Signals and Systems 2

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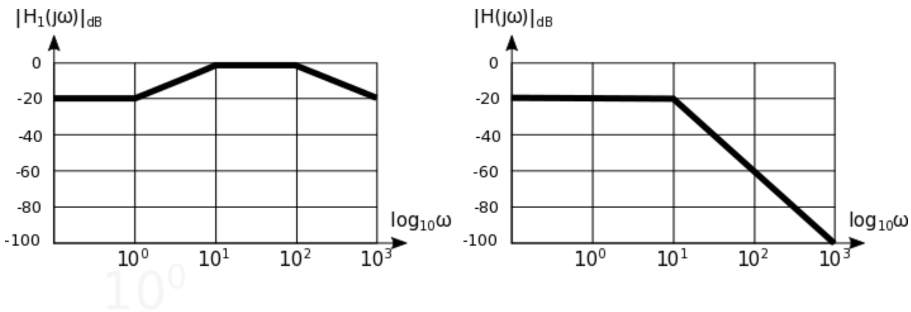
Tutorial #3: Filters and Bode diagrams

1. Consider an ideal passband filter whose frequency response is: $H(j\omega) = \begin{cases} 1, & \omega_c \leq |\omega| \leq 3\omega_c \\ 0, & \text{otherwise} \end{cases}$
   Determine, using the properties of the Fourier transform and the known transform pairs, a function $g(t)$ such that: $h(t) = \frac{\omega_c}{\pi} \frac{\sin(\omega_c t)}{\omega_c t} \cdot g(t)$

2. Consider a real lowpass filter with a passband frequency $f_p = 1000$ Hz, and a supression frequency $f_s = 1200$ Hz. The impulse response of the filter is $h(t)$. We want to convert such system into a bandpass filter with impulse response $g(t) = 2h(t)\cos(4000\pi t)$. Determine the passband and supression frequencies of the resulting bandpass filter.

3. Consider an LTI system with frequency response $H(j\omega)$, built from cascading two subsistems with frequency responses $H_1(j\omega)$ and $H_2(j\omega)$, respectively. The figure shows the asymptotic approximation of the Bode diagrams corresponding to the first subsystem $H_1(j\omega)$ and the total (cascading) system $H(j\omega)$. Sketch the Bode diagram corresponding to the second subsystem $H_2(j\omega)$.

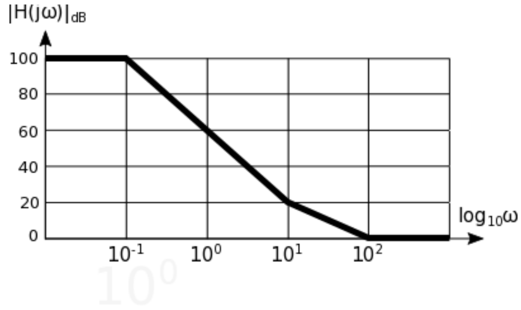


4. The output of a causal LTI system is related with the input by the differential equation: $\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 2y(t) = x(t)$.
   (a) Determine the frequency response of the system.
   (b) Sketch the Bode diagram (only magnitude) of the system.
   (c) if $x(t) = e^{-t}u(t)$, determine the output of the system $y(t)$.

5. Sketch the asymptotic approximations of the Bode diagrams (only magnitude) corresponding to the systems below:
   (a) $H(j\omega) = j\omega + a$
   (b) $H(j\omega) = 10 \cdot \frac{j\omega+0.1}{j\omega+10}$
   (c) $H(j\omega) = 0.1 \cdot \frac{j\omega+10}{j\omega+0.1}$

6. Sketch the asymptotic approximations of the Bode diagrams (only magnitude) corresponding to the system $H(j\omega) = \frac{10(j\omega)^2+2j\omega+0.1}{(j\omega)^2+11j\omega+10}$

7. The figure below shows the asymptotic approximation of the Bode diagram (only magnitude) of an LTI system. Determine the transfer function of the inverse of this system.



8. We want to build a low-pass filter whose specifications in the frequency domain are:
   (a) cut-off frequency $\omega_c = 100$ rad/s;
   (b) no ripple at all in the bandpass;
   (c) supression band below -35dB at $\omega > 1000$ rad/s.

   Determine the transfer function of a possible filter satisfying the specifications.