Signals and Systems 2

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Tutorial #5: Laplace transform

1. Consider a system with transfer function $H(s) = \frac{1}{s^2+3s+2}$. It is possible to build such system as a cascading of two first-order filters $H_1(s)$ and $H_2(s)$, or as the parallel of two first-order filters $H_3(s)$ and $H_4(s)$. Determine the transfer functions of the four filters.

2. Determine the Laplace transform of $x(t) = e^{-a|t|}$, by considering the signal as a sum of two symmetric subsignals $x_1(t) = e^{-at}\, u(t)$ and $x_2(t) = e^{at}\, u(-t)$. Plot a zero/pole diagram and the region of convergence of $X_1(s)$, $X_2(s)$ and $X(s)$.

3. Determine the Laplace transform of the following signals. Plot the corresponding zero-pole diagrams, including the regions of convergence.
   (a) $x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$
   (b) $x(t) = e^{t}\sin(2t)\, u(-t)$

4. Consider the transfer function $H(s) = \frac{1}{(s-1)(s+3)}$. Sketch all the possible regions of convergence. For each case, reason whether:
   (a) The impulse response is right-sided, left-sided or bilateral.
   (b) The system is causal.
   (c) The system is stable.

5. Determine the magnitude of the Fourier transform of $H(s) = \frac{s^2-s+1}{s^2+s+1}, \mathrm{Re}\{s\} > -\frac{1}{2}$.

6. Consider a continuous, causal and stable LTI system, whose input $x(t)$ and output $y(t)$ are related by the differential equation: $\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 5y(t) = 2x(t)$. Determine its step response $s(t)$.

7. Consider an RLC circuit in which the relation between input and output is given by the differential equation: $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{1}{LC}y(t) = \frac{1}{LC}x(t)$. Determine the relation between R, L and C, so that the step response of the system does not present any oscillation.

8. For each of the following second-order differential equations describing causal and stable LTI systems, determine the transfer function $H(s)$ and whether the impulse response is underdamped, critically damped or overdamped.

   (a) $\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 4y(t) = x(t)$

   (b) $5\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 5y(t) = 7x(t)$

   (c) $\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 20\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = x(t)$

   (d) $5\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 4\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 5y(t) = 3\frac{\mathrm{d}x(t)}{\mathrm{d}t} + 7x(t)$

9. The signal $y(t) = e^{-2t}\,u(t)$ is the output of a causal all-pass system for which the system function is $H(s) = \frac{s-1}{s+1}$.

   (a) Find and sketch two possible inputs $x(t)$ that could produce $y(t)$.

   (b) What is the input $x(t)$ if it is known that $\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$?