Signals and Systems 2

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Tutorial #1: Signals and systems

1. Write two formulas for the signal below, one as a sum of step functions, another as a product of step functions.
$$x(t) = \begin{cases} 1, & a \leq t \leq b \\ 0, & \text{otherwise} \end{cases}$$

2. Simplify the following signals, using the properties of the basic signals. Be careful and distinguish between multiplication and convolution.
   (a) $\delta(t-10) * [\delta(t+10) + 2e^{-t}\, u(t)]$
   (b) $\cos(100\pi t)\, [\delta(t) + \delta(t-0.02)]$
   (c) $\frac{d}{dt}[e^{-2(t-2)}\, u(t-2)]$
   (d) $\int_{-\infty}^{t} \cos(100\pi\tau)\, [\delta(\tau) + \delta(\tau - 0.02)]\, \mathrm{d}\tau$
   (e) $\delta(t-1) * \delta(t-2) * \delta(t)$
   (f) $e^{-(t-4)}\, u(t-4)\, \delta(t-5)$
   (g) $\int_{-\infty}^{t-5} \delta(\tau - 1)\, \mathrm{d}\tau$
   (h) $e^{-4t}\, u(t)\, [\delta(t+1) + \delta(t-1)]$
   (i) $\int_{-\infty}^{\infty} e^{-4\tau}\, u(\tau)\, \delta(t-\tau)\, \mathrm{d}\tau$

3. Use the convolution integral to calculate the output of the system $y(t) = x(t) * h(t)$ for each case:
   (a) $x(t) = h(t) = u(t)$
   (b) $x(t) = h(t) = e^{-at}\, u(t)$
   (c) $x(t) = e^{-at}\, u(t)$, $h(t) = e^{-bt}\, u(t)$, $a \neq b$
   (d) $x(t) = \begin{cases} 1, & 0 \leq t \leq a \\ 0, & \text{otherwise} \end{cases}, \quad h(t) = \begin{cases} 1, & 0 \leq t \leq b \\ 0, & \text{otherwise} \end{cases}, \quad a < b$

4. Determine $h(t)$ such that: $e^{-(t-4)}\,u(t-4) * h(t) = 2e^{-t}\,u(t)$

5. Consider a system with impulse response $h(t) = \begin{cases} e^{-0.1(t-2)}, & 2 \leq t \leq 12 \\ 0, & \text{otherwise} \end{cases}$

   (a) Reason whether the system is stable or not.

   (b) Reason whether the system is causal or not.

   (c) Determine the output $y(t)$ for an input $x(t) = \delta(t-2)$

6. Consider an input signal $x(t) = u(t) - 2u(t-4) + u(t-6)$ fed into a system with impulse response $h(t) = \begin{cases} t+1, & -1 \leq t < 0 \\ t-1, & 0 < t \leq 1 \\ 0, & \text{otherwise} \end{cases}$. **Without** determining the output $y(t)$:

   (a) Determine $y(0)$

   (b) Determine all the values of t for which the output is $y(t) = 0$

   (c) Determine the value of t for which y(t) has the largest negative value.

   (d) Reason whether the system is stable or not.

   (e) Reason whether the system is causal or not.

7. Consider a system defined by the input/output relation: $y(t) = \int\limits_{t-2}^{t+2} x(\tau)\mathrm{d}\tau$

   (a) Determine $h(t)$

   (b) Reason whether the system is stable or not.

   (c) Reason whether the system is causal or not.

8. Consider three subsystems: $h_1(t) = u(t+3)$, $\quad h_2(t) = u(t-5)$, $\quad h_3(t) = \delta(t-t_d)$, and an overall system comprised of parallel $h_1(t) - h_2(t)$ in cascade with $h_3(t)$.

   (a) Determine the impulse reponse of the overall system.

   (b) Determine the values of t_d for which the overall system is causal.

   (c) Reason whether each subsystem and the overall system are stable or not.