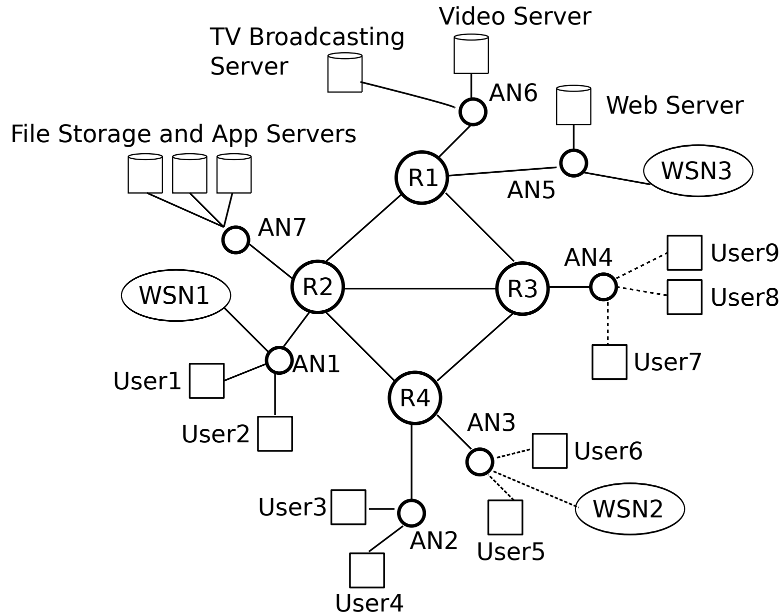

Figure 1: Basic Network

Seminar 1: Exercises

Exercise 1

Consider the link between R2 (Router 2) and AN1 (Access Network 1) in Figure 1. It has a length equal to $d = 2$ Km, and its propagation speed is equal to $3 \cdot 10^8$ m/s. R2 sends packets of size $L = 11000$ bits to AN1, and they are transmitted at a rate $R = 100$ Mbps.

1. Calculate the propagation delay, D_p, that suffers a packet between R2 and AN1.
2. Calculate the transmission delay, D_s, for a packet transmitted by R2.
3. Calculate the time between a packet starts to be transmitted at R2 until it is completely received at AN1.
4. If the processing delay at AN1 is negligible, i.e., $D_c = 0$, and R2 transmits packets continuously, i.e., without any delay between two consecutive packets. How many packets are transmitted in 10 seconds?

5. If each packet has a header of $L_h = 200$ bits. How many "data" bits / second are successfully received by AN1?

Exercise 2

Let us consider the "wireless" link between WSN2 and AN3. Packets of $L_d = 100$ Bytes with $L_h = 10$ Bytes are now transmitted by WSN2 at a rate of $R = 50$ Kbps. The distance between WSN2 and AN3 is of 10 meters, so the propagation delay can be considered negligible.

If WSN2 wants to transmit a file of size $F = 1$ MByte, and the time between two consecutively transmitted packets is of $T = 1$ second:

1. How many packets WSN2 has to transmit to AN3?
2. What is the file transfer delay between WSN2 and AN3?
3. If the maximum "acceptable" file transfer delay is of 100 seconds, what is the maximum size of the files that can be transmitted between WSN2 and AN3?

Exercise 3

Consider the link between AN6 and R1 in Figure 1. The traffic sent over that link is measured during 10 seconds. The obtained measures (Figure 2) show that there are two traffic flows, one from the TV Broadcasting Server to User 1 and one from the Video Server to User 7. The packets send by the TV Broadcasting Server have a size of $L_{\mathrm{U1}} = 450$ bits and the packets send by the Video Server have a size of $L_{\mathrm{U7}} = 10000$ bits.

