ON THE IMPLEMENTATION OF
EQUALITY OF OPPORTUNITY

Guillem López-Casasnovas

Héctor Pifarré i Arolas

Frederic Udina

January, 2014

CRES-UPF Working Paper #201401-79

Noname manuscript No.
(will be inserted by the editor)

On the Implementation of Equality of Opportunity

Guillem Lopez-Casasnovas, Héctor Pifarré i Arolas, Frederic Udina

the date of receipt and acceptance should be inserted later

Acknowledgements The authors thank Carmen Herrera, Humberto Llavador, Antonio Villar for insightful comments.

Abstract In a context of scarce resources, deciding which groups to target with public intervention becomes relevant. We extend Roemer's framework of equality of opportunity (EOp) with a more general implemention in the context of a realistic "second best world", when complete equal opportunities are unachievable under both a public budget constraint and a given, ex ante, allocation of resources disregarding EOp.Our approach consists of attaching varying importance to the potential conflicting objectives for compensating the individual types that are worst off and individual types that can benefit the most with intervention. We do so by accounting for beneficiaries' capabilities of benefiting from the public policy, a feature that is particularly relevant in application to areas such as higher education or health care.

1 Introduction

While Roemer's theory of equality of opportunity (Roemer 1998) has gained ground in numerous applications, we argue that an unexplored area has been its application to too-real situations of *scarce or limited resources*. Given a number of groups of individuals facing *unfair* inequalities and a dearth of resources to compensate all individuals, which group should be targeted first? Roemer suggests to compensate groups that are worst off; we argue that this choice is not inherent to the ethical criterion of EOp. As situations of scarce resources are most frequent and real, our work contributes to the theory of EOp by first proposing a separation of the ethical criterion from the implementation, then addressing the latter with our approach, whereby the social decision is broadened vis a vis defining targeted groups.[1]

To contextualize our argumentation, we briefly review Roemer's proposal. In his book, Roemer formalizes external circumstances and effort, individual responsibility,

Address(es) of author(s) should be given

[1] Equality of opportunities has been studied according to different definitions by a number of works. For a comparison between Roemer's and Van der gaer's approach see Ooghe, Schokkaert and Van de gaer (2007).

and public compensation, in a way that recovers the concept of individual responsibility. According to this conceptualization, individuals' outcomes are attributed to both effort and circumstances. Roemer proposes effort as the only legitimate source of differences, implying that inequalities rooted in circumstances should be compensated. Possibly, the major strength of Roemer's work is providing a framework for society to consider inequality, while leaving the responsibility for most of the normative content for society.

Many authors have taken advantage of the flexibility of the framework and have applied Roemer's strategies for EOp in to different fields. These include education (Peragine and Serlenga 2007 and 2009, Bratti 2008, Blanco and Villar 2010), healthcare (Williams and Cookson 2000, Fleuerbaey and Shockhaert 2009) and development aid (Llavador and Roemer 2001).

Returning to Roemer's original contribution, the normative choice open to society is the identification of tolerable sources of inequality; in Roemer's terminology, this is the distinction between circumstances and effort. The framework offers a very large degree of flexibility, since it is possible to encompass, within Roemer's theoretical construction, ethical criteria that are often considered to be on opposite ends of conceptions of justice. We recover utilitarianism when society judges individual outcomes as determined solely by effort and embrace Rawlsian ideals when outcomes are judged as dependent only on circumstances.

However, in a context of scarce resources where social justice according to EOp might not be achievable, the question of which groups to compensate becomes an equally important dimension for social deliberation. Given a number of groups of individuals facing *unfair* inequalities, which group should we target first? Roemer's answer is *the group which is the worst off.* As much as this Ralwsian choice might be defendable, it is by no means the only possibility. Society might also concern itself with the degree to which groups are expected to improve when given public help. Taking this consideration to the extreme would lead to the utilitarian decision to target *the group which would benefit the most.* Our contribution is in recognizing the importance of choices in the implementation and to offer an approach that allows the inclusion of both extremes and the continuum in between, effectively extending society's choice to cover all aspects of the application oWith these assumptions, functions $v^t\left(\pi, \phi^t\right)$ are continuous and non decreasing in both arguments, not bounded on the second argument but bounded for $\pi \in [0,1]$. Function $v\left(\pi, \Phi\right) = \min_t v^t\left(\pi, \phi^t\right)$ also has these properties. Function $R\left(\omega\right) = \max_{\Phi} \int_0^1 v\left(\pi, \Phi\right) d\pi$ is increasing and not bounded as ω grows.

Proof If $\Phi = \Phi_\omega$ is the optimal policy for a budget ω, a type t has $\phi^t = 0$ if and only if $v^t\left(\pi, \phi^t_\omega\right) > v\left(\pi, \Phi_\omega\right)$ for all values of π except possible for some isolated ones. This may occur either because type t has some initial assignment x^t_0 or because the distribution of effort does not contain the zero effort. The only case where ϕ^t_ω does not increase when ω grows is when $\phi^t_\omega = 0$.

