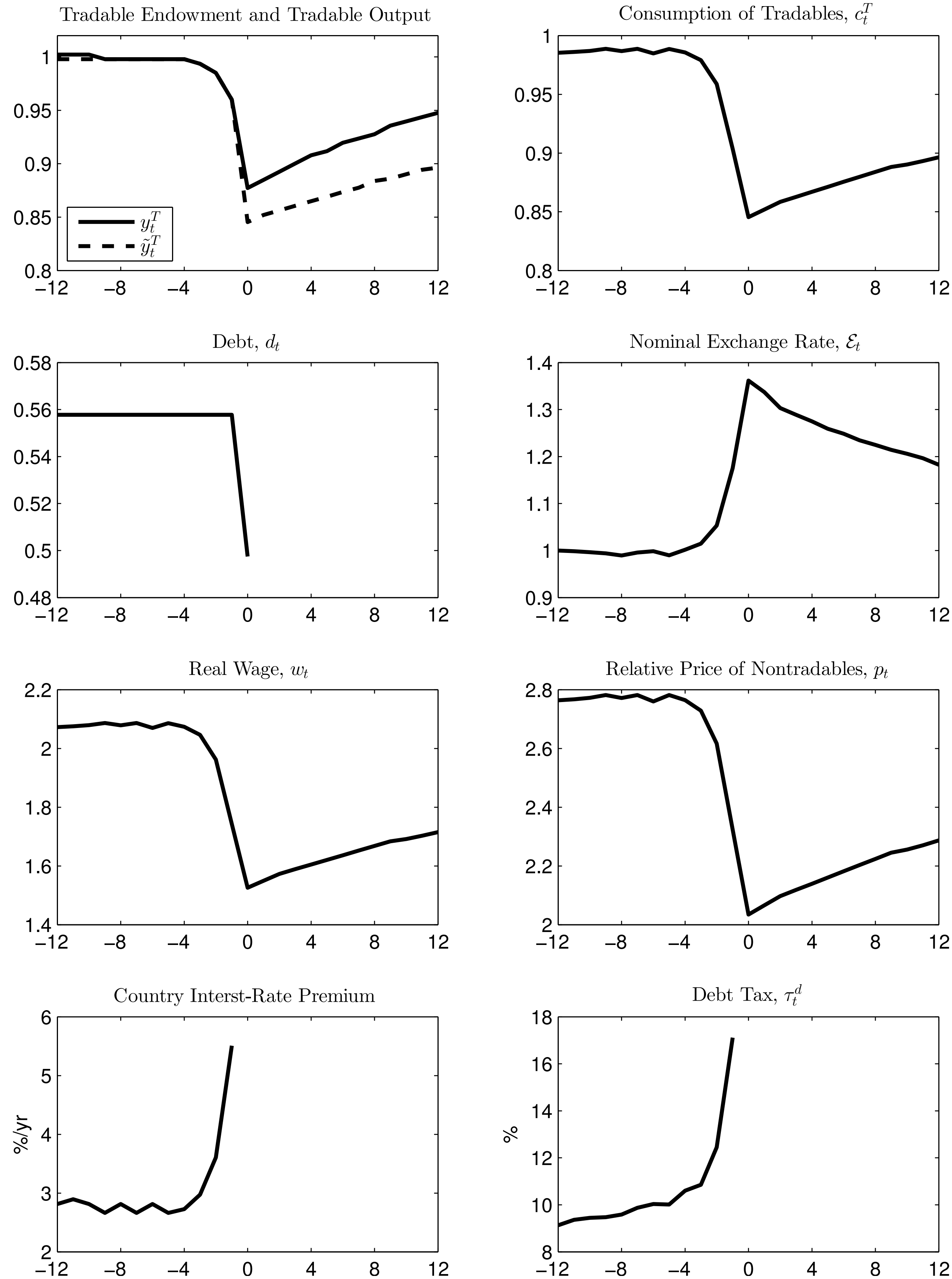
Figure 2 displays the dynamics around a typical default episode. The model predicts that optimal defaults occur after a sudden contraction in tradable output. As shown in the upper left panel, y_t^T is at its mean level of unity until three quarters prior to the default. Then a string of three negative shocks drives y_t^T 12 percent (or 1.3 standard deviations) below trend.[7] At this point (period 0), the government finds it optimal to default, triggering a loss of output $L(y_t^T)$, as shown by the difference between the solid and the broken lines in the upper left panel. After the default, tradable output begins to recover. Thus, the period of default coincides with the trough of the contraction in the tradable endowment, y_t^T. The same is true for GDP measured in terms of tradables. Therefore, the model captures the empirical regularity regarding the cyclical behavior of output around default episodes identified by Levy-Yeyati and Panizza (2011), according to which default marks the end of a contraction and the beginning of a recovery.

As can be seen from the right panel of the top row of the figure, the model predicts that the country does not smooth out the temporary decline in the tradable endowment. Instead, the country sharply adjusts the consumption of tradables downward, by about 14 percent. The contraction in traded consumption is actually larger than the contraction in traded output so that the trade balance (not shown) improves. In fact, the trade balance surplus is large enough to generate a slight decline in the level of external debt. These dynamics seem at odds with the quintessential dictum of the intertemporal approach to the balance of payments according to which countries should finance temporary declines in income by external borrowing. The country deviates from this prescription because foreign lenders raise the interest rate premium prior to default. This increase in the cost of credit discourages borrowing and induces agents to postpone consumption.

Both the increase in the country premium and the contraction in tradable output in

[7] One may wonder whether a fall in traded output of this magnitude squares with a default frequency of only 2.6 per century. The reason why it does is that it is the sequence of output shocks that matters. The probability of traded output falling from its mean value to 1.3 standard deviations below mean in only three quarters is much lower than the unconditional probability of traded output being 1.3 standard deviations below mean.

Figure 2: A Typical Default Episode Under Optimal Exchange-Rate Policy



the quarters prior to default cause a negative wealth effect that depresses the desired consumption of nontradables. In turn the contraction in the demand for nontradables puts downward pressure on the price of nontradables. However, firms in the nontraded sector are reluctant to cut prices given the level of wages, for doing so would generate losses. Thus, given the real wage, the decline in the demand for nontradables would translate into involuntary unemployment. In turn, unemployment would put downward pressure on nominal wages. However, due to downward nominal wage rigidity, nominal wages cannot decline to a point consistent with clearing of the labor market. To avoid unemployment, the government finds it optimal to devalue the currency sharply by about 35 percent (see the right panel on row 2 of figure 2). The devaluation lowers real wages (left panel of row 3 of the figure) which fosters employment, thereby preventing that a crisis that originates in the external sector spreads into the nontraded sector. The model therefore captures the Twin Ds phenomenon as an equilibrium outcome.

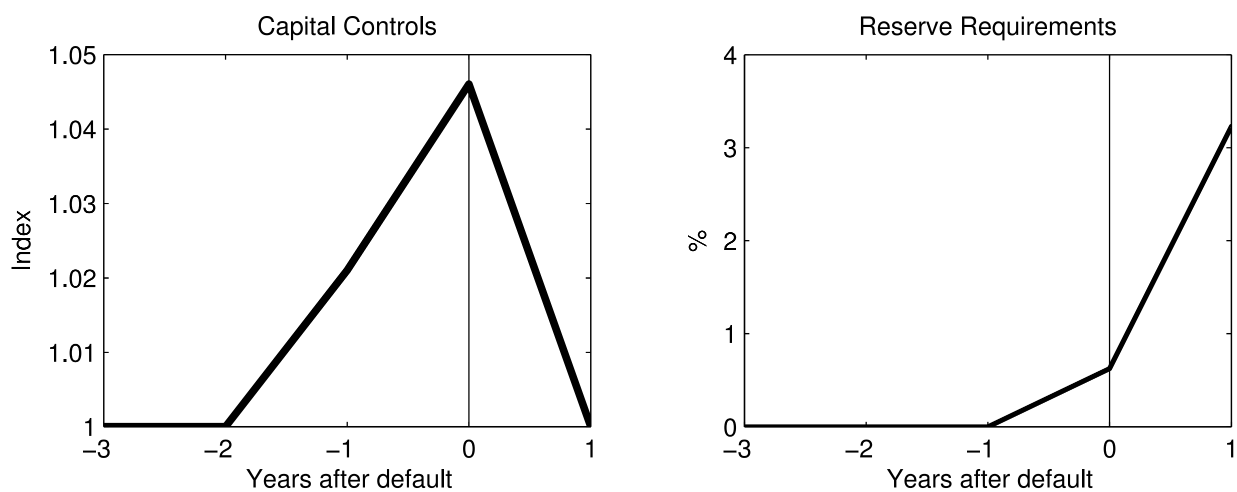
The large nominal exchange-rate depreciation that accompanies default is associated with a sharp real depreciation of equal magnitude, as shown by the collapse in the relative price of nontradables (see the right panel on the third row of figure 2). The fact that the nominal and real exchange rates decline by the same magnitude may seem surprising in light of the fact that nominal product prices are fully flexible. Indeed, the nominal price of nontradables remains stable throughout the crisis, which may convey the impression that nominal prices in the nontraded sector are rigid. The reason why firms find it optimal not to change nominal prices is that the devaluation reduces the real labor cost inducing firms to cut real prices. In turn, the decline in the real price of nontradables is brought about entirely by an increase in the nominal price of tradables (i.e., by the nominal devaluation). The predicted stability of the nominal price of nontradables is in line with the empirical findings of Burstein, Eichenbaum, and Rebelo (2005) who report that the primary force behind the observed large depreciation of the real exchange rate that occurred after the large devaluations in Argentina (2002), Brazil (1999), Korea (1997), Mexico (1994), and Thailand (1997) was the slow adjustment in the nominal prices of nontradable goods.

