



Figure 1: Basic Network

Seminar 4: Exercises

Exercise 1

The transmitter associated to the link between R4 and R3 in Figure 1 is modeled by an M/M/1/K queue. The aggregate arrival rate to this link is $\lambda = 24000$ packets/second, the packet size follow an exponential distribution with average $E[L] = 8000$ bits and the transmission rate is $R = 200$ Mbps. If the operator of the network wants to guarantee a blocking probability lower than $P_b < 10^{-3}$:

1. Derive an expression to find K in terms of P_b .
2. What is the numerical value of K ?
3. What is the option that gives the best performance? Justify the results:
 - Previous case.
 - $K' = \lceil K/2 \rceil$ and $R' = 2R$.
 - $K'' = \lceil K/4 \rceil$ and $R'' = 4R$.

Exercise 2

Router R2 receives packets directed to the hosts in AN1 and AN7 from R1, R3 and R4. Namely, it receives $\lambda_{R1} = 1200$ packets/second from R1, $\lambda_{R3} = 2400$ packets/second from R3 and $\lambda_{R4} = 900$ packets/second from R4. The expected packet size is $E[L] = 12000$ bits, and the packet size follows an exponential distribution.

The probability that a packet received in R2 is directed to AN1 is α_1 and to AN7 is α_7 .

The links/transmitters that interconnect R2 with AN1 and AN7 have the following characteristics:

- Link from R2 to AN1: $R = 20$ Mbps, $K = \infty$ (M/M/1).
- Link from R2 to AN7: $R = 40$ Mbps, $K = \infty$ (M/M/1).

Then:

1. Is the arrival process at each queue Poisson? Justify the answer based on the properties of Poisson processes.
2. Find the values of α_7 and α_1 that result in the same expected waiting delay ($E[D_q]$) for both queues.
3. Considering previous values for α_7 and α_1 , calculate the expected queue occupancy ($E[N_q]$), the system occupancy $E[N]$ and the total delay ($E[D]$) for each queue.

Exercise 3

User 1 is sending a data flow of $B = 8$ Mbps to User 3, where packets are generated following a Poisson process. The data flow can follow three different paths: a) R2→R4, b) R2→R3→R4, c) R2→R1→R3→R4. The background traffic in each link of those routes follows a Poisson distribution:

- R2→R1: 10 Mbps
- R2→R3: 70 Mbps
- R2→R4: 90 Mbps
- R1→R3: 20 Mbps
- R3→R4: 5 Mbps

where $R_i \rightarrow R_j$ denotes the background traffic passing through routers R_i and R_j . The transmission rate of all links is $R=100$ Mbps, and packets sizes are exponentially distributed with mean $E[L] = 250$ Bytes. Propagation delays are the same for all links and equal to $10 \mu s$. In addition, all processing delays are negligible, and buffers are large enough to be considered of infinite size. Therefore, all interfaces can be modelled as M/M/1 queues. Note that since the hops User1→AN1, AN1→R2, and R4→AN2, AN2→User3 are the same in all routes, they do not affect the route election.

1. Represent the sequence of interfaces that are traversed by the data flow in all 3 cases.
2. Calculate the end to end delay in the 3 cases.
3. Explain why the optimal routing depends on the background traffic.

Solution to Ex1

1. The blocking probability for a M/M/1/K queue is:

$$P_b = \frac{(1-a)a^K}{1-a^{K+1}}$$

Goal: $K = f(P_b, a)$, so:

$$\begin{aligned}P_b - P_b a^{K+1} &= (1-a)a^K \\P_b &= (1-a)a^K + P_b a^{K+1} = a^K(1-a + P_b a) \\ \ln(P_b) &= K \ln(a) + \ln(1-a + P_b a) \\ K &= \left\lceil \frac{\ln(P_b) - \ln(1-a + P_b a)}{\ln(a)} \right\rceil \\ K &= \left\lceil \frac{\ln(a + \frac{1-a}{P_b})}{\ln(a)} \right\rceil\end{aligned}$$

2. With $a = \frac{\lambda}{\mu} = \frac{24}{25} = 0.96$, $K = \lceil 90.94 \rceil = 91$ packets.
3. We need to calculate P_b for each of the three cases to compare them:
 - Case 1: $P_b = 1 \cdot 10^{-3}$
 - Case 2: now, $a = 24000 \cdot \frac{8000}{400 \cdot 10^6} = 0.480$, and $K' = \lceil \frac{91}{2} \rceil = 46$.

$$P_b = \frac{a^{46}(1-a)}{1-a^{47}} = 1.13 \cdot 10^{-15}$$

- Case 3:

$$a = 24000 \cdot \frac{8000}{800 \cdot 10^6} = 0.240, K'' = 23$$

$$P_b = \frac{a^{23}(1-a)}{1-a^{24}} = 4.22 \cdot 10^{-15}$$

Solution to Ex2

1. First all arrivals are aggregated into one (Poisson) arrival process, and then are randomly assigned to the individual queues, which again results in a Poisson process.