Now we proceed by induction on the number of types T. For $T = 1$ the proposition is clearly true. For T, choose a budget ω_0 large enough to be a sufficient budget for types $t = 1, \ldots, T-1$ while also satisfying $v\left(\pi, \Phi_{\omega_0}\right) > v^T\left(\pi, 0\right)$ for all π in some interval of positive length. This is also a sufficient budget for all types. □

f the theory of EOp. To that effect, we first reformulate Roemer's framework by separating the definition of the ethical criterion (EOp) from the implementation method (maxmin). Then, we address the implementation, restating the problem as the minimization of the distance to an unattainable social objective which allows us, through the choice of different metrics, to achieve the desired flexibility.

Our work is very closely related to the literature on the so-called bankruptcy problems, which proposes solutions to the allocation of a divisible good [2] amoung agents when the total amount of the good is insuffient to cover all their demands. Although the problem in the bankruptcy solution literature (for an extensive survey of the literature, see Thompson 2003) resembles ours to a large extent, our formulation diverges in that we follow Roemer's formalization of EOp. In the same way that the problem is closely related, our particular proposal to implement EOp is similar to the one discussed by Herrero and Villar (1994) in an application to the allocation of a public budget to different objectives when the available funds are insufficient to completely satisfy all the objectives. In their work, the authors study the properties of a number of sharing rules inspired by both the bankruptcy and axiomatic negotiations literatures. An important distinction between our work and the rules suggested in the bankruptcy literature (and Herrero and Villar 1994) is that we introduce concerns for recipients' capacities to benefit from transfers.

The paper is organized as follows. In the first section, we formally review the framework presented in Roemer (1998). Our deviation consists in separating the ethical criterion from the implementation method. The second section is devoted to presenting our method. Preliminary, we impose further assumptions on the advantage functions and effort decisions. We continue by dividing between impediments to achieving social justice from budgetary restrictions and from complexities of the compensation policy. We tackle them separately for clarity. First, in order to convey the basic intuition, we use a simplified case to present how budgetary limitations might preclude the social planner from achieving EOp regardless of the implementation method. We then illustrate how simple policies can only attain EOp in very specific settings. Since these situations are not the focus of our work, we offer a modified social objective which allows us to ignore the concerns regarding the complexity of policies. In this modified setting, we present our implementation method and discuss how the choice of different metrics allows us to cover the full spectrum of social decisions regarding which groups to intervene.

2 The formalization of EOp

In this section we briefly introduce the basic elements of the theory required for our contribution. The problem, in short, is the following. Consider a set of individuals who can achieve a certain advantage, perhaps income, or health or any other relevant dimension of welfare. Suppose the advantage is a function of the amount of resources consumed by individuals, the effort they exert (individually), and their circumstances. Our goal is to decide on the distribution of a given public budget across individuals to achieve a particular social outcome which meets the ethical criteria of EOp. While we endorse all the concepts developed by Roemer, it is important to highlight that a major distinction in our work lies in defining the targeted ethical critiera as independent from the method of implementation. This modification, which has implications we discuss

[2] The literature tends to focus on divisible goods; nevertheless, for an exploration of the problem under indivisible goods see Herrero and Martinez (2008)

later, is the basis for extending society's choice set to cover the selection of the recipients of compensation, which is the basic contribution of our work.

The basis of EOp's normative structure lies in the distinction between circumstances and effort. Define *circumstances* to be that which society judges beyond the will of the individual. *Effort* is constituted by all the actions that society judges as within the will of the individual. It follows from the characterization of circumstances, that we may create a classification of individuals by types. A *type* is a subset of individuals who share the same circumstances relevant to the attainment of the chosen dimension of advantage.

We denote $\mathcal{T} = \{1, \ldots, T\}$as the set of T types in which we divide population. Note, the difference between "effort" and "circumstance" is entirely a collective decision by which society judges what lies within the realm of an individual's responsibilities. The relation between resources, effort and advantage is given by the advantage function. The *advantage* of an individual is a function of the effort and resources consumed, indexed by the type.

We denote $u^t(x, e)$ the function of advantage for type t, where x are the resources consumed and e is the effort[3]. As stated before, society must choose an allocation of resources (potentially) dependent on effort and type. While we know that the aim of the policy is to attain EOp, let it be for now any rule which satisfies the following definition:

Definition 1 (Policy) A policy is a T-tuple of functions which specify, for each type, the resources devoted as a function of effort. We denote it $\phi = \left(\phi^1, \ldots, \phi^T\right)$ and call each function ϕ^t an allocation rule. Then, $\phi^t(e)$ is the amount of resources a type t individual receives if she exerts effort e.