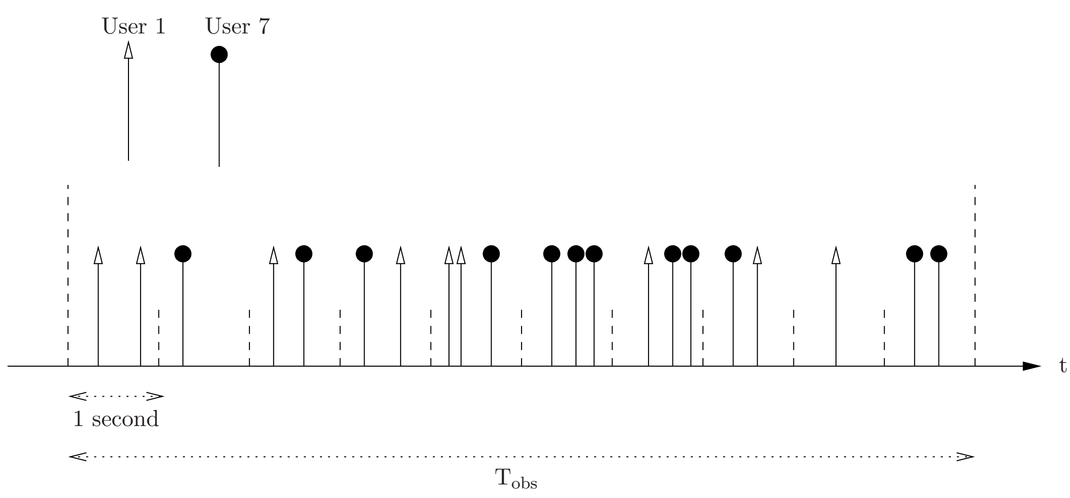

Figure 2: Traffic Measurements in the link between AN6 and R1

1. If P is the random variable of the number of packets observed in the link between AN6 and R1 in 1 second, compute the Histogram of P, $\mathrm{E}[P]$, $\mathrm{V}[P]$ and its $\mathrm{CV}[P]$. To compute the variance, use the second moment of P, i.e., $\mathrm{E}[P^2]$.

2. Repeat the previous point, but now for the random variables P_{U1} and P_{U7}, which respectively are the number of packets directed to User 1 and User 7 observed in 1 second.

3. From the measurements, it is observed that the time between two packets follows an exponential distribution with an expected value equal to $\mathrm{E}[\tau] = \frac{1}{\mathrm{E}[P]}$. Remember that the exponential distribution has the following probability density function, $f_\tau(t) = \frac{1}{\mathrm{E}[P]} e^{-\frac{1}{\mathrm{E}[P]}t}$, and its variance is $\mathrm{V}[\tau] = \frac{1}{\mathrm{E}^2[P]}$. Then, calculate the value of $\mathrm{E}[\tau]$, $\mathrm{E}[\tau^2]$ and $\mathrm{CV}[\tau]$.

4. If L is the random variable of the packet sizes observed in the link between AN6 and R1, compute the Histogram of L, $\mathrm{E}[L]$, $\mathrm{V}[L]$ and $\mathrm{CV}[L]$. To compute the variance, use the second moment of L, i.e., $\mathrm{E}[L^2]$.

5. Packets are transmitted at rate $R = 100$ Mbps over the link between AN6 and R1. Then, the transmission time of each packet is $D_{\mathrm{s}} = \frac{L}{R}$. Is D_{s} a random variable? If yes, compute the Histogram of D_{s}, $\mathrm{E}[D_{\mathrm{s}}]$, $\mathrm{V}[D_{\mathrm{s}}]$ and its $\mathrm{CV}[D_{\mathrm{s}}]$. To compute the variance, use the second moment of D_{s}, i.e., $\mathrm{E}[D_{\mathrm{s}}^2]$.

Solution to Exercise 1

1. The propagation delay a packet suffers is given by

$$D_p = \frac{d}{u} = \frac{2 \cdot 10^3}{3 \cdot 10^8} = \frac{20}{3} \cdot 10^{-6} \approx 6.67\ \mu s \tag{1}$$

2. The transmission delay is given by

$$D_s = \frac{L}{R} = \frac{11000}{100} \cdot 10^{-6} = 110\ \mu s \tag{2}$$

3. The time it takes to receive a packet, D_r, equals the sum of the transmission and propagation delays

$$D_r = D_s + D_p = 116.67\ \mu s \tag{3}$$

4. The time it takes to receive N packets, $D_r(N)$, is given by the equation

$$D_r(N) = ND_s + D_p \tag{4}$$

   Therefore, the number of packets received in 10 seconds, i.e., $D_r(N) = 10$, is

$$N = \left\lfloor \frac{10 - D_p}{D_s} \right\rfloor = \left\lfloor \frac{10 - 6.67 \cdot 10^{-6}}{110 \cdot 10^{-6}} \right\rfloor = 90909 \text{ packets} \tag{5}$$

   where $\lfloor \cdot \rfloor$ denotes the floor function.

