

Figure 1: Basic Network

# Seminar 1: Exercises

## Exercise 1

Consider the link between R2 (Router 2) and AN1 (Access Network 1) in Figure 1. It has a length equal to  $d = 2$  Km, and its propagation speed is equal to  $3 \cdot 10^8$  m/s. R2 sends packets of size  $L = 11000$  bits to AN1, and they are transmitted at a rate  $R = 100$  Mbps.

1. Calculate the propagation delay,  $D_p$ , that suffers a packet between R2 and AN1.
2. Calculate the transmission delay,  $D_s$ , for a packet transmitted by R2.
3. Calculate the time elapsed since a single packet starts to be transmitted at R2 until it is completely received at AN1.
4. If the processing delay at AN1 is negligible, i.e.,  $D_c = 0$ , and R2 transmits packets continuously, i.e., without any delay between two consecutive packets. How many packets are transmitted in 10 seconds?

5. If each packet has a header of  $L_h = 200$  bits. How many "data" bits / second are successfully received by AN1 given R2 transmits packets continuously?

## Exercise 2

Consider the link between AN6 and R1 in Figure 1. The traffic sent over that link is measured during 10 seconds. The obtained measures (Figure 2) show that there are two traffic flows, one from the TV Broadcasting Server to User 1 and one from the Video Server to User 7. The packets send by the TV Broadcasting Server have a size of  $L_{U1} = 450$  bits and the packets send by the Video Server have a size of  $L_{U7} = 10000$  bits.