It is then reasonable to believe, in turn, that the effort exerted by individuals is dependent on the policy. For the purposes of this section, we keep the individual's response general, but we elaborate on this later. Given a policy, the individuals of a given type t generate a distribution of effort, given by a cumulative probability function $F^t_{\phi^t}$, so $F^t_{\phi^t}(e)$ is the cumulated probability up to and including e. In some cases we may assume that effort has a discrete probability distribution, in others we assume a continuous distribution with a convex support, say an interval (infinite or not) with a density function $f^t_{\phi^t}$.

We now specify the notation for the resources constraint. Let ω be the per capita disposable resources. The amount of (per capita) resources assigned to type t is

$$\omega^t = \sum_e \phi^t(e) P(e) \text{ or } \omega^t = \int_R \phi^t(e)\, dF^t_{\phi^t}(e)$$

depending on whether effort is a discrete or a continuous variable. Denote $\alpha^t$the proportion of type t in the population. Then the global constraint is

$$\omega = \sum_t \alpha^t \omega^t. \tag{2.1}$$

Let $\rho^t = \omega^t/\omega$ be the per capita share of the resource for type t. Then $\sum_t \rho^t = 1$.

[3] Indexing by type is equivalent to including another variable or vector of variables which includes type characteristics, i.e., $u(C, x, e)$, where C is the vector of individual characteristics that constitute a type (see Roemer, 2002 for a complete exposition of this notation).

A key contribution by Roemer is to formalize comparison of effort exerted by different types. Given an expectation of higher levels of effort from more advantaged types, how might we *fairly* compare them to disadvantaged types? Roemer's solution is to draw a distinction between the *level* and the *degree* of effort as formalized in this definition.

Definition 2 (Effort level, effort degree and indirect advantage) For $\pi \in (0,1)$, let $e^t\left(\pi, \phi^t\right)$ be the *level of effort* exerted by an individual of the type t in the π^{th}quantile of effort of the type . We call π a *degree of effort*. These levels and degrees are characterized by the equations

$$\pi = \int_0^{e^t\left(\pi, \phi^t\right)} dF^t_{\phi^t}, \qquad t \in \mathcal{T}$$

in the case of continuous effort distribution, and similarly for the discrete case.

The *indirect advantage function* gives the advantage of an individual of type t receiving the resources determined by the policy ϕ and exerting the π degree of effort of the type distribution of effort, and is defined by

$$v^t\left(\pi, \phi^t\right) = u^t\left(\phi^t\left(e^t\left(\pi, \phi^t\right)\right), e^t\left(\pi, \phi^t\right)\right)$$

2.1 The definition of equality of opportunity and its implementation

Having presented the concepts, we introduce the concept of equality of opportunity. We first establish the definition of equality of opportunity and then discuss at length the method for implementation.

Definition 3 (Social criteria and social justice) The ethical criterion of *equality of opportunity* (EOp) states that given any two individuals, independent of type, exerting the same *degree* of effort (individuals in the same position in their respective type distributions), these individuals must achieve the same level of advantage. That is,

$$\forall i, j \in \mathcal{T},\ \forall \pi \in (0,1)\,, \quad v^i\left(\pi, \phi^i\right) = v^j\left(\pi, \phi^j\right) \tag{2.2}$$

By *strong social justice* according to a given criterion we indicate, the state in which the advantages of all individuals satisfy the requirements of the chosen ethical criterion, in this case, EOp.

By separately stating the ethical criterion of EOp, we deviate from Roemer (1998) in a nontrivial way. Roemer's presentation refers to EOp as in (2.2). However, when implementating EOP - i.e. the choice of policy - Roemer defines the resulting policy from his proposed methodology as an EOp policy regardless of whether it actually achieves the EOp in definition 3. In a sense, one could argue that Roemer has made EOp a criterion subsidiary to the method, as what is actually treated as EOp is the outcome of his rule for choosing the policy. We proceed differently by separating the definition of the criterion from its implementation. When we refer to EOp, we are considering the criterion in definition 3. Our implementation is an attempt to approach social justice, that is, the outcomes whattained.ich fulfill the ethical criterion. The main consequence of adopting our strategy is allowance for the possibility that EOp is not actually achieved, which is ruled out by the nature of Roemer's procedure.

If we proceed according to this separation of *goal* and *method,* we can still discuss Roemer's rule for a choice of policy as a particular method. Recall that we seek a distribution of resources that leads to the implementation of EOp. Ideally, we aim at achieving this for every percentile of effort, whereby all types acomplish the same level of advantage. The method proposed by Roemer is the maximization of the minimum advantage among types for every π of effort.

$$\max_{\phi} \min_{t \in \mathcal{T}} v^t \left(\pi, \phi^t\right)$$

Note that this implementation method is Rawlsian in its conception. It centers the attention on the type that is worst off, for every π of effort. This is the essence of Roemer's implementation and what follows are technical considerations. Given limitations on the sophistication of the implementation policy, the solution to the program for a given quantile might not correspond with the solution that equalizes the advantage among other quantiles. Therefore, we encounter the problem of potentially obtaining as many policies as quantiles. Roemer proposes a solution consisting of assigning a weight equivalent to the population weight for every quantile and solving for this modified problem. Effectively, it concedes the same importance to every quantile, so in this sense, it becomes utilitarianist across quantiles.

