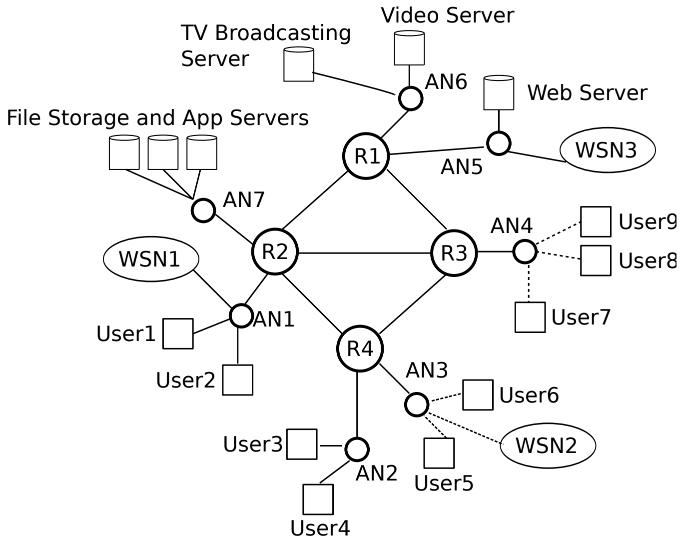

Figure 1: Basic Network

Seminar 4: Exercises

Exercise 1

The transmitter associated to the link between R4 and R3 in Figure 1 is modeled by an M/M/1/K queue. The aggregate arrival rate to this link is $\lambda = 24000$ packets/second, the packet size follow an exponential distribution with average $E[L] = 8000$ bits and the transmission rate is $R = 200$ Mbps. If the operator of the network wants to guarantee a blocking probability lower than $P_b < 10^{-3}$:

1. Derive an expression to find K in terms of P_b.
2. What is the numerical value of K?
3. What is the option that gives the best performance? Justify the results:
   - Previous case.
   - $K' = \lceil K/2 \rceil$ and $R' = 2R$.
   - $K'' = \lceil K/4 \rceil$ and $R'' = 4R$.

Exercise 2

Router R2 receives packets directed to the hosts in AN1 and AN7 from R1, R3 and R4. Namely, it receives $\lambda_{R1} = 1200$ packets/second from R1, $\lambda_{R3} = 2400$ packets/second from R3 and $\lambda_{R4} = 900$ packets/second from R4. The expected packet size is $E[L] = 12000$ bits, and the packet size follows an exponential distribution.

The probability that a packet received in R2 is directed to AN1 is α_1 and to AN7 is α_7.

The links/transmitters that interconnect R2 with AN1 and AN7 have the following characteristics:

- Link from R2 to AN1: $R = 20$ Mbps, $K = \infty$ (M/M/1).
- Link from R2 to AN7: $R = 40$ Mbps, $K = \infty$ (M/M/1).

Then:

1. Is the arrival process at each queue Poisson? Justify the answer based on the properties of Poisson processes.
2. Find the values of α_7 and α_1 that result in the same expected waiting delay ($E[D_q]$) for both queues.
3. Considering previous values for α_7 and α_1, calculate the expected queue occupancy ($E[N_q]$), the system occupancy $E[N]$ and the total delay ($E[D]$) for each queue.

Exercise 3

User 1 is sending a data flow of B = 8 Mbps to User 3, where packets are generated following a Poisson process. The data flow can follow three different paths: a) R2→R4, b) R2→R3→R4, c) R2→R1→R3→R4. The background traffic in each link of those routes follows a Poisson distribution:

- R2→R1: 10 Mbps
- R2→R3: 70 Mbps
- R2→R4: 90 Mbps
- R1→R3: 20 Mbps
- R3→R4: 5 Mbps

where Ri→Rj denotes the background traffic passing through routers Ri and Rj. The transmission rate of all links is R=100 Mbps, and packets sizes are exponentially distributed with mean $E[L] =$ 250 Bytes. Propagation delays are the same for all links and equal to 10 μs. In addition, all processing delays are negligible, and buffers are large enough to be considered of infinite size. Therefore, all interfaces can be modelled as M/M/1 queues. Note that since the hops User1→AN1, AN1→R2, and R4→AN2, AN2→User3 are the same in all routes, they do not affect the route election.

1. Represent the sequence of interfaces that are traversed by the data flow in all 3 cases.

2. Calculate the end to end delay in the 3 cases.
3. Explain why the optimal routing depends on the background traffic.

Solution to Ex1

1. The blocking probability for a M/M/1/K queue is:

$$P_b = \frac{(1-a)a^K}{1-a^{K+1}}$$

Goal: $K = f(P_b, a)$, so:

$$\begin{aligned}
P_b - P_b a^{K+1} &= (1-a)a^K \\
P_b &= (1-a)a^K + P_b a^{K+1} = a^K(1-a+P_b a) \\
ln(P_b) &= K ln(a) + ln(1-a+P_b a) \\
K &= \left\lceil \frac{ln(P_b) - ln(1-a+P_b a)}{ln(a)} \right\rceil \\
K &= \left\lceil \frac{-ln(a + \frac{1-a}{P_b})}{ln(a)} \right\rceil
\end{aligned}$$