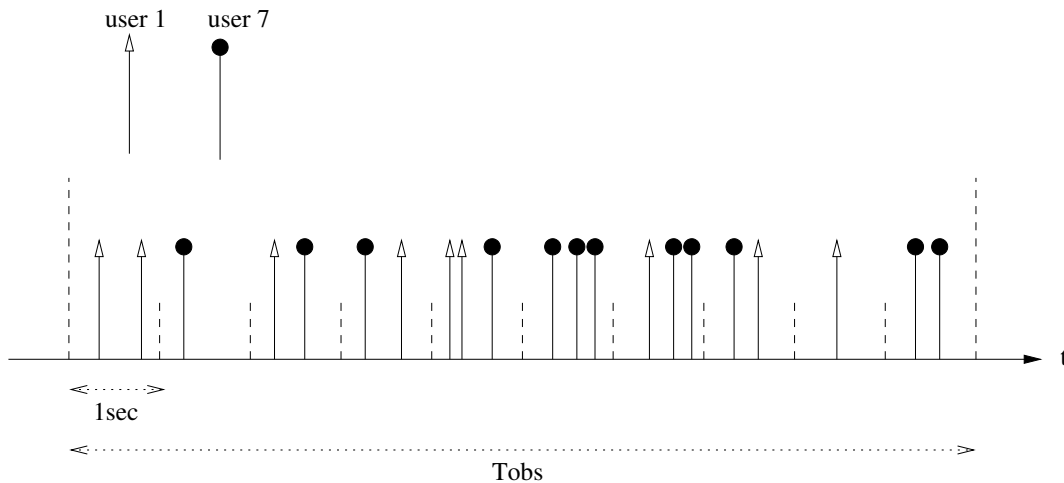


Figure 2: Traffic Measurements in the link between AN6 and R1

1. If  $P$  is the random variable of the number of packets observed in the link between AN6 and R1 in 1 second, compute the Histogram of  $P$ ,  $E[P]$ ,  $V[P]$  and its  $CV[P]$ . To compute the variance, use the second moment of  $P$ , i.e.,  $E[P^2]$ .
2. Repeat the previous point, but now for the random variables  $P_{U1}$  and  $P_{U7}$ , which respectively are the number of packets directed to User 1 and User 7 observed in 1 second.
3. From the measurements, it is observed that the time between two packets follows an exponential distribution with an expected value equal to  $E[\tau] = \frac{1}{E[P]}$ . Remember that the exponential distribution has the following probability density function,  $f_\tau(t) = \frac{1}{E[P]} e^{-\frac{1}{E[P]}t}$ , and its variance is  $V[\tau] = \frac{1}{E^2[P]}$ . Then, calculate the value of  $E[\tau]$ ,  $E[\tau^2]$  and  $CV[\tau]$ .
4. If  $L$  is the random variable of the packet sizes observed in the link between AN6 and R1, compute the Histogram of  $L$ ,  $E[L]$ ,  $V[L]$  and  $CV[L]$ . To compute the variance, use the second moment of  $L$ , i.e.,  $E[L^2]$ .
5. Packets are transmitted at rate  $R = 100$  Mbps over the link between AN6 and R1. Then, the transmission time of each packet is  $D_s = \frac{L}{R}$ . Is  $D_s$  a random variable? If yes, compute

the Histogram of  $D_s$ ,  $E[D_s]$ ,  $V[D_s]$  and its  $CV[D_s]$ . To compute the variance, use the second moment of  $D_s$ , i.e.,  $E[D_s^2]$ .

## Solution to Exercise 1

1. The propagation delay a packet suffers is

$$D_p = \frac{d}{v_{\text{prop}}} = \frac{2 \cdot 10^3}{3 \cdot 10^8} = \frac{20}{3} \cdot 10^{-6} \approx 6.67 \mu\text{s}. \quad (1)$$

2. The transmission delay is given by

$$D_s = \frac{L}{R} = \frac{11000}{100} \cdot 10^{-6} = 110 \mu\text{s}. \quad (2)$$

3. The time it takes since a packet begins to be transmitted at R2 until it is completely received at AN 1,  $D_r$ , equals the sum of the transmission and propagation delays

$$D_r = D_s + D_p = 116.67 \mu\text{s}. \quad (3)$$

4. The number of packets transmitted in 10 seconds by R2 is simply:

$$\frac{10}{D_s} = 90909 \text{ packets}. \quad (4)$$

where you can notice the propagation delay is not included. The propagation delay just represents a delay between what happens at the transmitter and what happens at the receiver, and so it does not affect to the number of packets transmitted by R2.

5. The number of packets that are successfully received in 1 second by AN1 is

$$\frac{1}{D_s} = 9090.9 \text{ packets} \quad (5)$$

and so the number of data bits per second is

$$9090.9 \cdot (11000 - 200) = 98.181 \text{ Mbps}. \quad (6)$$

## Solution to Exercise 2

The number of packets counted in each second is  $P(t_i) = [2, 1, 2, 2, 3, 3, 3, 2, 1, 2]$ , with  $i = 1 \dots 10$ . As we can observe, the state space of  $P$  is  $\mathcal{P} = \{1, 2, 3\}$ .

The probability to observe 1, 2 or 3 packets in each interval is:

$$P\{P = 1\} = \frac{2}{10} \quad (7)$$

$$P\{P = 2\} = \frac{5}{10} \quad (8)$$

$$P\{P = 3\} = \frac{3}{10} \quad (9)$$

$$(10)$$

which is the histogram of  $P$ .

Once we have the histogram of  $P$ , we can calculate all the other parameters:

$$E[P] = \frac{2}{10} + 2\frac{5}{10} + 3\frac{3}{10} = 2.1 \text{ packets/second} \quad (11)$$

$$E[P^2] = \frac{2}{10} + 2^2\frac{5}{10} + 3^2\frac{3}{10} = 4.9 \text{ (packets/second)}^2 \quad (12)$$

$$V[P] = E[P^2] - E^2[P] = 0.49 \text{ (packets/second)}^2 \quad (13)$$

$$CV[P] = \frac{\sqrt{V[P]}}{E[P]} = 0.33 \quad (14)$$

If we now only consider the packets from User 1, we have:

$$P\{P_{U1} = 0\} = \frac{3}{10} \quad (15)$$

$$P\{P_{U1} = 1\} = \frac{5}{10} \quad (16)$$

$$P\{P_{U1} = 2\} = \frac{2}{10} \quad (17)$$

$$(18)$$

which is the histogram of  $P_{U1}$ .