$$\max_{\phi} \int_0^1 \min_{t \in \mathcal{T}} v^t \left(\pi, \phi^t\right) \; d\pi \tag{2.3}$$

For all our contribution, we limit ourselves to the case of constant policies**,** that is, policies that depend on the type but not on effort. By doing so, we simplify the presentation; however, our methodology would remain unharmed with the introduction of more sophisticated policies. The remaining of the paper is devoted to constructing and justifying our proposed implementation.

3 A new implementation under unattainable objectives

As emphasized prior, we retain EOp as an objective, as stated in (2.2), and autonomously develop our method. Our main objective is to develop a method which allows society to choose targeted types when EOp is unattainable due to the scarcity of public funds. The choice is made by deciding on a tradeoff between the two (potentially) conflicting objectives highlighted in the introduction: allocating resources to the types who are worst off versus directing resources towards types that would benefit most from the resources.

Once EOp is stated independent of its implementation method, the direct implication is that there is no guarantee that EOp is achieved. In order to convey the relevance of our proposal, we first clarify the circumstances under which EOp might not be achieved, independent of the method chosen to implement it. In our setting, which includes a number of additional assumptions with respect to Roemer's presentation, this occurs for two reasons. First, the social planner may be unable to compensate for existing initial differences across types with a limited budget. Second, restriction to constant policies imposes heavy requirements on the advantage functions if EOp is to be achieved. In order to accurately describe the implications of both issues, we address each separately then consider when both might occur.

3.1 Assumptions

Throughout this section, we assume the following regarding effort decisions, the advantage function, and the behavior of individuals.

1. Achievement functions $u^t(x, e)$ are defined for all nonnegative values of their arguments, unbounded for any fixed positive value of any of its arguments, and twice differentiable with continuity, i.e. C^2 functions. First partial derivatives are strictly positive and second derivatives with respect to the same argument twice are strictly negative.
2. Both resources and effort are needed and sufficient for obtaining a positive achievement: $u^t(x, e) = 0$ if and only if $x = 0$ or $e = 0$.
3. The assignment policy is determined by Roemer's method, so it is the optimal solution of (2.3). Furthermore, we restrict to constant policies.
4. Denote e^* the individual choice of effort of any given type. Then, effort depends positively on the amount of resources allocated to this type: $\forall x, \frac{de^*}{dx} > 0$

We clarify here the extent to which our second assumption is restrictive. We state that advantage requires a positive amount of resources, but we do not require the resources to be fully determined by the social planner. Resources originate from other public interventions (not aimed at achieving EOp), privately by individuals ("other resources"), or as allocated by the social planner. We impose a restrictive assumption further along by requiring "other resources" to be unaffected by the social planner to obtain EOp. This simplifying assumption precludes important considerations such as crowding-out of private resources.

In applications, however, it is natural to think that there is the possibility that agents actually exert less effort when they are allocated more resources (a type of "crowding out" effect). This is important for our implementation method since we rely on the possibility of achieving social justice. If agents diminish their efforts in a way that fully offsets the advantage gains garnered from allocation of further resources, our method would no longer result in a well-defined problem. Our claim, however, is that our implementation method can be applied even if there is some "crowding out" as long as there is an arbitrarily large public budget such that social justice can be attained.

Finally, our fourth assumption is made to simplify the otherwise complex issue of effort responses. Strictly from a modeling standpoint, a rationalization for this assumption is that effort is the result of optimizing $e^* = \arg\max_e u(x, e)$. In this context, it is possible to impose restrictions on the cross derivatives of the function $u(x, e)$ that would deliver the assumption we have imposed as a result. Calsamiglia (2009) has explored these types of models. We can establish the conditions on effort responses that would guarantee the validity of our method; it would complicate our exposition and require making behavioural assumptions that are likely dependent on the context to which EOp is applied. As a practical application of the theory of EOp is beyond of the scope of this paper, we have chosen to make this simple albeit overly parsimonious assumption.

3.2 Insufficient budget

We portray in detail the context of an insufficient budget. In order to focus solely on this issue, we present a limit case by ruling out the possibility of conflicts in the policies required by each quantile by assuming a single level of effort for each type (although not necessarily the same one). That is, the distribution of effort is characterized by a single effort for each type with the immediate implication that EOp is achieved by completely equalizing advantages across types. This is the simplest scenario where the availability of public funds represents a limitation in the achievement of EOp. Having established the aim of our exercise, suppose now that all the resources available for individuals are publicly provided and distributed according to the ethical objective of EOp. Then, as we formalize in proposition 1, strong social justice is always achieved in our context.

