# USE CASE 1 <br> Performance Analysis of Cellular Networks 

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## 1 Scenario

Consider a cellular network giving service to a number of inhabitants that make an average of 3 calls/day. Each mobile operator has 3 carriers (each carrier is formed by 8 channels) and performs frequency re-use. Let's consider that a cell can have two levels of density: low ( 1000 inhab $/ \mathrm{Km}^{2}$ ) and high (5000 inhab $/ \mathrm{Km}^{2}$ ). Each hexagonal cell is of radius $R=300 \mathrm{~m}$. Thus, of area $2.6 R^{2}=0.234 \mathrm{Km}^{2}$. Let's consider as well that a cell can have three levels of mobility: no mobility $(\alpha=0)$, low ( $\alpha=0.2$ ) and high mobility ( $\alpha=0.7$ ) with $\lambda_{h o}=\alpha \cdot \lambda$. Each user spends 120 s on average in each cell $\left(t_{4}-t_{3}\right.$ in Figure $1)$. We assume this time is the same for new and handover calls (!).


Figure 1: Schematic showing a cellular user mobility pattern.

What is the offered load (traffic) per cell?
The number of inhabitants per cell is $234 / 1170$, respectively for the high and low density cases. Each user makes 3 calls per day on average. Thus, $\lambda_{H}=0.008125$ calls $/ \mathrm{s}$ and $\lambda_{L}=0.040625$ calls $/ \mathrm{s}$, respectively. Given the time spent by a user in a cell ( 120 s ), we have that:

- Low density case: $A=0.008125 \cdot 120=0.975$ Erlangs
- High density case: $A=0.040625 \cdot 120=4.875$ Erlangs


## 2 Performance Analysis

### 2.1 Low Density Case

Parameters:

- Number of channels $C=8$
- $\lambda=0.008125$ calls $/ \mathrm{s}$ (Poisson arrivals)
- $X=120 \mathrm{~s}$

Model of the cell:


Figure 2: Markov Chain of the system.

The Markov Chain in Figure 2 corresponds to a birth and death model. We can solve it by computing the equilibrium/balance equations. If we are only interested in the blocking probability we can look it up at the Erlang-B tables. ${ }^{1}$

What is the blocking probability ( $B P$ )?
Probability to be in the N state: $P(N)=7.64 \cdot 10^{-6}$.

What is the dropping probability (DP)?
New and handover calls are treated equally, so the dropping probability is the probability to be in the N state: $P(N)=7.64 \cdot 10^{-6}$.

What is the probability that a cell is not serving any call?
Probability to be in the state $0: P(0)=0.377$.

What is the Grade of Service (GoS) defined as GoS=BP + 10DP?

[^0]$$
\mathrm{GoS}=8.4 \cdot 10^{-5} .
$$

What is the probability the cell is serving $0,1,2 \ldots N$ calls?
Figure 3 shows the probability associated to each state of the Markov Chain depicted in Figure 2. Note how the probability to have more than 4 calls at the same time is very small.


Figure 3: Steady-state distribution of the system (low density).

### 2.2 High Density Case

In Lab 1.

### 2.3 Guard Channels

Now, consider that we reserve a number of guard channels to be used by handover calls only with the goal to minimize the GoS.

New model of the cell:


Figure 4: Markov Chain of the system with guard channels.

We are now interested in knowing the optimal number of guard channels. Let's define optimal as the number of guard channels that minimize the GoS. What is the optimal number of guard channels for each case?

The blocking, dropping probabilities and the GoS for each case while varying the number of guard channels are depicted in Figure 5. Note that the optimal number of guard channels does not depend solely on the mobility level but also
on the traffic load coming from new calls. Given these results, how many guard channels will you decide to use if you do not know the conditions (low/high density and low/high mobility) that the cell will experience?


Figure 5: Blocking, dropping probabilities and GoS for varying number of guard channels.

### 2.4 Delaying Calls

Now let's consider that we are able to delay calls until a resource (channel) is freed. What is the model of the cell in this case with only new calls?.

Model of the cell:


Figure 6: Markov Chain of the system with call delaying.

What is the model of the cell in this case with new and handover calls considering that handover calls cannot be delayed?


Figure 7: Markov Chain of the system with call delaying with new and handover calls. In the example $C=2$ and the buffer of new calls is also equal to 2 .


[^0]:    ${ }^{1}$ http://www.sis.pitt.edu/ dtipper/2720/erlang-table.pdf

