Network Engineering

Lecture 7. Modelling a Network Interface 2

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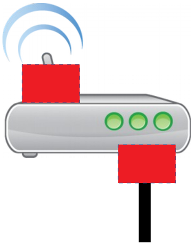
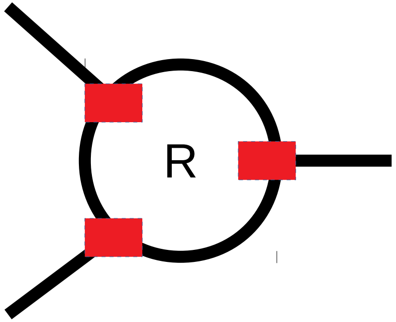
Routers, Access Networks (Switchs/APs), Hosts

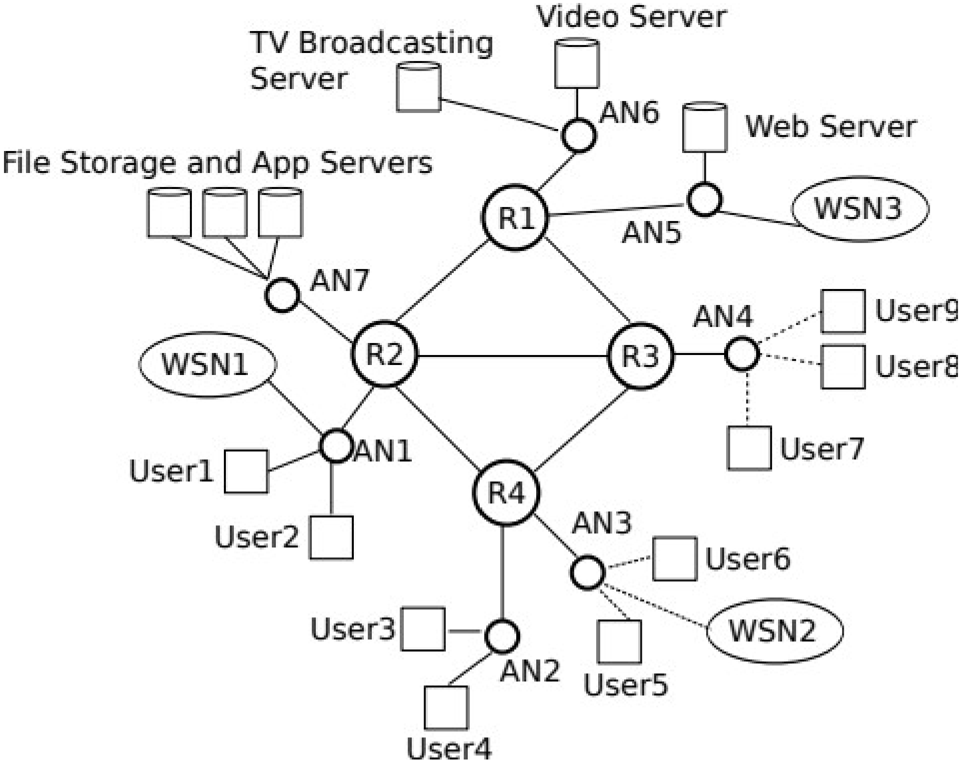
host

AN=switch



Router



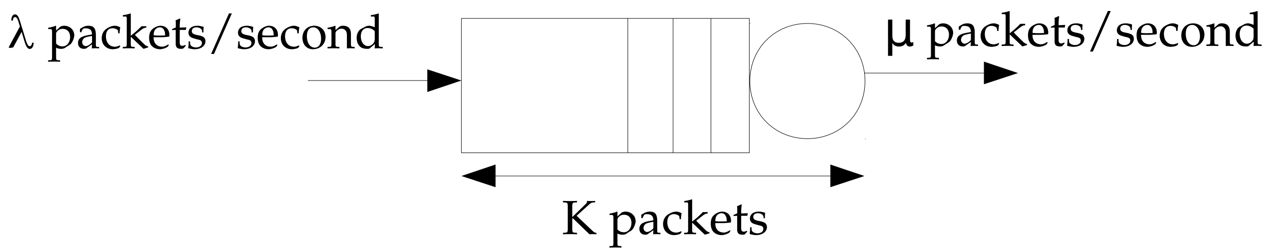


Note

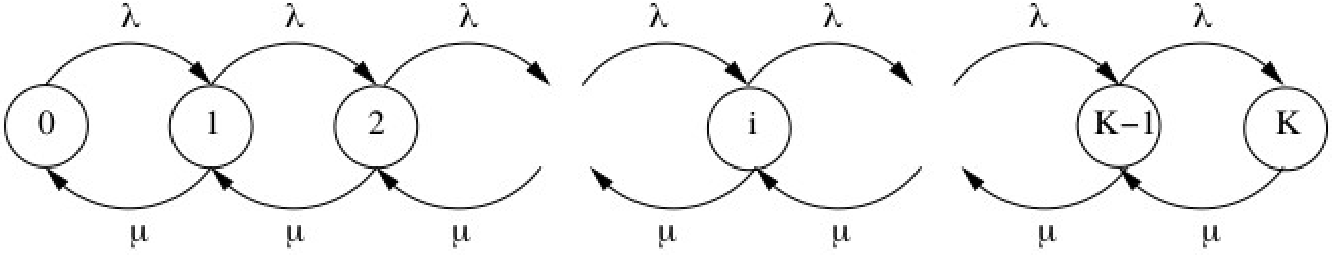
- Network interfaces have usually buffers of size larger than 100 packets.
- We need to find close-form expressions where the buffer size is a parameter.

Model of a network interface: M/M/1/K

- Buffer + 1 transmitter
- Poisson packet arrivals, Exponentially distributed packet sizes
- Markov process: number of packets in the network interface
- Performance metrics:
  - Packet losses (buffer overflow)
  - Delay (total, in the buffer)



M/M/1/K



Local Balance Equations

$$\pi_0 \lambda = \pi_1 \mu$$

$$\pi_1 \lambda = \pi_2 \mu$$

$$\cdots$$

$$\pi_{i-1}\lambda = \pi_i \mu \rightarrow \pi_i = \left(\frac{\lambda}{\mu}\right)\pi_{i-1} = a\pi_{i-1} \rightarrow \pi_i = a^i \pi_0$$

$$\cdots$$

$$\pi_{K-1}\lambda = \pi_K \mu$$

Equilibrium distribution