Proposition 1 *Under a single nonzero effort level in each type and assuming assumptions 1, 2, 3 and 4, strong social justice is achieved; advantages for all types is identical for any amount ω of available resources.*

Proof Under a single effort level for a given type, functions v^t 2 are now simply

$$v^t\left(\phi^t\right) = u^t\left(\phi^t, e^t\left(\phi^t\right)\right)$$

attained.

where $e^t(\phi^t)$ denotes the effort level applied by all individuals of type t as a response to receiving ϕ^t resources. By 4 , v^t as a function of its single argument is monotone and strictly increasing with continuity. It can be shown that, if two types have different advantages, it is possible to reassign resources to reduce the difference. To reassign: choose i, j such that $v^i(\phi^i)$ is minimum among all types and $v^j(\phi^j)$ is maximum among types. Since functions v^t are continuous and increasing, there exists $\delta > 0$ such that

$$v^i\left(\phi^i\right) < v^i\left(\phi^i + \delta\alpha^i\right) < v^j\left(\phi^j - \delta\alpha^j\right) < v^j\left(\phi^j\right).$$

Then we can adjust ϕ^i by $\phi^i + \delta\alpha^i$ and ϕ^j by $\phi^j - \delta\alpha^j$ to obtain a new policy that still satisfies the restriction 2.1. It may be that several types share the same minimum given by $v^j\left(\phi^j\right)$. To achieve a policy that improves Roemer's criterion, reassignment may require repeating the application. □

What poses a threat to the achievement of EOp is the existence of initial differences, not the availability of budgetary resources. After all, if types did not have initial levels of achievement, it would always be possible to equalize them at the bottom by assigning zero advantage for all of them. Proposition 1 formalizes the idea that in the absence of initial levels of advantage, EOp is achieved regardless of the budget.

Assume now that the types have some initial advantage independent of the policy selected. We consider the case where this is due to some arbitrary initial assigment of resources that is given without necessarily respecting any justice requirement. It can be interpreted as public funds distributed in a previous stage according to some other criterion (for instance, utilitarism), as privately-provided resources (for instance, provisions by the family) or a combination of both. Regardless of the origin, we assume that all types hold some initial amount of resources (with at least one amount strictly positive) and that this does not depend on posterior public resources allocated. Then, in our context, it is strictly a matter of the the size of the public budget whether EOp can be reached or not.

Proposition 2 *Assuming a single nonzero effort level in each type and assumptions 1 and 2, let x_0^t, $t \in \mathcal{T}$, be some nonnegative initial assignments (where at least one assignment is positive, and at least one is zero). Then there are values ω_S and ω_L for the amount of resources to be assigned such that the following hold.*

(i) For $\omega < \omega_S$, equality of achievement among types is unattainable.

(ii) For $\omega \geq \omega_L$, equality of achievement among types is achieved.

(iii) If strong social justice has been achieved (all types have equal advantage) any marginal increment of resources is distributed such that

$$\forall i, j \in \mathcal{T}, \ \frac{\partial u^i}{\partial x}\frac{d\phi^i}{d\omega} + \frac{\partial u^i}{\partial e}\frac{de^i}{d\phi^i}\frac{d\phi^i}{d\omega} = \frac{\partial u^j}{\partial x}\frac{d\phi^j}{d\omega} + \frac{\partial u^j}{\partial e}\frac{de^j}{d\phi^j}\frac{d\phi^j}{d\omega}$$

Proof If (i) were false, letting ω go to zero will give us equality of advantages among $u^t\left(x_0^t + 0, e^t\right)$ with some x_0^t positive and some zero, which is impossible given assumption 2. To prove (ii), apply proposition 1 with advantage functions $u_*^t\left(x, e\right) = u\left(x_0^t + x, e\right) - u\left(x_0^t, e\right)$. We have a budget ω_* with equality of achievements that we may assume to be the minimal one. Then take $\omega_L = \omega_* + \sum_t x_0^t$. (iii) follows differentiating with respect to ω (using the chain rule) the equality $u^i\left(\phi^i, e^i\left(\phi^i\right)\right) = u^j\left(\phi^j, e^j\left(\phi^j\right)\right)$ (see proof of prop. 1) where now $\phi^t = \phi^t\left(\omega\right)$ □

We extract a number of valuable insights from proposition 2. First, the amount public funds available matters for the equality of achievements. Second, once a state of strong social justice is reached, further resources are allocated so that every type obtains a share. The size of the portion depends on the ability of types of transforming resources into achievement. Having characterized the issue of the scarcity of resources to the extent that is needed for our exercise, we now turn to the problem posed by limitations on the complexity of the policy.