2. The expected delay $E[D_q] = \frac{a}{\mu - \lambda}$ should be equal for both queues. With the according values for the individual a, μ, λ we get:

$$\begin{aligned}
E[D_{q1}] &= E[D_{q7}] \\
\frac{\frac{\alpha_1 \lambda}{\mu_1}}{\mu_1 - \alpha_1 \lambda} &= \frac{\frac{\alpha_7 \lambda}{\mu_7}}{\mu_7 - \alpha_7 \lambda} \\
\frac{\alpha_1}{\mu_1^2 - \alpha_1 \mu_1 \lambda} &= \frac{\alpha_7}{\mu_7^2 - \alpha_7 \mu_7 \lambda} \\
\frac{(1 - \alpha_7)}{\mu_1^2 - (1 - \alpha_7) \mu_1 \lambda} &= \frac{\alpha_7}{\mu_7^2 - \alpha_7 \mu_7 \lambda} \\
(1 - \alpha_7) \cdot (\mu_7^2 - \alpha_7 \mu_7 \lambda) &= \alpha_7 \cdot (\mu_1^2 - (1 - \alpha_7) \mu_1 \lambda)
\end{aligned}$$

$$\begin{aligned}
\mu_7^2 - \alpha_7 \mu_7 \lambda - \alpha_7 \mu_7^2 + \alpha_7^2 \mu_7 \lambda - \alpha_7 \mu_1^2 + \alpha_7 \mu_1 \lambda - \alpha_7^2 \mu_1 \lambda &= 0 \\
\alpha_7^2 (\mu_7 \lambda - \mu_1 \lambda) - \alpha_7 (\mu_7 \lambda + \mu_7^2 + \mu_1^2 - \mu_1 \lambda) + \mu_7^2 &= 0 \\
\Rightarrow \alpha_7 &= 0.683 \\
\alpha_1 &= 0.316
\end{aligned}$$

Check:

$$E[D_{q1}] = \frac{\frac{0.316 \cdot 4500 \frac{1}{s}}{\mu_1}}{\mu_1 - 0.316 \cdot 4500 \frac{1}{s}} = 0.0035s$$

$$E[D_{q7}] = \frac{\frac{0.683 \cdot 4500 \frac{1}{s}}{\mu_7}}{\mu_7 - 0.683 \cdot 4500 \frac{1}{s}} = 0.0035s$$

3. The queue occupancy $E[N_q]$ can be derived from the waiting delay $E[D_q]$, using Little's Theorem:

$$E[N_{qi}] = \lambda_i \cdot E[D_{qi}].$$

Therefore,

$$E[N_{q1}] = 4.98, E[N_{q7}] = 10.76.$$

The system occupancy is the queue occupancy plus the server/link occupancy, which in turn is the system offer a in a M/M/1/ ∞ system:

$$E[N_i] = E[N_{qi}] + a_i$$

$$E[N_1] = 5.83, E[N_7] = 11.68$$

Finally, applying Little again, we get for the total delay:

$$E[D_i] = \frac{E[N_i]}{\lambda_i}$$

$$\Rightarrow E[D_1] = 0.0041s, E[D_7] = 0.0038s$$

Solution to Ex3

```
clear
clc
%% EX1
B=8E6;
EL=250*8;
lambda = B/EL
Dp = 10E-6;

C=100E6;
mu = C/EL

% Route 1
lambdaR2R4 = 90E6/EL;
DR2R4 = 1 / (mu-lambda-lambdaR2R4);
DRoute1 = DR2R4 + Dp;

% Route 2
lambdaR2R3 = 70E6/EL;
DR2R3 = 1 / (mu-lambda-lambdaR2R3);
lambdaR3R4 = 5E6/EL;
DR3R4 = 1 / (mu-lambda-lambdaR3R4);
DRoute2 = DR2R3 + DR3R4 + 2*Dp;

% Route 3
lambdaR2R1 = 10E6/EL;
DR2R1 = 1 / (mu-lambda-lambdaR2R1);

lambdaR1R3 = 20E6/EL;
DR1R3 = 1 / (mu-lambda-lambdaR1R3);
lambdaR3R4 = 5E6/EL;
DR3R4 = 1 / (mu-lambda-lambdaR3R4);

DRoute3 = DR2R1 + DR1R3 + DR3R4 + 3*Dp;

disp('Results');
DRoute1
DRoute2
DRoute3
```