Signals and Systems 2

Giovanni Geraci — Universitat Pompeu Fabra, Barcelona

Tutorial #2: Fourier transform

1. Consider the signal $x(t) = \frac{20\sin(2\pi t)}{\pi t}$
   (a) Find A and θ such that $x(t) = A\,sinc(\theta)$, where *sinc* is the normalized variant.
   (b) Make a carefully labeled sketch of $x(t)$ for $t \in [-2, 2]$

2. Determine the CTFT of the following signals, using the Fourier transform definition.
   (a) $x(t) = \delta(t+1) + \delta(t-1)$
   (b) $x(t) = \frac{\mathrm{d}}{\mathrm{d}t}[u(-t-2) + u(t-2)]$

3. Determine the CTFT of the periodic impulse train: $x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

4. Determine the CTFT of the following signals, using known Fourier transform pairs:
   (a) $x(t) = \begin{cases} 1, & -2 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$
   (b) $x(t) = u(t+3)\,u(3-t)$
   (c) $x(t) = \delta(t+2) + 2\delta(t) + \delta(t-2)$

5. Determine the inverse CTFT of the following spectra, using known Fourier transform pairs:
   (a) $X(j\omega) = \frac{1}{-0.1+j\omega}$
   (b) $X(j\omega) = \frac{j\omega}{-0.4+j\omega}$
   (c) $X(j\omega) = \frac{j\omega}{4+2j\omega}$
   (d) $X(j\omega) = 2 + 2\cos(\omega)$
   (e) $X(j\omega) = \frac{1}{1+j\omega} - \frac{1}{2+j\omega}$
   (f) $X(j\omega) = \frac{1}{1+j\omega} + \frac{1}{-2+j\omega}$
   (g) $X(j\omega) = j\delta(\omega - 100\pi) - j\delta(\omega + 100\pi)$
   (h) $X(j\omega) = 4\pi\delta(\omega) + 2\pi\delta(\omega - 10\pi) + 2\pi\delta(\omega + 10\pi)$

6. Consider the ideal delay system $y(t) = x(t - \frac{1}{2})$.
   (a) Derive the system's frequency response from the Fourier transform and its properties.
   (b) Plot the magnitude and phase of the frequency response.

7. Consider the LTI system $y(t) = x(t+1) + 2x(t) + x(t-2)$.

   (a) Determine the impulse response of the system.

   (b) Determine the frequency response of the system by explicit (integral) Fourier transform of the impulse response.

   (c) Determine the output of the system for the input $x(t) = e^{j\omega t}$ and demonstrate that $y(t) = H(j\omega)e^{j\omega t}$.

8. Use the convolution property to determine the inverse Fourier transform of $Y(j\omega) = \left(\frac{sin(2\omega)}{\omega/2}\right)\left(\frac{\sin\omega}{\omega/2}\right)$

9. Determine the Fourier transform of the following signals:

   (a) $x(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

   (b) $x(t) = 20\frac{\sin(200\pi(t-10))}{\pi(t-10)}$

   (c) $x(t) = e^{-4t}\,u(t) - e^{-4t}\,u(t-10)$

10. Determine the Fourier transform of $h(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\sin(4\pi t)}{\pi t}\right)$, and its magnitude.

11. Given the Fourier transform pair $x(t) \leftrightarrow X(j\omega)$, determine the Fourier transform of the following signals (in terms of $X(j\omega)$):

    (a) $x(1-t) + x(-1-t)$

    (b) $x(3t-6)$

    (c) $\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,x(t-1)$

12. The impulse response of an LTI system is given by $h(t) = \frac{4\sin(\omega_b t)}{\pi t}$, and the input to the system is $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

    (a) Determine $X(j\omega)$ and plot it over the range: $-6\pi \leq \omega \geq 6\pi$.

    (b) Determine the frequency response of the system and plot its magnitude for the case $\omega_b = 5\pi$.

    (c) Determine the output of the system when $\omega_b = 5\pi$.

    (d) Determine the values of ω_b so that the output is a constant C, and determine the value of C.

13. Consider the triangular function $x(t) = \begin{cases} 1-|t|, & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$. Using the fact that $x(t)$ can be expressed as the convolution of two rectangular pulses, determine $X(j\omega)$.