3.3 Simplicity of the policy

In the presentation of his method, Roemer addresses the possibility of conflicts in the policies necessary to obtain strong social justice for every given quantile. In our context, this is the second main reason why strong social justice might not be achieved. We now exclude initial differences in resources and focus on illustrating the problems posed by restricting to the particular case of constant policies. In a context of limitations on the availability of information to design compensation schemes, we believe there is a gain in designing a method that works under the simplest possible policies. We find that very stringent requirements are needed to be able to attain the ethical objective if we restrict ourselves to constant policies when dealing with multiple efforts. Through Example 1, we show how, under very particular homotheticity properties (elaborated later on) between both types and quantiles of effort, strong social justice is attained. We provide the example as an illustration of the degree to which special circumstances are necessary to completely fulfill the desired ethical criterion.

Example 1 Assume:

(i) there are just two types $t = A, B$ each with a continuum of effort levels, and achievement functions of the form $u^t(\varphi, e) = \lambda^t \varphi^{\alpha^t} e^{1-\alpha^t}$.

(ii) there is a policy φ that assigns a constant amount of resources $\varphi^A + \varphi^B = 1$ (total amount of resources per capita is normalized to 1) and achieves strong social justice (equality of indirect achievments), that is $\forall \pi \in (0,1)$, $v^A\left(\pi, \varphi^A\right) = v^B\left(\pi, \varphi^B\right)$.

Then, the frontier of advantages is homothetic on the quantiles. If e^t_π denotes the π quantile in the effort distribution of type t,

$$\forall \phi \in (0,1), \quad \forall \pi, \pi' \in (0,1), \quad \frac{u^A\left(\phi, e^A_\pi\right)}{u^B\left(1-\phi, e^B_\pi\right)} = \frac{u^A\left(\phi, e^A_{\pi'}\right)}{u^B\left(1-\phi, e^B_{\pi'}\right)} \tag{3.1}$$

Furthermore, this assumption forces the effort quantiles by $e^A_\pi = c_1\left(e^B_\pi\right)^{c_2}$ for some constants c_1, c_2.

Proof Simple substitution of u^t into the social justice condition 3.1 gives

$$\left(e^B_\pi\right)^{1-\alpha^B} = \frac{\lambda^A\left(\varphi^A\right)^{\alpha^A}}{\lambda^B\left(\varphi^B\right)^{\alpha^B}}\left(e^A_\pi\right)^{1-\alpha^A}$$

and this gives both results. □

We do not wish to restrict our contribution to such special cases nor relax our restrictions on policies, so our strategy is to develop a weaker definition of social justice as an objective. In particular, we relax the requirements on the definition of social justice by only accepting differences that arise from the simplicity of the policy.

3.4 The method

In application, the feasible level of complexity of policies may become an issue. We tackle this problem by proposing a weakened version of strong social justice which is enough for our objective. Namely, we consider a new social objective in which only inequalities derived from the simplicity of the policy are tolerated. To that effect, recall that proposition 2 which establishes that all types receive a positive amount any marginal increase in available public funds once social justice is met. We build our new definition of social justice around this notion by defining a state of *weak social justice* in which all types receive a share of any marginal increase in the public budget. In the particular situation where proposition 2 is established, this coincides with the achievement of strong social justice. This is no longer the case when given multiple levels of effort. However, it is still possible to find an amount of resources (total budget) such that by using Roemer's implementation method, all types obtain a portion of a marginal increase of the public funds. We define this amount to be such that, if the optimal policy is chosen by Roemer's method, all types would receive a positive share of any marginal increase of the budget. By choosing Roemer's implementation, we relax our definition of social justice in the same way that Roemer did in his contribution. However, as emphasized earlier, we depart from Roemer in allowing only for differences in advantage arising from the simplicity our policy. We maintain that given exogenous initial differences across types, our relaxed version of social justice might not be achieved. The formalization of this explanation is given in the following definition.

Definition 4 (Sufficient budget and weak social justice): Assume the policy is decided by solving Roemer's program. A sufficient budget is such that, for any budget in excess of the sufficient budget, all types would receive a strictly larger assignment under the decided policy associated with the larger budget. Formally, a total budget ω is *sufficient,* if, for all types t and for all $\bar{\omega} > \omega$, $\phi^t_{\bar{\omega}} > \phi^t_{\omega}$, where ϕ_ω denotes the assignment under the optimal policy in assumption 3. Denote by *weak social justice* the state in which the total budget is at least *sufficient*.

The definition captures the intuition that if every type receives a share of the additional pie, it is not possible to move closer to the social justice state. Therefore, a total budget is sufficient if it compensates for initial inequalities among types. We now prove that such a budget exists under fairly general conditions.

Proposition 3 *Assuming assumptions 1 , 2 , 3 and 4 and compact support effort distribution for each type (or finite number of values if effort levels are discrete), there exists a (finite) sufficient budget.*

Proof With these assumptions, functions $v^t\left(\pi, \phi^t\right)$ are continuous and non decreasing in both arguments, not bounded on the second argument but bounded for $\pi \in [0, 1]$. Function $v\left(\pi, \Phi\right) = \min_t v^t\left(\pi, \phi^t\right)$ also has these properties. Function $R\left(\omega\right) = \max_\Phi \int_0^1 v\left(\pi, \Phi\right) d\pi$ is increasing and not bounded as ω grows.