Finally, the bottom right panel of figure 2 shows that the government increases the tax on debt prior to the default from 9 to 17 percent. It does so as a way to make private agents internalize an increased sensitivity of the interest rate premium with respect to debt. The debt elasticity of the country premium is larger during the crisis because foreign lenders understand that the lower is output the higher the incentive to default, as the output loss, that occurs upon default, $L(y_t^T)$, decreases in absolute and relative terms as y_t^T falls.

The predicted increase in the debt tax around the typical default episode is implicitly present in every default model à la Eaton-Gersovitz. However, because the literature has limited attention to economies in which consumption, borrowing, and default decisions are all centralized, such taxes never surface. By analyzing the decentralized version of the Eaton-Gersovitz economy, the present analysis makes their existence explicit.

It follows that the behavior of debt taxes around default provides a dimension, distinct from the Twin Ds phenomenon, along which one can assess the plausibility of the predicted default dynamics. The variable τ_t^d, which in the model abstractly refers to a tax on debt, can take many forms in practice. Here, we examine two prominent ones, namely, capital control taxes and reserve requirements. The first measure is based on annual data on a capital control index constructed by Fernández et al. (2015). The index covers the period 1995 to 2011 for 91 countries. We combine this data with the default dates used in figure 1. The intersection of the data sets on capital controls and default dates yields 22 default episodes in 17 countries. The left panel of figure 3 displays the median behavior of the capital control index starting three years prior to the default date. For each default episode, the capital control index is normalized to unity in year -3. The figure shows that on average countries tighten capital controls as they move closer to default.

Figure 3: Capital Controls and Reserve Requirements Around Default: Empirical Evidence



Source. Own calculations based on data on capital controls from Fernández et al. (2015) and on reserve requirements from Federico et al. (2014).

The second empirical measure of borrowing restrictions we examine comes from a dataset on reserve requirements produced by Federico, Végh, and Vuletin (2014). The dataset contains quarterly observations on various measures of legal reserve requirements for 52 countries (15 industrialized and 37 emerging) covering the period 1970 to 2011. Of the 52 countries in the dataset, Federico et al. classify 30 as active users of reserve requirements as a macro prudential policy instrument. We cross the reserve requirement data for active users with the default dates used in figure 1. This step delivers reserve requirement data for 14 default episodes in 8 different countries. The right panel of figure 3 displays the median change in reserve requirements relative to year -3. The figure shows that, on average, defaults are accompanied by a tightening of reserve requirements. Taken together, the empirical evidence examined here provides support for the predictions of the model above and beyond its ability to capture the Twin Ds phenomenon.

5 Default And Unemployment Under Fixed Exchange Rates

The analysis of optimal default under fixed exchange rates is of interest because sovereign debt crises have been observed in the context of currency pegs or monetary unions. Prominent recent examples are countries in the periphery of Europe, such as Greece and Cyprus, in the aftermath of the global contraction of 2008. Formally, we now assume that

$$\epsilon_t = 1. \tag{33}$$

This policy specification can be interpreted either as a currency peg or as a monetary union. We assume that the government sets the default and debt taxation policies in an optimal fashion.

Definition 3 (Peg-Constrained Optimal Equilibrium) *An optimal-policy equilibrium under a currency peg is a set of processes* $\{c_t^T, h_t, w_t, d_{t+1}, \lambda_t, q_t^d, \tau_t^d, q_t, I_t\}_{t=0}^{\infty}$ *that maximizes*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t))) \tag{28}$$

subject to (18)-(23), (25), (27),

$$w_t \geq \gamma w_{t-1}, \tag{34}$$

$$(h_t - \bar{h})(w_t - \gamma w_{t-1}) = 0, \tag{35}$$

and to the constraint that if $I_{t-1} = 1$*, then* I_t *is an invariant function of* y_t^T*,* d_t*, and* w_{t-1}*, and if* $I_{t-1} = 0$*, then* $I_t = 0$ *except when reentry to credit markets occurs exogenously, and the no-Ponzi-game debt limit, given the initial conditions* d_0*,* w_{-1}*, and* I_{-1}*.*

Note that now the default decision depends not only on y_t^T and d_t, as in the case in which the devaluation rate was a policy instrument available to the government, but also on the past real wage w_{t-1}. This is because, under a currency peg, the competitive equilibrium conditions (i.e., the constraints faced by the policy planner) always include the past wage. Consequently, by equation (27) the price of debt, q_t, depends on the triplet (y_t^T, d_{t+1}, w_t).

Our strategy to characterize the peg-constrained optimal-policy equilibrium is again to consider a less constrained maximization problem and then show that the solution to this problem also satisfies the constraints of the peg-constrained optimal-policy problem listed in definition 3. The less constrained problem consists in dropping conditions (20)-(22) and (35) from the set of constraints in definition 3 and choosing processes $\{c_t^T, h_t, w_t, d_{t+1}, q_t, I_t\}$ to maximize the utility function (28). To see that the solution to this less restrictive problem satisfies the constraints dropped from the definition of the optimal-policy equilibrium, set λ_t to satisfy (20). If $I_t = 1$, the set q_t^d to satisfy (22) and set τ_t^d to satisfy (21). If $I_t = 0$, then, by the convention (17) $\tau_t^d = 0$, and set q_t^d to satisfy (21).

It remains to show that (35) is also satisfied. The proof is by contradiction. Suppose, contrary to what we wish to show, that the solution to the less constrained problem implies $h_t < \bar{h}$ and $w_t > \gamma w_{t-1}$ at some date $t' \geq 0$. Consider now a perturbation to the allocation that solves the less constrained problem consisting in a small increase in hours at time t' from $h_{t'}$ to $\tilde{h}_{t'}$, where $h_{t'} < \tilde{h}_{t'} \leq \bar{h}$. Clearly, this perturbation does not violate the resource constraint (18), since hours do not enter in this equation. From (23) we have that the real wage falls to $\tilde{w}_{t'} \equiv \frac{A_2(c_{t'}^T, F(\tilde{h}_{t'}))}{A_1(c_{t'}^T, F(\tilde{h}_{t'}))} F'(\tilde{h}_{t'}) < w_{t'}$. Because A_1, A_2, and F' are continuous functions, expression (34) is satisfied provided the increase in hours is sufficiently small. In period $t'+1$, restriction (34) is satisfied because $\tilde{w}_{t'} < w_{t'}$. We have therefore established that the perturbed allocation satisfies the restrictions of the less constrained problem. Finally, the perturbation is clearly welfare increasing because it raises the consumption of nontradables in period t' without affecting the consumption of tradables in any period or the consumption of nontradables in any period other than t'. It follows that an allocation that does not satisfy the slackness condition (35) cannot be a solution to the less constrained problem. This completes the proof that the allocation that solves the less constrained problem is also feasible in the optimal-policy problem. It follows that the allocation that solves the less constrained problem is indeed the optimal allocation.