5. The number of packets that are successfully received in 1 second is

$$N = \left\lfloor \frac{1 - D_p}{D_s} \right\rfloor = \left\lfloor \frac{1 - 6.67 \cdot 10^{-6}}{110 \cdot 10^{-6}} \right\rfloor = 9090 \text{ packets} \tag{6}$$

   and the number of data bits per second is

$$N_{bps} = N(L - L_h) = 9090 \cdot (11000 - 200) = 98.172 \text{ Mbps} \tag{7}$$

Solution to Exercise 2

1. The number of packets WSN2 has to transmit to AN3 is

$$N = \left\lceil \frac{F}{L_d} \right\rceil = \left\lceil \frac{1024^2}{800} \right\rceil = 1311 \text{ packets} \tag{8}$$

   where $\lceil \cdot \rceil$ denotes the ceil function and is due to zero padding.

2. The size of a packet in bits is

$$L = 8 \cdot (L_h + L_d) = 8 \cdot (100 + 10) = 880 \text{ bits} \tag{9}$$

Therefore, the time it takes to transmit a packet is

$$D_s = \frac{L}{R} = \frac{880}{50} \cdot 10^{-3} = 17.6 \ ms \tag{10}$$

The file transfer delay between WSN2 and AN3 is given by the equation

$$D_f = ND_s + (N-1)T = 1311 \cdot 17.6 \cdot 10^{-3} + 1310 = 1333.1 \ s \quad (\approx 22\text{mins}) \tag{11}$$

3. We want the delay transfer to be at most 100 seconds, i.e., $D_f = 100$. Therefore, the maximum file size that can be transmitted between WNS2 and AN3 is

$$N = \left\lfloor \frac{D_f + T}{D_s + T} \right\rfloor = \left\lfloor \frac{100 + 1}{17.6 \cdot 10^{-3} + 1} \right\rfloor = \left\lfloor \frac{101 \cdot 10^3}{17.6 + 1 \cdot 10^3} \right\rfloor = 99 \text{ packets} \tag{12}$$

or in bits

$$N_b = N \cdot L_d = 99 \cdot 800 = 79200 \text{ bits} \quad (77.344 \text{ KByte}) \tag{13}$$

Solution to Exercise 3

The number of packets counted in each second is $P(t_i) = [2, \ 1, \ 2, \ 2, \ 3, \ 3, \ 3, \ 2, \ 1, \ 2]$, with $i = 1 \dots 10$. As we can observe, the state space of P is $\mathcal{P} = \{1, \ 2, \ 3\}$.

The probability to observe 1, 2 or 3 packets in each interval is:

$$\mathrm{P}\{P = 1\} = \frac{2}{10} \tag{14}$$

$$\mathrm{P}\{P = 2\} = \frac{5}{10} \tag{15}$$

$$\mathrm{P}\{P = 3\} = \frac{3}{10} \tag{16}$$

$$\tag{17}$$

which is the histogram of P.

Once we have the histogram of P, we can calculate all the other parameters:

$$\mathrm{E}[P] = \frac{2}{10} + 2\frac{5}{10} + 3\frac{3}{10} = 2.1 \text{ packets/second} \tag{18}$$

$$\mathrm{E}[P^2] = \frac{2}{10} + 2^2\frac{5}{10} + 3^2\frac{3}{10} = 4.9 \text{ (packets/second)}^2 \tag{19}$$

$$\mathrm{V}[P] = \mathrm{E}[P^2] - \mathrm{E}^2[P] = 0.49 \text{ (packets/second)}^2 \tag{20}$$

$$\mathrm{CV}[P] = \frac{\sqrt{\mathrm{V}[P]}}{\mathrm{E}[P]} = 0.33 \tag{21}$$

If we now only consider the packets from User 1, we have:

$$\mathrm{P}\{P_{\mathrm{U1}} = 0\} = \frac{3}{10} \tag{22}$$
$$\mathrm{P}\{P_{\mathrm{U1}} = 1\} = \frac{5}{10} \tag{23}$$
$$\mathrm{P}\{P_{\mathrm{U1}} = 2\} = \frac{2}{10} \tag{24}$$
$$\tag{25}$$

which is the histogram of P_{U1}.