If $\Phi = \Phi_\omega$ is the optimal policy for a budget ω, a type t has $\phi^t = 0$ if and only if $v^t\left(\pi, \phi^t_\omega\right) > v\left(\pi, \Phi_\omega\right)$ for all values of π except possible for some isolated ones. This may occur either because type t has some initial assignment x^t_0 or because the distribution of effort does not contain the zero effort. The only case where ϕ^t_ω does not increase when ω grows is when $\phi^t_\omega = 0$.

Now we proceed by induction on the number of types T. For $T = 1$ the proposition is clearly true. For T, choose a budget ω_0 large enough to be a sufficient budget for types $t = 1, \ldots, T-1$ while also satisfying $v\left(\pi, \Phi_{\omega_0}\right) > v^T\left(\pi, 0\right)$ for all π in some interval of positive length. This is also a sufficient budget for all types. □

Having proved existence, we select a particular *sufficient* budget. Recall, we seek to approach *weak social justice* by means of our selected policy. However, this new and more lax requirement of social justice could be achieved by a number of budgets. Our natural choice of budget, which in turn characterizes the advantages in a particular *weak social justice* state, is that which requires the least amount of resources.

Definition 5 (Minimum sufficient budget): Given initially allocated resources $x^t \geq 0$, whereby some are positive, the infimum of all socially acceptable budgets will be named the *minimum sufficient* budget and the corresponding optimal policy will be named the *minimum sufficient policy* (Note that this is also a sufficient budget).

How do we choose a policy when the budget is short of a sufficient budget? Roemer's method remains valid, however, it presumes priority for compensating types that are the worst off, a judgement that is not implied by EOp. We propose a method that allows society to decide which types to compensate by taking a stand on the tradeoff between the effectiveness of the public intervention and helping the worst off.

We suggest that, given an *insufficient budget*, we can obtain our policy from the minimization of the distance to the state of *weak social justice* generated by the *minimum sufficient policy.*

Definition 6 (Socially-selected policy): Let ω_0 be the minimum sufficient budget for a given set of initially allocated resources $\{x^t\}$, let ϕ_0 be the corresponding minimum sufficient policy , and let $v^t(\pi, \phi_0)$ the associated indirect achievement functions. We denote by $\mathbf{v}(\pi, \phi_0)$ the vector of all types' achievements for degree π. For a smaller budget ω we say that the *socially-selected* policy is the solution for the program

$$\min_{\phi} \int_0^1 \|\mathbf{v}(\pi, \phi_0) - \mathbf{v}(\pi, \phi)\|_p \, d\pi \tag{3.2}$$

where $\|.\|_p$ denotes a norm (see below) that measures the distance to the minimum sufficient policy and $\mathbf{v}(\pi, \phi)$ is the vector of achievements under policy ϕ.

It is clear that the choice of distance has profound implications on the resulting policy. Consider a p-norm; then, 3.2 becomes

$$\min_{\phi} \int_0^1 \left(\sum_{t=1}^{t} \left| v^t(\pi, \phi_0) - v^t\left(\pi, \phi^t\right) \right|^p \right)^{\frac{1}{p}} d\pi \tag{3.3}$$

It is desirable to formulate the problem in this fashion as every metric comprised in the p-norm corresponds to a choice of weights in the trade off between effectiveness of public resources and compensating those who are worse off. For instance, for $p = 1$, 3.3 becomes the minimization of the sum of differences.

$$\min_{\phi} \int_0^1 \left(\sum_{t=1}^{t} \left| v^t(\pi, \phi_0) - v^t\left(\pi, \phi^t\right) \right| \right) d\pi$$

The choice of this metric entails a utilitarian policy or simply, targeting *the type that could benefit the most*. At the margin, resources are allocated to the type with a larger partial derivative of the advantage function with respect to resources. Since in general, there is no guarantee that an interior solution is achieved, this could lead to abandoning types with lower abilities to transform resources into advantage.

On the other extreme, when $p \to \infty$, 3.3 is then:

$$\min_{\phi} \int_0^1 \left(\max_t \left| v^t(\pi, \phi_0) - v^t\left(\pi, \phi^t\right) \right| \right) d\pi$$

The solution allocates resources to types farther away from the minimum sufficient policy advantages, disregarding any consideration regarding the effectiveness of the public funds .We acknowledge, however, a non-desired consequence that follows from our definition of weak social justice. We cannot prove that in general, when limited to simple constant policies, our sufficient policy leads to the equality of achievements for every quantile across types[4]

[4] Through our example 1 we have shown that in general, when limited to simple constant policies, there is no guarantee that our sufficient policy leads to the equality of achievements for every quantile across types.There can be quantiles for given types that result in higher levels of achievement than others, leading to the counterintuitive result that a Ralwsian policy would prioritize their compensations. This is entirely a result based on the limitations we have imposed to our policy and its actual relevance depends on how different advantages are in weak social justice.