We now pose the peg-constrained optimal-policy equilibrium in recursive form. This representation is of great convenience for the quantitative analysis that follows. For a government in good financial standing at the beginning of period t, the value of continuing to service its debt, denoted $v^c(y_t^T, d_t, w_{t-1})$, is given by

$$v^c(y_t^T, d_t, w_{t-1}) = \max_{\{c_t^T, d_{t+1}, h_t, w_t\}} \left\{ U\left(A\left(c_t^T, F(h_t)\right)\right) + \beta \mathbb{E}_t v^g(y_{t+1}^T, d_{t+1}, w_t) \right\} \tag{36}$$

subject to

$$c_t^T + d_t = y_t^T + q(y_t^T, d_{t+1}, w_t)d_{t+1}, \tag{37}$$

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{23}$$

$$w_t \geq \gamma w_{t-1}, \tag{34}$$

$$h_t \leq \bar{h}, \tag{25}$$

where $v^g(y_t^T, d_t, w_{t-1})$ denotes the value function associated with entering period t in good financial standing, for an economy with tradable output y_t^T, external debt d_t, and past real wage w_{t-1}.

The value of being in bad financial standing in period t, denoted $v^b(y_t^T, w_{t-1})$, is given by

$$v^b(y_t^T, w_{t-1}) = \max_{\{h_t, w_t\}} \left\{ U\left(A\left(y_t^T - L(y_t^T), F(h_t)\right)\right) + \beta \mathbb{E}_t \left[\theta v^g(y_{t+1}^T, 0, w_t) + (1-\theta) v^b(y_{t+1}^T, w_t)\right] \right\}, \tag{38}$$

subject to

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{23}$$

$$w_t \geq \gamma w_{t-1}, \tag{34}$$

$$h_t \leq \bar{h}. \tag{25}$$

The value of being in good standing in period t is given by

$$v^g(y_t^T, d_t, w_{t-1}) = \max\left\{v^c(y_t^T, d_t, w_{t-1}), v^b(y_t^T, w_{t-1})\right\}. \tag{39}$$

Note that now the values of default, continuation, and good standing, $v^b(y_t^T, w_{t-1})$, $v^c(y_t^T, d_t, w_{t-1})$, and $v^g(y_t^T, d_t, w_{t-1})$, respectively, depend on the past real wage, w_{t-1}. This is because under downward nominal wage rigidity and a suboptimal exchange-rate policy, the past real wage, by placing a lower bound on the current real wage, can prevent the labor market from clearing, thereby causing involuntary unemployment and suboptimal consumption of nontradable goods.

Under a currency peg, the default set is defined as

$$D(d_t, w_{t-1}) = \left\{y_t^T : v^b(y_t^T, w_{t-1}) > v^c(y_t^T, d_t, w_{t-1})\right\}. \tag{40}$$

The price of debt must satisfy the condition that the expected return of lending to the domestic country equals the opportunity cost of funds. Formally,

$$\frac{\text{1-Prob}\left\{y_{t+1}^T \in D(d_{t+1}, w_t)\right\}}{q_t} = 1 + r^*. \tag{41}$$

Next, we characterize numerically the dynamics implied by the model under a currency peg. The calibration of the model is as shown in table 1. Relative to the case of optimal devaluations, the equilibrium under a currency peg features an additional state variable, namely the past real wage, w_{t-1}. We discretize this state variable with a grid of 125 points, equally spaced in logs, taking values between 1.25 and 4.25. This additional endogenous state variable introduces two computational difficulties. First, it significantly expands the number of points in the discretized state space, from 40 thousand to 5 million. Second, it introduces a simultaneity problem that can be a source of nonconvergence of the numerical algorithm. The reason is that the price of debt, $q(y_t^T, d_{t+1}, w_t)$, depends on the current wage, w_t. At the same time, the price of debt determines consumption of tradables, which, in turn, affects employment and the wage rate itself. To overcome this source of nonconvergence, we develop a procedure to find the exact policy rule for the current wage given the pricing function $q(\cdot, \cdot, \cdot)$ for each possible debt choice d_{t+1}. With this wage policy rule in hand, the debt policy rule is found by value function iteration. This step delivers a new debt pricing function, which is then used in the next iteration.

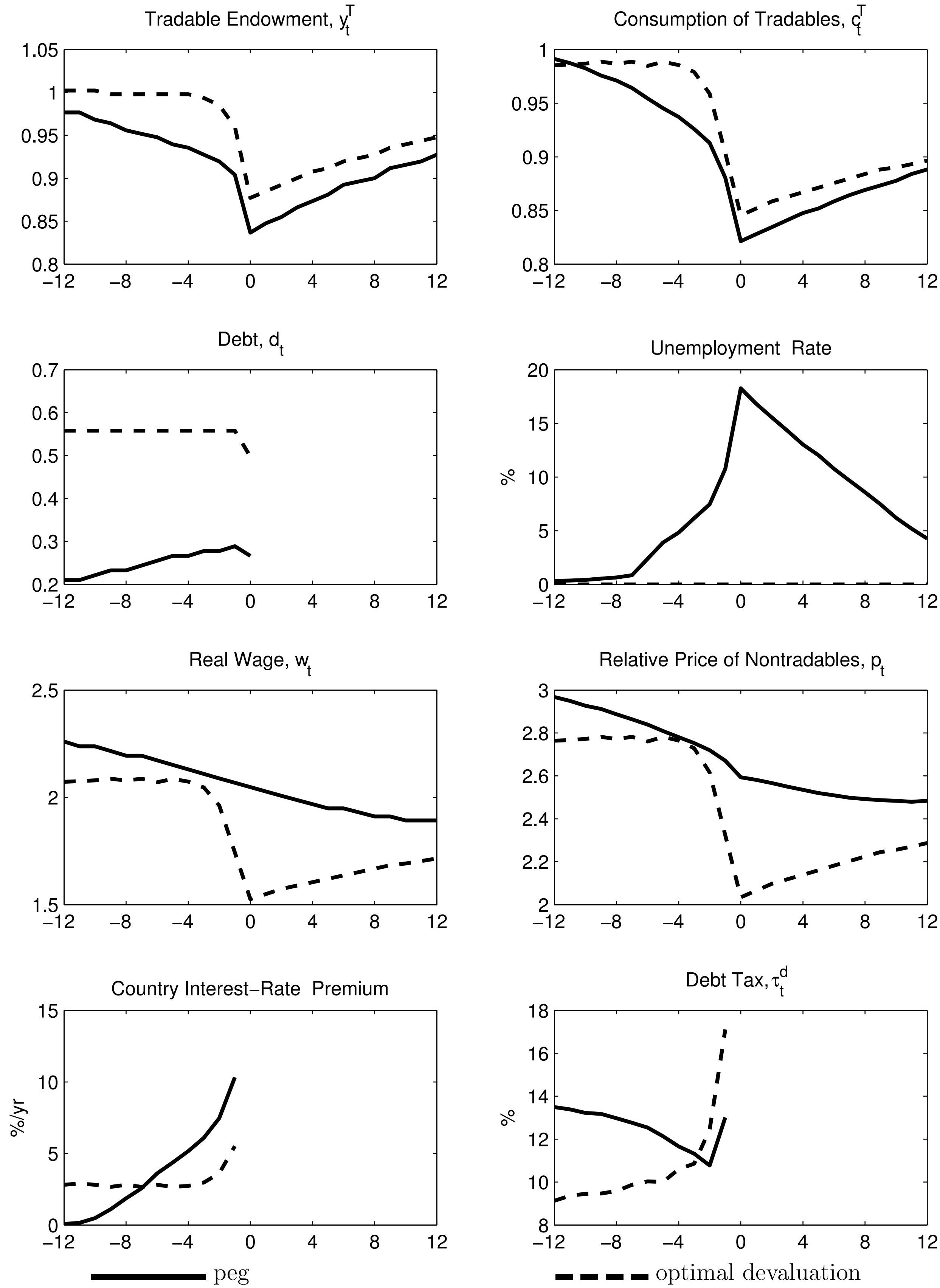
5.1 Typical Default Episodes With Fixed Exchange Rates

Figure 4 displays with solid lines the model dynamics around typical default episodes. The typical default episode is constructed in the same way as in the case of optimal devaluations. To facilitate comparison, figure 4 reproduces with broken lines the typical default dynamics under the optimal devaluation policy.

The top panels of the figure show that, as in the case of optimal exchange-rate policy, default occurs after a string of negative output shocks and a significant contraction in tradable consumption. However, unlike the case of optimal devaluation policy, the contraction in aggregate demand leads to massive involuntary unemployment, which reaches 20 percent in the period of default. Involuntary unemployment is caused by a failure of real wages to decline in a context of highly depressed aggregate demand (see the left panel of row 3 of figure 4). In turn, the downward rigidity of the real wage is due to the fact that nominal wages are downwardly rigid and that the nominal exchange rate is fixed.