Once we have the histogram of  $P_{U1}$ , we can calculate all the other parameters:

$$E[P_{U1}] = 0.9 \text{ packets/second} \quad (19)$$

$$E[P_{U1}^2] = 1.3 \text{ (packets/second)}^2 \quad (20)$$

$$V[P_{U1}] = E[P_{U1}^2] - E^2[P_{U1}] = 0.49 \text{ (packets/second)}^2 \quad (21)$$

$$CV[P_{U1}] = \frac{\sqrt{V[P_{U1}]}}{E[P_{U1}]} = 0.778 \quad (22)$$

If we now only consider the packets from User 7, we have:

$$P\{P_{U7} = 0\} = \frac{2}{10} \quad (23)$$

$$P\{P_{U7} = 1\} = \frac{5}{10} \quad (24)$$

$$P\{P_{U7} = 2\} = \frac{2}{10} \quad (25)$$

$$P\{P_{U7} = 3\} = \frac{1}{10} \quad (26)$$

$$(27)$$

which is the histogram of  $P_{U7}$ .

Once we have the histogram of  $P_{U7}$ , we can calculate all the other parameters:

$$E[P_{U7}] = 1.2 \text{ packets/second} \quad (28)$$

$$E[P_{U7}^2] = 2.2 \text{ (packets/second)}^2 \quad (29)$$

$$V[P_{U7}] = E[P_{U7}^2] - E^2[P_{U7}] = 0.76 \text{ (packets/second)}^2 \quad (30)$$

$$CV[P_{U7}] = \frac{\sqrt{V[P_{U7}]}}{E[P_{U7}]} = 0.726 \quad (31)$$

From the measurement, it is observed that the time between two packets follows an exponential distribution, with average  $E[\tau] = \frac{1}{2.1} = 0.4762$  seconds, variance  $V[\tau] = \frac{1}{2.1^2} = 0.2268$  seconds<sup>2</sup>, second moment  $E[\tau^2] = \frac{1}{2.1} = 0.4535$  seconds<sup>2</sup> and  $CV[\tau] = 1$ .

Regarding the packet sizes, as all the packets sent by the same user have the same size, the histogram of the packet sizes is simply the number of packets of each user over the total number of packets counted.

$$P\{L = 450\} = 0.42857 \quad (32)$$

$$P\{L = 10000\} = 0.57143 \quad (33)$$

$$(34)$$

The other requested parameters are:

$$E[L] = 5907.1 \text{ bits} \quad (35)$$

$$E[L^2] = 5.723 \cdot 10^7 \text{ bits}^2 \quad (36)$$

$$V[L] = E[L^2] - E^2[L] = 2.233 \cdot 10^7 \text{ bits}^2 \quad (37)$$

$$CV[L] = \frac{\sqrt{V[L]}}{E[L]} = 0.8. \quad (38)$$

Finally, it is important to see that the transmission delay of each packet over the target link depends on both the packet size and the transmission rate. As the transmission rate is fixed, all the randomness of  $D_s$  will be only related to  $L$ . Therefore, the results for  $D_s$  will be exactly the same as for  $L$  but scaled by  $R$ . Then, we have:

$$P\left\{D_s = \frac{450}{R}\right\} = 0.42857 \quad (39)$$

$$P\left\{D_s = \frac{10000}{R}\right\} = 0.57143. \quad (40)$$

The other requested parameters are:

$$E[D_s] = 5.907 \cdot 10^{-5} \text{ seconds/packet} \quad (41)$$

$$E[D_s^2] = 5.723 \cdot 10^{-9} \text{ (seconds/packet)}^2 \quad (42)$$

$$V[D_s] = E[D_s^2] - E^2[D_s] = 2.2337 \cdot 10^{-9} \text{ (seconds/packet)}^2 \quad (43)$$

$$CV[D_s] = \frac{\sqrt{V[D_s]}}{E[L]} = 0.8. \quad (44)$$