$$\pi_0 = \frac{1}{\sum_{j=0}^{K} a^j} = \frac{1}{\frac{1-a^{K+1}}{1-a}} = \frac{1-a}{1-a^{K+1}}.$$

$$\pi_i = a^i \pi_0 = \frac{(1-a)a^i}{1-a^{K+1}}.$$

M/M/1/K – Performance metrics

Blocking Probability

$$P_b = \pi_K = \frac{(1-a)a^K}{1-a^{K+1}}$$

Delay

$$E[D_s] = \frac{E[N_s]}{\lambda(1-P_b)} = \frac{1-\pi_0}{\lambda(1-P_b)} = \frac{E[L]}{R}$$

$$E[D] = \frac{E[N]}{\lambda(1-P_b)}$$

$$E[D_q] = \frac{E[N_q]}{\lambda(1-P_b)} = E[D] - E[D_s]$$

M/M/1/K – Performance metrics

$$
\begin{aligned}
E[N] &= \sum_{q=0}^{K} \pi_q q = \sum_{q=0}^{K} \frac{(1-a)a^q}{1-a^{K+1}} q \\
&= \frac{(1-a)}{1-a^{K+1}} \sum_{q=0}^{K} q a^q = \frac{a(1-a)}{1-a^{K+1}} \sum_{q=0}^{K} q a^{q-1} \\
&= \frac{a(1-a)}{1-a^{K+1}} \sum_{q=0}^{K} \frac{d}{da} a^q = \frac{a(1-a)}{1-a^{K+1}} \frac{d}{da} \sum_{q=0}^{K} a^q \\
&= \frac{a(1-a)}{1-a^{K+1}} \frac{d}{da} \left(\frac{1-a^{K+1}}{1-a} \right) = \\
&= \frac{a(1-(K+1)a^K + Ka^{K+1})}{(1-a)(1-a^{K+1})} = \\
&= \frac{a(1-(K+1)a^K + (K+1)a^{K+1} - a^{K+1})}{(1-a)(1-a^{K+1})} = \\
&= \frac{a(1-a^{K+1} - (K+1)a^K(1-a))}{(1-a)(1-a^{K+1})} = \\
&= \frac{a}{1-a} - \frac{(K+1)a^{K+1}}{1-a^{K+1}}
\end{aligned}
$$