The right panel on the third row of figure 4 displays the behavior of the relative price

Figure 4: A Typical Default Episode Under A Currency Peg

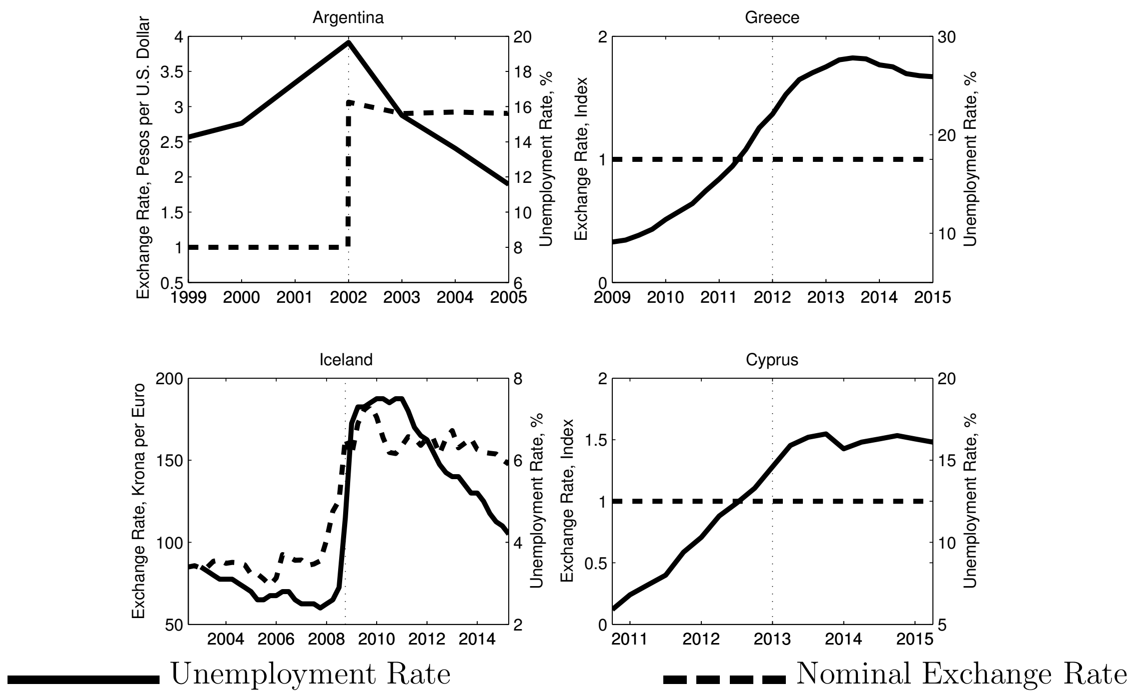


of nontradables. A fall in this variable means that the real exchange rate depreciates as tradables become more expensive relative to nontradables. Under the optimal policy, the real exchange rate depreciates sharply around the default date, inducing agents to switch expenditure away from tradables and toward nontradables. This redirection of aggregate spending stimulates the demand for labor (since the nontraded sector is labor intensive) and prevents the emergence of involuntary unemployment. Under the currency peg, by contrast, the real exchange rate depreciates insufficiently, inducing a much milder expenditure switch toward nontradables, and thus failing to avoid unemployment. The reason why the relative price of nontradables is reluctant to decline under the peg is that real wages, and hence the labor cost faced by firms, stay too high due to the combination of downward nominal wage rigidity and a currency peg.

As in the case of optimal exchange-rate policy, the default takes place in the context of an increase in the debt tax. This tightening in borrowing conditions aims to induce private borrowers to internalize the heightened sensitivity of the country interest rate to the level of debt.

One prediction of the model highlighted by the preceding analysis is that, all other things equal, defaults are characterized by larger recessions when they take place under fixed exchange rates than when they are accompanied by a devaluation. It is natural to ask whether this prediction is borne out in the data. One difficulty in addressing this question is that there are few cases in which default takes place in the context of a fixed exchange rate. The typical default falls into the Twin Ds category. A second difficulty is that the size of the contraction around default depends not only on the exchange-rate regime, but also, among other factors, on the size of the shock that triggers the default. So, in principle, a default event with devaluation could be associated with a larger recession than a default event with fixed exchange rates if the shock that triggered the former is sufficiently larger than the one that caused the latter. One way to at least partially control for this factor is to study default events with and without devaluation that happened around the same time and that were conceivably caused by a common set of external shocks. The Great Contraction of 2008 provides a suitable natural environment for this purpose. Following this global crisis, there have been two defaults that were not followed by a devaluation, namely Greece in 2012 and Cyprus in 2013, and one that was followed by a devaluation, namely Iceland in 2009. In addition, we include in the comparison the 2002 Argentine default because it is a recent well-studied event and because our model was calibrated using some long-run regularities of the Argentine economy. Figure 5 displays with a solid line the unemployment rate and with a broken line the nominal exchange rate around the default date, which is indicated with a vertical dotted line. In all four cases, default was associated with rising levels of

Figure 5: Default, Devaluation, and Unemployment: Argentina, Cyprus, Greece, and Iceland



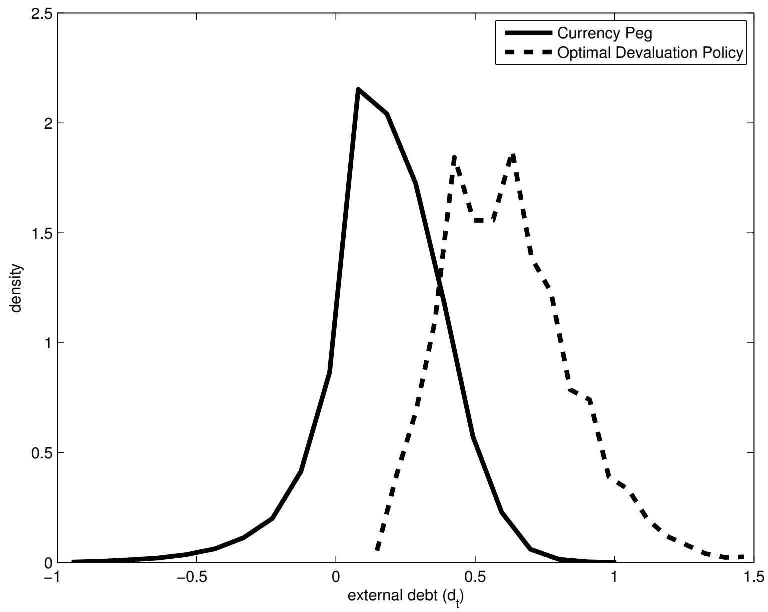
Note. Vertical line indicates the year of default. Own calculations based on data from INDEC (Argentina), EuroStat, and the Central Bank of Iceland.

unemployment. But the unemployment dynamics post default were different across peggers and nonpeggers. In Argentina and Iceland, the default cum devaluation was followed by an improvement in unemployment. By contrast, Greece and Cyprus, both of which stayed in the eurozone post default, experienced no decline in unemployment. We view this evidence as consistent with the predictions of the model that devaluation around default reduces unemployment.

5.2 Debt Sustainability Under A Currency Peg

Under a currency peg the economy can support less debt than under the optimal devaluation policy. Figure 6 displays with a solid line the distribution of external debt under a currency peg, conditional on the country being in good financial standing. For comparison, the figure also displays, with a broken line, the distribution of debt under the optimal devaluation policy. The median debt falls from 0.6 (60 percent of tradable output) under the optimal devaluation policy to 0.2 (20 percent of tradable output) under a currency peg. This reduced debt capacity is a consequence of the fact that, all other things equal, the benefits from

Figure 6: Distribution of External Debt



Note. Debt distributions are conditional on being in good financial standing.

defaulting are larger under a currency peg than under optimal devaluation policy. The reason is that under a currency peg, default has two benefits. One is to spur the recovery in the consumption of tradables, since the repudiation of external debt frees up resources otherwise devoted to servicing the external debt. The second, related to the first, is to lessen the unemployment consequences of the external crisis. Recall that in equilibrium c_t^T is a shifter of the demand for labor (see equation 23). The first benefit is also present under optimal devaluation policy and is the one stressed in real models of default in the Eaton-Gersovitz tradition . But the second is not, for the optimal devaluation policy, by itself, can bring about the first-best employment outcome.