2. With $a = \frac{\lambda}{\mu} = \frac{24}{25} = 0.96$, $K = \lceil 90.94 \rceil = 91$ packets.
3. We need to calculate P_b for each of the three cases to compare them:
   - Case 1: $P_b = 1 \cdot 10^{-3}$
   - Case 2: now, $a = 24000 \cdot \frac{8000}{400 \cdot 10^6} = 0.480$, and $K' = \lceil \frac{91}{2} \rceil = 46$.

   $$P_b = \frac{a^{46}(1-a)}{1-a^{47}} = 1.13 \cdot 10^{-15}$$

   - Case 3:

   $$a = 24000 \cdot \frac{8000}{800 \cdot 10^6} = 0.240, K'' = 23$$

   $$P_b = \frac{a^{23}(1-a)}{1-a^{24}} = 4.22 \cdot 10^{-15}$$

Solution to Ex2

1. First all arrivals are aggregated into one (Poisson) arrival process, and then are randomly assigned to the individual queues, which again results in a Poisson process.
2. The expected delay $E[D_q] = \frac{a}{\mu - \lambda}$ should be equal for both queues. With the according values

for the individual a, μ, λ we get:

$$\begin{aligned} E[D_{q1}] &= E[D_{q7}] \\ \frac{\frac{\alpha_1 \lambda}{\mu_1}}{\mu_1 - \alpha_1 \lambda} &= \frac{\frac{\alpha_7 \lambda}{\mu_7}}{\mu_7 - \alpha_7 \lambda} \\ \frac{\alpha_1}{\mu_1^2 - \alpha_1 \mu_1 \lambda} &= \frac{\alpha_7}{\mu_7^2 - \alpha_7 \mu_7 \lambda} \\ \frac{(1-\alpha_7)}{\mu_1^2 - (1-\alpha_7)\mu_1 \lambda} &= \frac{\alpha_7}{\mu_7^2 - \alpha_7 \mu_7 \lambda} \\ (1-\alpha_7) \cdot (\mu_7^2 - \alpha_7 \mu_7 \lambda) &= \alpha_7 \cdot (\mu_1^2 - (1-\alpha_7)\mu_1 \lambda) \end{aligned}$$

$$\begin{aligned} \mu_7^2 - \alpha_7 \mu_7 \lambda - \alpha_7 \mu_7^2 + \alpha_7^2 \mu_7 \lambda - \alpha_7 \mu_1^2 + \alpha_7 \mu_1 \lambda - \alpha_7^2 \mu_1 \lambda &= 0 \\ \alpha_7^2(\mu_7 \lambda - \mu_1 \lambda) - \alpha_7(\mu_7 \lambda + \mu_7^2 + \mu_1^2 - \mu_1 \lambda) + \mu_7^2 &= 0 \\ \Rightarrow \alpha_7 &= 0.683 \\ \alpha_1 &= 0.316 \end{aligned}$$

Check:

$$E[D_{q1}] = \frac{\frac{0.316 \cdot 4500 \frac{1}{s}}{\mu_1}}{\mu_1 - 0.316 \cdot 4500 \frac{1}{s}} = 0.0035s$$

$$E[D_{q1}] = \frac{\frac{0.683 \cdot 4500 \frac{1}{s}}{\mu_7}}{\mu_7 - 0.683 \cdot 4500 \frac{1}{s}} = 0.0035s$$

3. The queue occupancy $E[N_q]$ can be derived from the waiting delay $E[D_q]$, using Little's Theorem:

$$E[N_{qi}] = \lambda_i \cdot E[D_{qi}].$$

Therefore,

$$E[N_{q1}] = 4.98, E[N_{q7}] = 10.76.$$

The system occupancy is the queue occupancy plus the server/link occupancy, which in turn is the system offer a in a M/M/1/∞ system:

$$E[N_i] = E[N_{qi}] + a_i$$

$$E[N_1] = 5.83, E[N_7] = 11.68$$

Finally, applying Little again, we get for the total delay:

$$E[D_i] = \frac{E[N_i]}{\lambda_i}$$

$$\Rightarrow E[D_1] = 0.0041s, E[D_7] = 0.0038s$$

Solution to Ex3

```
clear
clc
%% EX1
B=8E6;
EL=250*8;
lambda = B/EL
Dp = 10E-6;

C=100E6;
mu = C/EL

% Route 1
lambdaR2R4 = 90E6/EL;
DR2R4 = 1 / (mu-lambda-lambdaR2R4);
DRoute1 = DR2R4 + Dp;

% Route 2
lambdaR2R3 = 70E6/EL;
DR2R3 = 1 / (mu-lambda-lambdaR2R3);
lambdaR3R4 = 5E6/EL;
DR3R4 = 1 / (mu-lambda-lambdaR3R4);
DRoute2 = DR2R3 + DR3R4 + 2*Dp;

% Route 3
lambdaR2R1 = 10E6/EL;
DR2R1 = 1 / (mu-lambda-lambdaR2R1);

lambdaR1R3 = 20E6/EL;
DR1R3 = 1 / (mu-lambda-lambdaR1R3);
lambdaR3R4 = 5E6/EL;
DR3R4 = 1 / (mu-lambda-lambdaR3R4);

DRoute3 = DR2R1 + DR1R3 + DR3R4 + 3*Dp;

disp('Results');
DRoute1
DRoute2
DRoute3
```