For the specific case $a = 1$:

$$E[N] = \frac{K}{2}$$

To find $E[N_q]$, we can use the relation

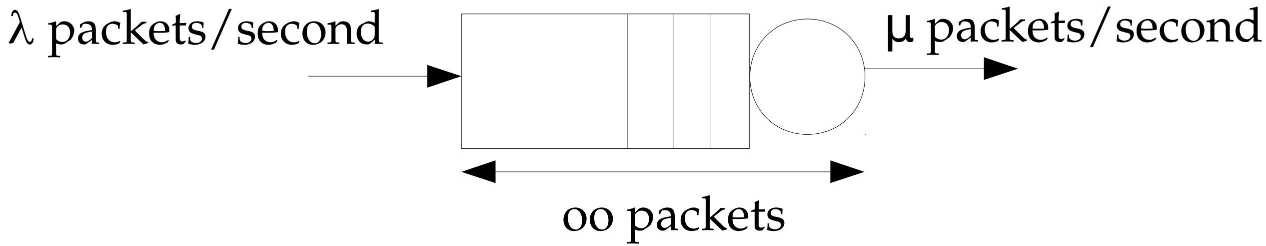
$$E[N_q] = E[N] - E[N_s]$$

where

$$E[N_s] = 1 - \pi_0$$

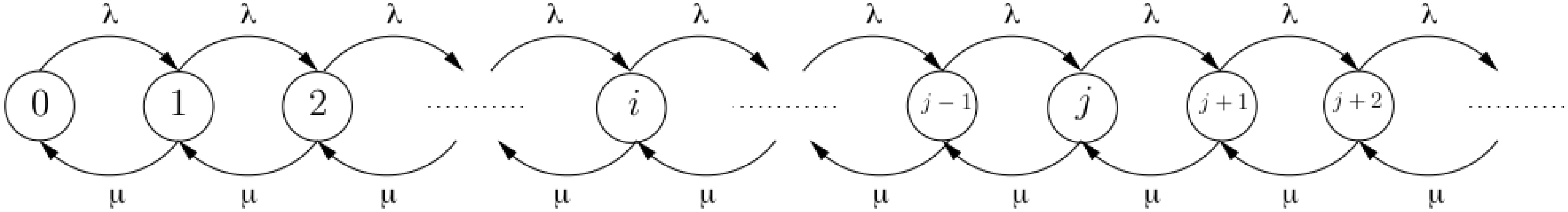
Model of a network interface: M/M/1

- Buffer (infinite) + 1 transmitter
- Poisson packet arrivals, Exponentially distributed packet sizes
- Markov process: number of packets in the network interface
- Performance metrics:
  - Delay (total, in the buffer)
  - There are no packet losses!



M/M/1

The system must be stable! a<1



Local Balance Equations

$$\pi_0\lambda = \pi_1\mu$$

$$\pi_1\lambda = \pi_2\mu$$

$$\dots$$

$$\pi_{i-1}\lambda = \pi_i\mu \rightarrow \pi_i = \left(\frac{\lambda}{\mu}\right)\pi_{i-1} = a\pi_{i-1} \rightarrow \pi_i = a^i\pi_0$$

$$\dots$$

Equilibrium distribution

$$\pi_0 = \frac{1}{\sum_{j=0}^{\infty} a^j} = \frac{1}{\frac{1}{1-a}} = 1 - a$$

$$\pi_i = a^i\pi_0 = (1-a)a^i$$

M/M/1 – Performance metrics

Blocking Probability P_b=0

Delay

$$E[D_s] = \frac{E[N_s]}{\lambda(1-P_b)} = \frac{a}{\lambda} = \frac{1}{\mu} \text{ seconds}$$