The model predicts that under fixed-exchange rates, the country on average defaults twice per century. This default frequency is slightly lower than that predicted under the optimal exchange rate policy, which was targeted in the calibration to be 2.6 times per century. This result may be surprising in light of the fact that ex ante peggers have a stronger incentive to default. The explanation is that the higher incentive to default under a peg implies a steeper supply of funds. This, in turn, induces the country to borrow less in the stochastic steady state. And with a lower external debt, the country has a reduced need to default. In general, the model does not predict a sharp difference in the frequency of default across peggers and optimal floaters.

It is of interest to contrast empirically the model's prediction that, all other things equal, peggers are able to support less debt in equilibrium than countries with optimal monetary policy, as shown in figure 6. To appropriately interpret this prediction it is important to keep in mind that it is valid in the stochastic steady state, that is, in a situation in which the economy has been under the same exchange rate regime for a long period of time and is expected to continue in the same exchange rate regime in the future. In the data, however, currency pegs rarely survive default. Thus, one does not get to observe the steady-state distribution of debt for countries that peg their currency. Instead the debt dynamics one observes for peggers are most likely contaminated by transitional effects. A case in point is the observed increase in external debt in the periphery of Europe following the adoption of the Euro in 2000. One piece of evidence suggesting that this increase in external debt was transitional in nature is the fact that at the time, the prevailing view was that the increase in debt was driven by the expectation that income levels in this group of relatively poor European countries would converge toward those observed in the core EU countries (see, for example, Blanchard and Giavazzi, 2002). Under this view, transitional effects linked to expected growth dynamics blur the effect stemming purely from the switch to a fixed exchange-rate regime. In this regard, it is of interest to contrast the dynamics of external debt in the periphery of Europe following the adoption of a currency peg in 2000 with those of Ecuador, a country that adopted a unilateral currency union with the United States in the same year, but that was not expected to converge to the country whose currency it was adopting. To this end, we calculate the net external debt position as a fraction of GDP of Ecuador and the mean across the GIPS (Greece, Ireland, Portugal, and Spain) countries from 2000 to 2010, using data on net foreign assets from the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). We find that during this period the GIPS countries display a fast accumulation of external debt from 20 to 100 percent of GDP, whereas Ecuador's external debt fell from 80 percent to 20 percent of GDP. We view the conditions in Ecuador surrounding its adoption of the U.S. dollar as more compatible with the theoretical environment laid out in the present paper, because arguably, there was less of an expectation that income per capita in Ecuador would converge to that of the United States.

6 Sensitivity Analysis

This section extends the model to allow for long-maturity debt and imperfect pass-through. It also analyzes the robustness of the central results of the paper to increasing the value of the subjective discount factor, β.

6.1 Long-Maturity Debt

The baseline model assumes that debt carries a maturity of one period. In this section we present a version of the model with long-maturity debt. We wish to show that the main result of the paper, namely, that the model predicts the Twin Ds phenomenon as the optimal outcome, is robust to this modification.

The specification of long-term debt follows Chatterjee and Eyigungor (2012). Assume that bonds have a random maturity. Specifically, with probability $\eta \in [0, 1]$ bonds mature next period and pay out one unit of the tradable consumption good. With probability $1 - \eta$ bonds do not mature and pay a coupon equal to $z > 0$ units of tradables. The country is assumed to hold a portfolio with a continuum of this type of bond. The realization of maturity is independent across bonds. Hence, if the country has d_t units of debt outstanding, a share η will mature each period with certainty and the remaining share $1 - \eta$ will not. The nonmaturing bonds trade at the price q_t per unit. Because a newly-issued bond is indistinguishable from an existing bond that did not mature, the ex-coupon price of old bonds and new bonds must be equal. If the debtor does not default, d_t units of debt pay $[\eta + (1 - \eta)(z + q_t)]d_t$ units of tradable consumption. If the debtor defaults, the bond pays zero. Absent default, the expected maturity of this type of bond is $1/\eta$ periods. Thus, the random-maturity model allows for bonds of arbitrary maturity. Furthermore, it nests the perpetuity model of debt (e.g., Hatchondo and Martínez, 2009) as a special case (for a proof, see Uribe and Schmitt-Grohé, 2015).

The main difference between the models with long-term and one-period debt is that long-term debt results in a state-contingent payoff, which may provide hedging against income risk to the borrower. Specifically, the payoff on the long-term bond, $\eta+(1-\eta)(z+q_t)$ depends on q_t, which is state contingent. In particular, in periods of low endowment, q_t is likely to be low, resulting in an ex-post low interest rate paid by the borrower. Because periods of low income are associated with low consumption, the long-term bond provides insurance against income risk. By contrast, the payoff on a one-period bond is unity and hence nonstate contingent, providing no insurance against income risk. Therefore, one should expect that all other things equal, the borrower will hold more debt if debt is long term rather than short term.

To embed this asset structure into the decentralized economy with downward nominal wage rigidity presented in section 2 consider first the household's problem. The household's sequential budget constraint is now given by

$$P_t^T c_t^T + P_t^N c_t^N + P_t^T \left[\eta + (1 - \eta)(z + q_t^d)\right] d_t = P_t^T \tilde{y}_t^T + W_t h_t + (1-\tau_t^d) P_t^T q_t^d d_{t+1} + F_t + \Phi_t, \quad (42)$$

where q_t^d now denotes the domestic price of long-term debt in period t. The optimality condition for the choice of debt becomes

$$(1-\tau_t^d)q_t^d\lambda_t = \beta\mathbb{E}_t\lambda_{t+1}\left[\eta+(1-\eta)(z+q_{t+1}^d)\right],$$

where, as before, λ_t/P_t^T denotes the Lagrange multiplier associated with the household's sequential budget constraint, now equation (42). All other optimality conditions associated with the household's problem are unchanged.

The firm's problem and the conditions characterizing the labor market are unaffected by the introduction of long-term debt. We continue to assume that in periods in which the country is in bad standing ($I_t = 0$), the government confiscates any payments of households to foreign lenders and returns the proceeds to households in a lump-sum fashion. The resulting sequential budget constraint of the government is

$$f_t = \tau_t^d q_t^d d_{t+1} + (1-I_t)[\eta+(1-\eta)(z+q_t^d)]d_t. \tag{43}$$

Consider now the participation constraint of foreign lenders. Let q_t denote the price of debt charged by foreign lenders. Then, q_t must satisfy the condition that the expected return of lending to the domestic country equal the opportunity cost of funds. Formally,

$$I_t\left[q_t - \frac{\mathbb{E}_t I_{t+1}[\eta+(1-\eta)(z+q_{t+1})]}{1+r^*}\right] = 0.$$

The market-clearing condition for traded goods takes the form

$$c_t^T = y_t^T - (1-I_t)L(y_t^T) + I_t\{q_t d_{t+1} - [\eta+(1-\eta)(z+q_t)]d_t\}.$$

A competitive equilibrium in the economy with long-term debt is a set of stochastic processes $\{c_t^T,\ h_t,\ w_t,\ d_{t+1},\ \lambda_t,\ q_t,\ q_t^d\}$ satisfying

$$c_t^T = y_t^T - (1-I_t)L(y_t^T) + I_t\{q_t d_{t+1} - [\eta+(1-\eta)(z+q_t)]d_t\}, \tag{44}$$

$$(1-I_t)d_{t+1} = 0, \tag{19}$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \tag{20}$$

$$(1-\tau_t^d)q_t^d\lambda_t = \beta\mathbb{E}_t\lambda_{t+1}[\eta+(1-\eta)(z+q_{t+1}^d)], \tag{45}$$

$$I_t(q_t^d - q_t) = 0, \tag{22}$$

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{23}$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{24}$$

$$h_t \leq \bar{h}, \tag{25}$$

$$(h_t - \bar{h}) \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0, \tag{26}$$

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}[\eta + (1-\eta)(z + q_{t+1})]}{1 + r^*} \right] = 0, \tag{46}$$

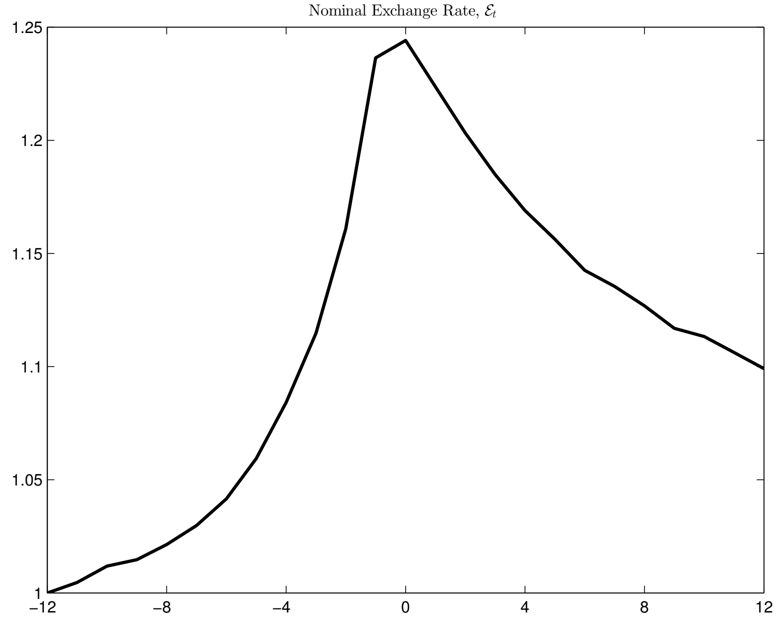
given processes $\{y_t^T, \epsilon_t, \tau_t^d, I_t\}$ and initial conditions w_{-1} and d_0.