Once we have the histogram of P_{U1}, we can calculate all the other parameters:

$$\mathrm{E}[P_{\mathrm{U1}}] = 0.9 \text{ packets/second} \tag{26}$$
$$\mathrm{E}[P_{\mathrm{U1}}^2] = 1.3 \text{ (packets/second)}^2 \tag{27}$$
$$\mathrm{V}[P_{\mathrm{U1}}] = \mathrm{E}[P_{\mathrm{U1}}^2] - \mathrm{E}^2[P_{\mathrm{U1}}] = 0.49 \text{ (packets/second)}^2 \tag{28}$$
$$\mathrm{CV}[P_{\mathrm{U1}}] = \frac{\sqrt{\mathrm{V}[P_{\mathrm{U1}}]}}{\mathrm{E}[P_{\mathrm{U1}}]} = 0.778 \tag{29}$$

If we now only consider the packets from User 7, we have:

$$\mathrm{P}\{P_{\mathrm{U7}} = 0\} = \frac{2}{10} \tag{30}$$
$$\mathrm{P}\{P_{\mathrm{U7}} = 1\} = \frac{5}{10} \tag{31}$$
$$\mathrm{P}\{P_{\mathrm{U7}} = 2\} = \frac{2}{10} \tag{32}$$
$$\mathrm{P}\{P_{\mathrm{U7}} = 3\} = \frac{1}{10} \tag{33}$$
$$\tag{34}$$

which is the histogram of P_{U7}.

Once we have the histogram of P_{U7}, we can calculate all the other parameters:

$$\mathrm{E}[P_{\mathrm{U7}}] = 1.2 \text{ packets/second} \tag{35}$$
$$\mathrm{E}[P_{\mathrm{U7}}^2] = 2.2 \text{ (packets/second)}^2 \tag{36}$$
$$\mathrm{V}[P_{\mathrm{U7}}] = \mathrm{E}[P_{\mathrm{U7}}^2] - \mathrm{E}^2[P_{\mathrm{U7}}] = 0.76 \text{ (packets/second)}^2 \tag{37}$$
$$\mathrm{CV}[P_{\mathrm{U7}}] = \frac{\sqrt{\mathrm{V}[P_{\mathrm{U7}}]}}{\mathrm{E}[P_{\mathrm{U7}}]} = 0.726 \tag{38}$$

From the measurement, it is observed that the time between two packets follows an exponential distribution, with average $\mathrm{E}[\tau] = \frac{1}{2.1} = 0.4762$ seconds, variance $\mathrm{V}[\tau] = \frac{1}{2.1^2} = 0.2268$ seconds^2, second moment $\mathrm{E}[\tau^2] = \frac{1}{2.1} = 0.4535$ seconds^2 and $\mathrm{CV}[\tau] = 1$.

Regarding the packet sizes, as all the packets sent by the same user have the same size, the histogram of the packet sizes is simply the number of packets of each user over the total number of packets counted.

$$\mathrm{P}\{L = 450\} = 0.42857 \tag{39}$$
$$\mathrm{P}\{L = 10000\} = 0.57143 \tag{40}$$
$$\tag{41}$$

The other requested parameters are:

$$\mathrm{E}[L] = 5907.1 \text{ bits} \tag{42}$$
$$\mathrm{E}[L] = 5.723 \cdot 10^7 \text{ bits}^2 \tag{43}$$
$$\mathrm{V}[L] = \mathrm{E}[L] - \mathrm{E}^2[L] = 2.233 \cdot 10^7 \text{ bits}^2 \tag{44}$$
$$\mathrm{CV}[L] = \frac{\sqrt{\mathrm{V}[L]}}{\mathrm{E}[L]} = 0.8 \tag{45}$$

Finally, it is important to see that the transmission delay of each packet over the target link depends on both the packet size and the transmission rate. As the transmission rate is fixed, all the randomness of D_s will be only related to L. Therefore, the results for D_s will be exactly the same as for L but scaled by R.

$$\mathrm{P}\{D_\mathrm{s} = \frac{450}{R}\} = 0.42857 \tag{46}$$
$$\mathrm{P}\{D_\mathrm{s} = \frac{10000}{R}\} = 0.57143 \tag{47}$$
$$\tag{48}$$

The other requested parameters are:

$$\mathrm{E}[D_\mathrm{s}] = 5.907 \cdot 10^{-5} \text{ seconds/packet} \tag{49}$$
$$\mathrm{E}[D_\mathrm{s}^2] = 5.723 \cdot 10^{-9} \text{ seconds/packet}^2 \tag{50}$$
$$\mathrm{V}[D_\mathrm{s}] = \mathrm{E}[D_\mathrm{s}^2] - \mathrm{E}^2[D_\mathrm{s}] = 2.2233 \cdot 10^{-9} \text{ seconds/packet}^2 \tag{51}$$
$$\mathrm{CV}[D_\mathrm{s}] = \frac{\sqrt{\mathrm{V}[D_\mathrm{s}]}}{\mathrm{E}[L]} = 0.8 \tag{52}$$