$$E[D] = \frac{E[N]}{\lambda(1-P_b)} = \frac{1}{\mu(1-a)} = \frac{1}{\mu-\lambda} \text{ seconds}$$

$$E[D_q] = \frac{E[N_q]}{\lambda(1-P_b)} = \frac{a^2}{\lambda(1-a)} = \frac{a}{\mu(1-a)} = \frac{a}{\mu-\lambda} \text{ seconds}$$

M/M/1 – Performance metrics

If $a < 1$, the system occupation is computed as follows:

$$E[N] = \sum_{q=0}^{\infty} \pi_q q = \frac{a}{1-a} \quad \text{packets.}$$

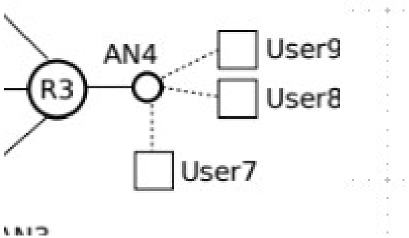
To find $E[N_q]$, we will use the relations

$$E[N_s] = 1 - \pi_0 = a$$

and

$$\begin{aligned} E[N_q] &= E[N] - E[N_s] \\ &= \frac{a}{1-a} - a = \frac{a - a + a^2}{1-a} = \frac{a^2}{1-a} \quad \text{packets.} \end{aligned}$$

Example – Can we appox. a M/M/1/K by a M/M/1 ?



User7 from Figure 4.1 is viewing online TV on his computer. The packets with the streaming data come from the *TV broadcasting server* and pass through the network elements AN6, R1, R3 and AN4. In this example, we will evaluate the delay that packets suffer in the AN4, which is a WLAN. We assume that User 8 and User 9 are not connected to the WLAN, so all the WLAN bandwidth is used by User7.

The AP sends packets to User 7 at a transmission rate of $R = 22$ Mbps. We assume that the service time, D_s, for each packet follows an exponential distribution, with average

$$D_s = DIFS + E[\mathrm{BO}] + \frac{L_h + E[L]}{R} + SIFS + \frac{L_{\mathrm{ACK}}}{R} \tag{5.60}$$

where DIFS$=$ 34 μs, SIFS$=$ 16 μs, $E[\mathrm{BO}] = 90$ μs are parameters of the WLAN.

$L_{\mathrm{ACK}} = 112$ bits is the length of MAC-layer ACK which is sent by the receiver after receiving a packet and $L_h = 230$ bits is the length of the MAC header which is added to each TV data packet. The packets have an average size of $E[L] = 4000$ bits.

If the *TV broadcasting server* send packets to User7 at a rate $\lambda = \frac{B_{\mathrm{stream}}}{E[L]}$ following a Poisson process, with $B_{\mathrm{stream}} = 8$ Mbps the bandwidth required by the TV flow, and the queue size at AN4 is $K = 10$ packets, compute $E[D_s]$, $E[D_q]$ and $E[D]$.

Solution: The AP can be modelled by a M/M/1/K queue, with $K = 10$, $\lambda = 500$ packets, $E[D_s] = 3.342$ ms and $\mu = 1/E[D_s] = 2992$ packets. The offered traffic is $a = \frac{\lambda}{\mu} = 0.668$ Erlangs.

First, we compute the Equilibrium Distribution. The results are shown in Table 5.1.

State	Value	State	Value
π_0	0.3356266	π_5	0.0447631
π_1	0.2243206	π_6	0.0299180
π_2	0.1499277	π_7	0.0199961
π_3	0.1002062	π_8	0.0133647
π_4	0.0669742	π_9	0.0089325
-	-	π_{10}	0.0059701

Table 5.1: Equilibrium Distribution for the WLAN Exercise

The blocking probability is

$$P_b = \pi_K = \pi_{10} = 5.9 \cdot 10^{-3}$$

.