Proposition 1 continues to hold. That is, when the government can choose ϵ_t and τ_t^d freely, stochastic processes $\{c_t^T, h_t, d_{t+1}, q_t\}$ can be supported as a competitive equilibrium if and only if they satisfy the subset of equilibrium conditions (19), (25), (44), and (46), given processes $\{y_t^T, I_t\}$ and the initial condition d_0. The proof mimics the one for Proposition 1, except that the proof that the Euler equation (45) holds must be modified. The reason is that now future values of q_t^d appear on the right-hand side of (45). We proceed as follows. In states in which the country is in good standing, set $q_t^d = q_t$. In states in which the country is in bad standing, set $q_t^d = q^{autarky}$, where $q^{autarky}$ is an arbitrary positive constant. Then, in any state, pick τ_t^d residually so as to satisfy the Euler equation (45). This is possible because at this point we know the processes I_t, λ_t, q_t^d, and q_t.

When ϵ_t and τ_t^d are unrestricted, an equilibrium under optimal policy is a set of processes $\{c_t^T,\ h_t,\ d_{t+1},\ q_t,\ I_t\}$ that maximizes the function (28) subject to (19), (25), (44), and (46), and to the constraint that if $I_{t-1} = 1$, then I_t is an invariant function of y_t^T and d_t and if $I_{t-1} = 0$, then $I_t = 0$ except when reentry to credit markets occurs exogenously. The set of processes must also satisfy the no-Ponzi-game debt limit. The initial values d_0 and I_{-1} are given.

Under optimal exchange-rate policy, full employment ($h_t = \bar{h}$) is optimal at all times. As in the case with one-period debt, this result follows directly from inspecting the constraints of the optimal policy problem. Similarly, by the arguments given in characterizing optimal policy under one-period debt, we obtain that the family of devaluation policies that support the optimal allocation is given by equation (30). It follows, in turn, that under the optimal exchange-rate policy the equilibrium is identical to that of the real economy with long-term debt studied in Chatterjee and Eyigungor (2012). This insight allows us to employ the following procedure to establish that the Twin Ds phenomenon obtains in equilibrium in the economy with long-maturity debt: First, compute the real allocation in the Chatterjee and Eyigungor (2012) economy. Associate the resulting process for c_t with

Figure 7: Optimal Devaluation Around the Typical Default Episode Under Long-Maturity Debt



c_t^T in the present model. Second, compute the process for the full-employment wage as $w_t^f = [\alpha(1-a)/a](c_t^T)^{1/\xi}$. Finally, use the optimal devaluation policy (32) to obtain the process for the optimal devaluation rate.

Figure 7 displays the behavior of the optimal nominal exchange rate around a typical default episode (the analogous to the right panel of the second row of figure 2) under the exact calibration used by Chatterjee and Eyigungor (2012). Specifically, following the notation in the present paper, we set $\beta = 0.95402$, $\delta_1 = -0.18819$, $\delta_2 = 0.24558$, $\rho = 0.948503$, $\sigma_\mu = 0.027092$, $r^* = 0.01$, $\sigma = 2$, $\eta = 0.05$, $z = 0.03$, and $\theta = 0.0385$. For the parameters that are particular to our model, we continue to use the values displayed in table 1, that is, $a = 0.26$, $\alpha = 0.75$, and $\xi = 0.5$. The assumed value of η implies that the average maturity of debt is 5 years. The figure shows that the government implements a large devaluation around the typical default episode. This prediction suggests that the Twin Ds phenomenon is robust to allowing for long-maturity debt.

6.2 Incomplete Exchange-Rate Pass Through

Thus far, we have assumed that the law of one price holds for tradable goods, $P_t^T = P_t^{T*}\mathcal{E}_t$. In this section, we will relax this assumption.

Continue to assume that P_t^{T*} is constant and normalized to unity. Let

$$\pi_t^T \equiv \frac{P_t^T}{P_{t-1}^T}$$

denote the domestic gross rate of inflation of tradable goods. Then, under the assumption of complete pass-through tradable inflation would equal the devaluation rate, that is, $\pi_t^T = \epsilon_t$. We introduce incomplete pass-through by imposing the following law of motion for π_t^T,

$$\pi_t^T = (\epsilon_t)^\eta \left(\pi_{t-1}^T\right)^{1-\eta} \tag{47}$$

with $\eta \in (0, 1]$. According to this expression, tradable prices display short-run deviations from the law of one price, in the sense that a one-percent devaluation in period t leads to an increase in the domestic price of tradables of η percent in period t, which is less than one percent. However, in the long run there is perfect pass-through in the sense that a one-percent devaluation in period t, all other things equal, leads asymptotically to a one-percent increase in the domestic price of tradables. The smaller is η the more incomplete is pass-through. The present formulation nests the case of perfect pass-through when $\eta = 1$.

The remaining elements of the model are unchanged. Then, we have that under incomplete pass-through a competitive equilibrium is a set of stochastic processes $\{c_t^T, h_t, w_t, d_{t+1}, \lambda_t, q_t, q_t^d, \pi_t^T\}$ satisfying

$$c_t^T = y_t^T - (1 - I_t)L(y_t^T) + I_t[q_t d_{t+1} - d_t], \tag{18}$$

$$(1 - I_t)d_{t+1} = 0, \tag{19}$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \tag{20}$$

$$(1 - \tau_t^d)q_t^d \lambda_t = \beta \mathbb{E}_t \lambda_{t+1}, \tag{21}$$

$$I_t(q_t^d - q_t) = 0, \tag{22}$$

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} = \frac{w_t}{F'(h_t)}, \tag{23}$$

$$h_t \leq \bar{h}, \tag{25}$$

$$I_t \left[q_t - \frac{\mathbb{E}_t I_{t+1}}{1 + r^*} \right] = 0, \tag{27}$$

$$\pi_t^T = (\epsilon_t)^\eta \left(\pi_{t-1}^T\right)^{1-\eta}, \tag{47}$$

$$w_t \geq \gamma \frac{w_{t-1}}{\pi_t^T}, \tag{48}$$

and

$$(h_t - \bar{h})\left(w_t - \gamma \frac{w_{t-1}}{\pi_t^T}\right) = 0, \tag{49}$$

given processes $\{y_t^T, \epsilon_t, \tau_t^d, I_t\}$ and initial conditions w_{-1}, d_0, and π_{-1}^T. One can readily establish that proposition 1 continues to hold under the present formulation of incomplete pass-through. This means that the Ramsey-optimal behavior of c_t^T, h_t, d_{t+1}, q_t, and I_t is the same as in the model with perfect pass-through.

The family of optimal devaluation policies now takes the form

$$\epsilon_t \geq \left[\gamma \frac{w_{t-1}}{w^f(c_t^T)} \left(\pi_{t-1}^T\right)^{\eta-1}\right]^{\frac{1}{\eta}}, \tag{50}$$

where the full-employment real wage, $w^f(c_t^T)$, continues to be given by $w^f(c_t^T) = \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h})$. The above expression shows that all other things equal, the more incomplete is pass-through (i.e., the smaller is η), the larger is the minimum devaluation required to maintain full employment in response to a contraction in c_t^T.