3.5 Discussion

We have assumed in this paper the standard case of continuous achievement function; this might be considered a limitation of our proposal as many applications require considering categorical data (for instance, health states or access to higher education). This issue has been studied in recent work that has explored extensions of the EOp framework categorical data

(Herrero and Villar 2012). Our methodology extends to these situations as well; when dealing with categorical data, our method requires minimizing the distance to the proportion, for each category and type, that is in accordance to the weakened definition of EOp.

Consider the case of access to higher education. In a simple case, advantage is defined as access or non-access to tertiary education. We note first that EOp entails equality of proportions of access across types. The reasoning is that EOp takes as relevant only relative effort, hence there is no basis for establishing that a certain type has exherted more effort overall than another. In this application, our method is unchanged but the advantage function is interpreted as the individual probability of access. A possible application would consist in using grades as a measure of effort since in many higher education systems, high school grades and access examination grades are used to rank students. The criterion of EOp would then require that any two students, irrespective of their types, ranked in the same quantile of their within type grade distribution should have the same probability of access. Considering the probability of access a continuous function (in both arguments) would then return us to the formulation we have adopted in our presentation.

On a more general note, a critique leveled against EOp is that it does not include the traditional tradeoff between efficiency and equity. Our work includes considerations regarding efficiency in the application of EOp, but only related to the attainment of social justice. Hence we acknowledge the possibility that pursuing the optimal policies according to our method could be inefficient from a broader standpoint. Nevertheless, we conceive the framework of EOp as providing the guidelines for the design of the policies of a public authority in charge exclusively of the attainment of equity. That is not to say that society, at a broader level, might still be allocating its resources considering the traditional balance between efficiency and equity. In the case of education, for instance, the relevant ministry might devote part of its budget to attain high levels of enrollment and some other fraction to equitable access. Our proposal is addressed exclusively at the latter objective.

4 Conclusion

We have argued that full capacity of decision over the normative content of the theory of equality of opportunity formulated by Roemer (1998) requires transfering decision power over its implementation. In order to apply the theory it is not only relevant for society to establish the legitimacy of inequalities; when the scarcity of resources results in the impossibility of attaining equality of opportunity, it is also crucial to decide which types to compensate first. In this work, we outlined the conditions in which this concern matters, namely when public funds are insufficient to compensate for initial differences across types. We then present a new method of implementation based on the intuitive idea of the minimization to an unnatainable objective. Through the choice of different metrics, our methodology allows us to include, with varying importance, the weight attached to the potentially conflicting objectives of compensating the types that are worst off and types that can benefit the most. The main contribution of this paper is, therefore, to extend the framework of EOp to allow societal control of the implementation process of equality of opportunities.

References

1. Calo-Blanco A, Villar A (2009) Education, Utilitarianism, and Equality of Opportunity. Fundación BBVA - Documentos de Trabajo No. 13.
2. Calsamiglia C (2009) Decentralizing Equality Of Opportunity. Int Econ Rev 50: 273-290
3. Checchi D, Peragine V, Serlenga L (2009) Income inequality ans opportunity inequality in Europe. Riv di Politica Economica, 98: 266-293
4. Fleuerbaey M, Shockhaert E (2009) Unfair inequalities in health and health care. J Health Econ 28: 73-90
5. Herrero C , Martínez R (2008) Egalitarian Rules in Claims Problems with Indivisible Goods. Soc Choice Welf 30: 603-617
6. Herrero C, Villar A (1994) La assistencia sanitaria como un problema de asignacion en presencia de objectivos inalcanzables. In: Lopez-Casasnovas G (ed.) Analisis economico de la sanidad. Servei catala de salut.
7. Herrero C, Villar A (2012) An Equity-based Proposal for the Evaluation of Health States. Fundación BBVA - Documentos de Trabajo No. 15.
8. Llavador H, Roemer JE (2001) An equal-opportunity approach to the allocation of international aid. J Dev Econ 64: 147-171
9. Ooghe E, Schokkaert E, Dirk Van de gaer D (2007) Equality of Opportunity versus Equality of Opportunity Sets. Soc Choice Welf 28: 209-230
10. Peragine V, Serlenga L (2008) Higher education and equality of opportunity in Italy. In: Bishop J and Zheng D (eds.) Research in Economic Inequality, vol 16, pp 67-97
11. Roemer JE (2002) Equality of opportunity: A progress report. Soc Choice Welf 19: 455-471.
12. Roemer JE (1998) Equality of opportunity. Harvard University Press, Cambridge, MA
13. Thomson W (2003) Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey. Math Soc Sci 45: 249-297
14. Williams A, Cookson R (2000) Equity in Health. In: Culyer A, Newhouse JP (eds.) Handbook of Health Economics. 1B: Elsevier Science B.V..