The expected system occupation is

$$E[N] = \sum_{k=0}^{K} \pi_k k = 1.8830 \text{ packets}$$

and the expected system delay can be obtained by applying the Little's Law

$$E[D] = \frac{E[N]}{\lambda(1 - P_b)} = 0.947 \cdot 10^{-3} \text{ seconds.}$$

The expected waiting delay is

$$E[D_q] = E[D] - E[D_s] = 0.613 \cdot 10^{-3} \text{ seconds.}$$

Alternatively, it can obtained by computing first $E[N_q] = \sum_{k=2}^{K} \pi_k(k-1) = 1.2186$ packets, and then applying Little's law.

From those results, we can observe that User7 will be able to watch the TV without suffering neither high packet losses nor high delays.

Example - Is $K = \infty$ a good approximation?

Here, we consider the same scenario as in the previous example. The goal now is to evaluate what is the impact of assuming that the buffer size is infinite in the performance metrics that we can obtain.

First, assuming that $K = \infty$, the $E[D]$ and $E[D_q]$ values for different TV stream bandwidth values are:

-	TV stream bandwidth (Mbps)		
Parameter	2	6	10
$E[D_q]$	6.7041e-05	3.3589e-04	0.0016968
$E[D]$	4.0122e-04	6.7007e-04	0.0020309

Table 5.2: $E[D_q]$ and $E[D]$ assuming $K = \infty$

Considering the case with the highest stream bandwidth, $B = 10$ Mbps, what is the value of K that gives similar values for $E[D_q]$ and $E[D]$ when they are compared with the case of $K = \infty$? The results obtained are shown in Table 5.3.

-	TV stream bandwidth (Mbps)		
Parameter	2	6	10
$E[D_q]$	6.7041e-05	3.3589e-04	0.0016968
$E[D]$	4.0122e-04	6.7007e-04	0.0020309

Table 5.2: $E[D_q]$ and $E[D]$ assuming $K = \infty$

K	P_b	$E[D_q]$	$E[D]$
5	1.0148e-01	5.4986e-04	8.8404e-04
10	0.0316381	0.0010332	0.0013674
15	0.0117574	0.0013343	0.0016685
20	0.0046219	0.0015082	0.0018423
25	0.0018554	0.0016024	0.0019366
30	7.5097e-04	1.6510e-03	1.9851e-03
40	1.2401e-04	1.6867e-03	2.0209e-03
50	2.0534e-05	1.6947e-03	2.0289e-03
60	3.4014e-06	1.6963e-03	2.0305e-03
70	5.6349e-07	1.6967e-03	2.0309e-03

$E[D_q]$ and $E[D]$ for a TV stream bandwidth value of 10 Mbps

Not a general case… it depends on the required accuracy with respect to the loss prob.

Example

- M/M/1/K and M/M/1 with multiple flows
  - The **aggregate flow** must satisfy Poisson distributed arrivals and exponentially distributed packet sizes with average λ and E[L] respectively.
- Calculate the system delay E[D] in a M/M/1 queue with two arriving traffic flows:
  - Transmission rate of the system: R=10 Mbps
  - Flow 1: Poisson arrivals with rate $\lambda 1$=100 packets / second
  - Flow 2: Poisson arrivals with rate $\lambda 2$=400 packets / second
  - Aggregate packet size is exponentially distributed with E[L]=8000 bits.

  - E[D] = 1 / (10E6/8000 – (100+400)) = 0.001333 seconds

Example

- Let's say that flow 2 is the background traffic, and flow 1 the 'target' traffic.
- How much does flow 2 affect flow 1 in terms of delay?
- Let's compute the delay of flow 1's packets without the presence of flow 2 packets:
  - E[D] only flow 1 = 1 / (10E6/8000 - (100)) = 0.000869 seconds
- So, packets from flow 1 see how their delay increases by ~ 0.5 ms due to the presence of flow 2's packets.

- What happens if now we add a flow 3 with $\lambda 3$=800 packets / second, and we try to compute again E[D]? And if $\lambda 3$=500 packets / second?