We continue to study the member of the family of optimal devaluation policies that fully stabilizes the nominal wage (i.e., the rule that implies that $W_t = W_{t-1}$ for all t). This policy rule now takes the form

$$\epsilon_t = \left[\frac{w_{t-1}}{w^f(c_t^T)} \left(\pi_{t-1}^T\right)^{\eta-1}\right]^{\frac{1}{\eta}}. \tag{51}$$

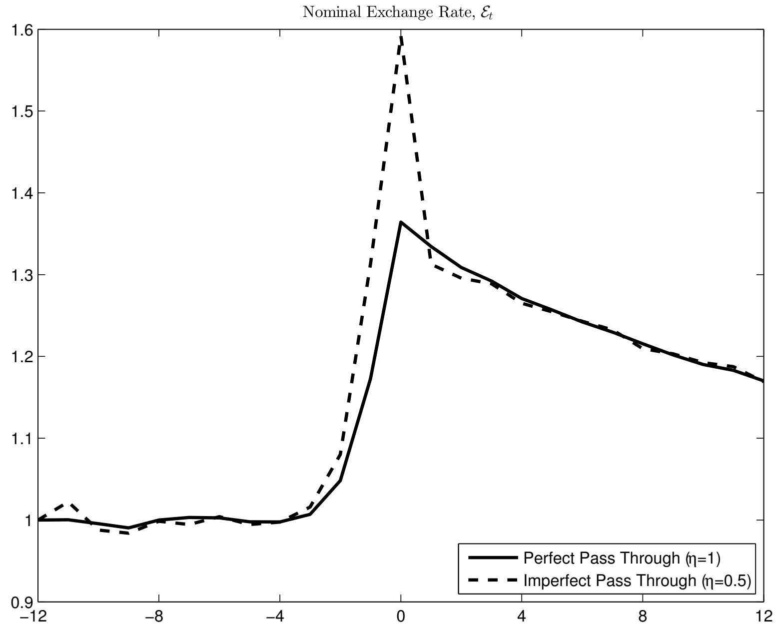
Because under full employment we have that $P_t^N F'(\bar{h}) = W_t$ and because under the specific optimal devaluation rule considered W_t is constant, it follows that P_t^N is also constant in equilibrium. So it continues to be true that even though the nominal price of nontradables is fully flexible, in equilibrium it behaves as if it was perfectly sticky.

Figure 8 compares the behavior of the level of the nominal exchange rate during a typical default episode in the economy with perfect pass-through (the solid line, reproduced from figure 2) and in the economy with imperfect pass-through (the broken line). In this latter case the parameter η takes the value 0.5. The figure shows that the nominal depreciation that takes place at the time of default is about twice as large under imperfect pass through than under perfect pass through. It follows that the Twin Ds phenomenon is more pronounced the lower the degree of pass-through.

6.3 Patience and the Twin Ds

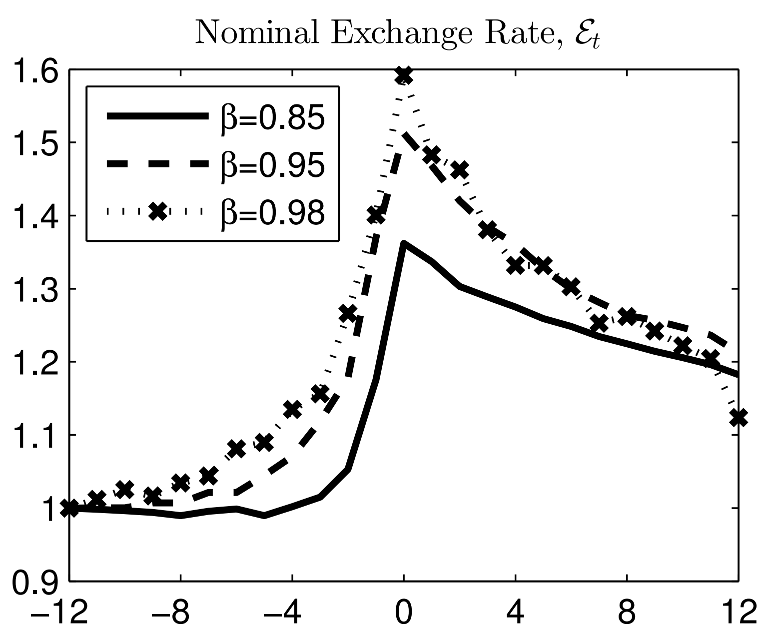
The calibration of the model features a value of β of 0.85. As mentioned earlier, this value is commonplace in the quantitative default literature, but low relative to the values used

Figure 8: The Nominal Exchange Rate During A Typical Default Episode Under Optimal Exchange-Rate Policy And Imperfect Pass-Through



in closed-economy business cycle studies. Here, we explore the sensitivity of the emergence of the Twin Ds as an optimal outcome to increasing the value of β. Figure 9 displays

Figure 9: Predicted Twin Ds And Patience



the predicted nominal exchange rate under optimal exchange-rate policy around the typical default episode for three values of β, 0.85, 0.95, and 0.98. In all cases, the model predicts that defaults are accompanied by large devaluations. In this sense, the Twin Ds prediction is robust to making households more patient. Indeed, the Twin Ds phenomenon is predicted to be more pronounced as β increases. The reason is that as agents become more patient, the costs of default (the output loss and financial autarky), which apply for a random period of time, have a larger present discounted value. As a result, it takes a deeper contraction in output for the country to choose to default. In turn, the larger the contraction, the larger the devaluation necessary to ensure that the real wage falls to the level that clears the labor market.

7 Conclusion

Sovereign defaults typically coincide with large devaluations of the domestic currency. In addition, the dynamics of the nominal exchange rate around defaults resemble more a one-time devaluation than a switch to a permanently higher rate of devaluation. For this reason, the typical devaluation around default does not seem to be driven by the objective of generating a higher stream of seignorage revenue. Furthermore, the fact that the majority of the default episodes observed since the 1970s involved economies in which much of the debt was either denominated in units of foreign currency or indexed discourages an explanation in which the chief objective of a devaluation is to deflate the real value of interest-bearing liabilities.

This paper proposes an explanation of the joint occurrence of default and devaluation in which the latter serves to correct a misalignment in relative prices. This explanation is motivated by the fact that in fixed exchange rate economies, contractions are characterized by a lack of downward adjustment in private nominal wages in spite of rising unemployment. A prominent example is the debt crisis in the periphery of Europe following the Great Contraction of 2008.

We formalize this explanation by embedding downward nominal wage rigidity into the Eaton-Gersovitz model of default. In this framework, default occurs in the context of highly depressed aggregate demand. In turn, weak demand for final products lowers the demand for labor, which puts downward pressure on real wages. In the absence of a devaluation, the required fall in the real wage necessitates a decline in the nominal wage. But this is ruled out by downward nominal wage rigidity. Thus, to avoid the emergence of large involuntary unemployment, the government chooses to combine the default with devaluation. For plausible calibrations of the model, the minimum devaluation rate consistent with full employment is found to be 35 to 40 percent.

By contrast, under a fixed exchange rate the government is unable to reduce the real value of wages by devaluing the domestic currency. Thus, default episodes are predicted to be accompanied by involuntary unemployment. Under plausible calibrations of the model, the unemployment rate increases by about 20 percentage points around the typical default.

Finally, the combination of nominal rigidities and a fixed exchange rate introduces an additional incentive to default into the Eaton-Gersovitz model. This incentive originates from the fact that the resources set free by default boost domestic demand and thus reduce slack in labor markets. Because of these elevated incentives to default, the model predicts that in the long run fixed-exchange-rate economies can support less external debt than economies with optimally floating rates.

Appendix

A.1 Proof of Proposition 2

It was shown in the body of the paper that under optimal policy $h_t = \bar{h}$ for all t. Taking this result into account, express the optimal policy problem of definition 2 in recursive form as follows. If the country is in good financial standing in period t, $I_{t-1} = 1$, the value of continuing to service the external debt, denoted $v^c(y_t^T, d_t)$, i.e., the value of setting $I_t = 1$, is given by

$$v^c(y_t^T, d_t) = \max_{\{c_t^T, d_{t+1}\}} \left\{ U\left(A\left(c_t^T, F(\bar{h})\right)\right) + \beta \mathbb{E}_t v^g(y_{t+1}^T, d_{t+1}) \right\} \tag{A.1}$$

subject to

$$c_t^T + d_t = y_t^T + q(y_t^T, d_{t+1}) d_{t+1}, \tag{A.2}$$

where $v^g(y_t^T, d_t)$ denotes the value of being in good financial standing.

The value of being in bad financial standing in period t, denoted $v^b(y_t^T)$, is given by

$$v^b(y_t^T) = \left\{ U\left(A\left(y_t^T - L(y_t^T), F(\bar{h})\right)\right) + \beta \mathbb{E}_t \left[\theta v^g(y_{t+1}^T, 0) + (1-\theta) v^b(y_{t+1}^T) \right] \right\}. \tag{A.3}$$

In any period t in which the economy is in good financial standing, it has the option to either continue to service the debt obligations or to default. It follows that the value of being in good standing in period t is given by

$$v^g(y_t^T, d_t) = \max\left\{ v^c(y_t^T, d_t), v^b(y_t^T) \right\}. \tag{A.4}$$

The government chooses to default whenever the value of continuing to participate in financial markets is smaller than the value of being in bad financial standing, $v^c(y_t^T, d_t) < v^b(y_t^T)$. Let $D(d_t)$ be the default set defined as the set of tradable-output levels at which the government defaults on a level of debt d_t. Formally,[8]

$$D(d_t) = \left\{ y_t^T : v^c(y_t^T, d_t) < v^b(y_t^T) \right\}. \tag{A.5}$$

We can then write the probability of default in period $t+1$, given good financial standing

[8]A well-known property of the default set is that if $d < d'$, then $D(d) \subseteq D(d')$. To see this, note that the value of default, $v^b(y_t^T)$, is independent of the level of debt, d_t. At the same time, the continuation value, $v^c(y_t^T, d_t)$ is decreasing in d_t. To see this, consider two values of d_t, namely d and $d' > d$. Suppose that d^* and c^{T*} are the optimal choices of d_{t+1} and c_t^T when $d_t = d'$, given y_t^T. Notice that given d^*, y_t^T, and $d_t = d$, constraint (A.2) is satisfied for a value of c_t^T strictly greater than c^{T*}, implying that $v^c(y_t^T, d_t) > v^c(y_t^T, d')$ for $d < d'$. This means that, for a given value of y_t^T, if it is optimal to default when $d_t = d$, then it must also be optimal to default when $d_t = d' > d$.

in period t, as

$$\text{Prob}\{I_{t+1} = 0 | I_t = 1\} = \text{Prob}\left\{y^T_{t+1} \in D(d_{t+1})\right\}.$$

Combining this expression with (12) and (29) yields

$$q(y^T_t, d_{t+1}) = \frac{\text{1-Prob}\left\{y^T_{t+1} \in D(d_{t+1}) | y^T_t\right\}}{1 + r^*}. \tag{A.6}$$

Equations (A.1)-(A.6) are those of the Eaton-Gersovitz model as presented in Arellano (2008).

We have therefore demonstrated that under optimal policy the equilibrium allocation in the decentralized economy with sticky wages is identical to the equilibrium allocation in the centralized real economy of Arellano (2008).

A.2 Decentralization From Real To Real

In section 3 of the body of the paper, we demonstrated the decentralization of the Eaton-Gersovitz model to a competitive economy with downward nominal wage rigidity. We established that debt taxes and devaluation policy make the decentralization possible. Consider now the question of decentralizing the standard Eaton-Gersovitz model to a real competitive economy. To make the competitive economy real, suppose that nominal wages are fully flexible ($\gamma = 0$). In this case, the devaluation rate, ϵ_t, disappears from the set of competitive equilibrium conditions. Specifically, ϵ_t drops from conditions (24) and (26). The economy thus becomes purely real, and exchange-rate policy becomes irrelevant. However, clearly debt taxes are still necessary to establish the equivalence between the optimal-policy problem and the standard default model, as they guarantee the satisfaction of the private-sector Euler equation (21). We therefore have the following result.

Proposition A.1 (Decentralization To A Real Economy) *Real models of sovereign default in the tradition of Eaton and Gersovitz (1981) can be decentralized to a real competitive economy via debt taxes.*

This result is of interest because it highlights the fact that debt taxes are present in all default models à la Eaton and Gersovitz even though they do not explicitly appear in the centralized analysis.

The need for debt taxes in the decentralization of Eaton-Gersovitz-style models arises from the fact that the government internalizes the effect of aggregate external debt on the country premium, whereas individual agents take the country premium as exogenously given. Kim and Zhang (2012) also consider the case of decentralized borrowing and centralized default. However, we characterize the debt tax scheme that results in an equilibrium allocation identical to that of a model with centralized borrowing and centralized default (the standard Eaton-Gersovitz-style setup). Specifically, both in the present setting and in Kim's and Zhang's borrowers do not internalize the fact that the interest rate depends on debt. However, in the present formulation households face debt taxes that make them internalize the effect of borrowing on the country interest rate. By contrast, in the formulation of Kim and Zhang, debt taxes are absent and hence the allocation under decentralized borrowing is different from the one under centralized borrowing.

A.3 Implementability of the Optimal-Policy Equilibrium

This appendix shows that any member of the family of devaluation policies (30) uniquely implements the optimal-policy equilibrium. We first prove that under any such exchange-rate rule, the equilibrium involves full employment at all times. The proof is by contradiction. Suppose that there is some period t such that $h_t < \bar{h}$ in equilibrium. Then, by the slackness condition (26), we have that $w_t = \gamma w_{t-1}/\epsilon_t$. Combining this expression with (30), we get that

$$w_t \leq w^f(c_t^T).$$

Now, using equations (23) and (31) we can write

$$\begin{aligned} w_t &= \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) \\ &> \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}) \\ &= w^f(c_t^T). \end{aligned}$$

The inequality holds because of the assumed properties of the functions $A(\cdot, \cdot)$ and $F(\cdot)$ and because $h_t < \bar{h}$. The above two expressions are clearly contradictory. We have therefore established that under every exchange-rate rule belonging to (30), the equilibrium must involve full employment. Because none of the remaining equilibrium conditions listed in definition 1 depend on the devaluation rate, any possible nonuniqueness cannot be induced by the monetary policy rule. In particular, if the real allocation in the Eaton-Gersovitz model is unique, so it is when implemented with the devaluation rule (30).

A.4 Optimal Devaluation Policy Without Debt Taxes

In the model of section 2, continue to assume that borrowing is decentralized, but suppose now that the government cannot set debt taxes optimally. Thus, the only policy instrument at the planner's disposal is exchange-rate policy. In general, the case without debt taxes is significantly more complex and potentially intractable. The reason is that without a fiscal instrument used to induce private agents to internalize the borrowing externality, the model may display multiple equilibria. The possibility of nonuniqueness is identified in Kim and Zhang (2012) and formally established in Ayres et al. (2015). (This multiplicity problem is not present in most of the existing default models (e.g., Arellano, 2008; Aguiar and Gopinath, 2006; Chatterjee and Eyigungor, 2012; Mendoza and Yue, 2012; etc.) because as demonstrated by proposition A.1 in appendix A.2, by centralizing both the default and borrowing decisions, these models implicitly assume the availability of optimal debt taxes.) We can show, however, that in the case in which the intra- and intertemporal elasticities of substitution equal each other ($\xi = 1/\sigma$), the full-employment devaluation policy is optimal even when the planner does not have access to debt taxes.

To see this, assume that debt taxes are not part of the set of policy instruments available to the government. Suppose then that the process $\{\tau_t^d\}$ is exogenous and arbitrary. In this case, one must expand the set of constraints of the optimal-policy problem stated in definition 2 to include competitive-equilibrium conditions (20)-(22). This is because τ_t^d can no longer be set residually to ensure the satisfaction of these constraints. But clearly, there are no longer guarantees that the solution to the expanded optimal-policy problem will feature $h_t = \bar{h}$ for all t, because the right-hand side of equation (20) in general depends on h_t. Notice that even if the government cannot set debt taxes optimally, it could still achieve full employment at all times by appropriate use of the devaluation rate. But the resulting allocation would in general be suboptimal. However, in the case in which the intra- and intertemporal elasticities of consumption substitution are equal to each other ($\xi = 1/\sigma$), full employment reemerges as optimal. This is because in this case competitive-equilibrium condition (20) is independent of h_t.

We have therefore established that full employment is optimal even if debt taxes are not available to the planner. The case $\xi = 1/\sigma$ is indeed quite relevant. As argued in the calibration of the model presented in section 4.1, empirical estimates of the intratemporal elasticity of substitution suggest that ξ is close to 0.5. At the same time, the typical value of the intertemporal elasticity of substitution used in quantitative business-cycle analysis for emerging countries is also 0.5, or $\sigma = 2$.

It follows immediately that when $\xi = 1/\sigma$, the family of optimal devaluation policies is given by expression (30). This means that large contractions in the domestic absorption of tradable goods will be accompanied by devaluations under the optimal exchange rate policy. Thus, if in any of the possible many equilibria default takes place during aggregate contractions (as is the case in the equilibrium selected by Kim and Zhang, 2012), the economy without optimal debt taxation policy will capture the Twin Ds phenomenon as an optimal outcome.

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