

**Preliminaries:**  $x_1 = 1$ st number of your NIA;  $x_2 = 2$ nd number of your NIA;  $x_3 = 3$ rd number of your NIA;  $x_4 = 4$ th number of your NIA, etc. If any number is zero, consider it equal to 1.

**Problem 1 (30 mins) - 5 points**

Wi-Fi networks use extenders, also called relays, to 'increase' the coverage area provided by a single Access Point connected to Internet. The use of extenders in single-band networks however adds another 'contender' to the network as the packet is transmitted two times (one by the node and one by the extender), which may cause further collisions. The goal of this exercise is to assess if in the Wi-Fi network of Figure 1, the use of an extender is beneficial or not for STA A. The contention window is  $CW = 8 \cdot x_3$ . Other parameters include:  $T_e = 0.9 \mu s$ , and  $L = 10000 \cdot x_5$  bits. Full-buffer traffic model is considered.

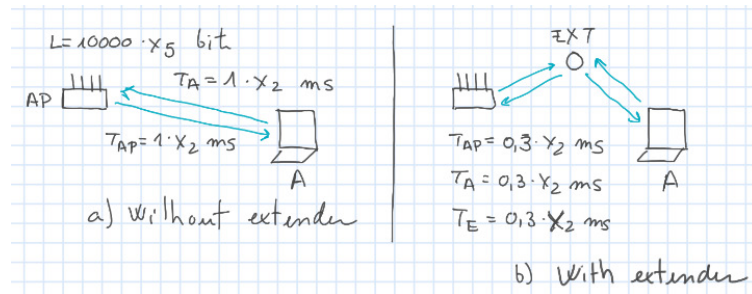


Figure 1: A Wi-Fi network with and without 'extender'.

- (1 point) Write your NIA, and calculate the different parameters that depend on it (transmission durations and CW). How many nodes are contending to access the channel without and with the extender?
- (2 points) Without the extender, calculate the transmission probability  $\tau$ , the slot probabilities  $p_0, p_1, p_{1+}$ , and the throughput of the Access Point. Note that the throughput of the Access Point is the aggregate throughput of the Wi-Fi network divided by the number of contenders.
- (2 points) Repeat previous point including the extender. Compare the results obtained in both cases, and justify them. Note that now the 'relevant' throughput is the one of the extender, as it forwards packets both from and to the AP. Therefore, since we are only interested in the throughput from the AP to A, we should first compute the throughput of the extender (dividing the Wi-Fi throughput by the number of contenders), and then divide it also by 2 because of the uplink / downlink transmissions.

**Problem 2 (30 mins) - 5 points**

We are interested in comparing the following two systems: a) M/M/1/1 and b) M/M/2/2, in terms of their throughput and delay.

- (2 points) Draw the Markov chain for both systems, write their local balance equations, and write their equilibrium distribution.
- (1.5 points) Calculate the throughput and average delay of each system if the traffic load is  $B = 4 \cdot x_2$  Mbps, and the transmission rate of a single server in each case is a)  $R_a = 4 \cdot x_3$  Mbps, and b)  $R_b = 2 \cdot x_3$  Mbps. Consider that  $E[L] = 1000 \cdot x_4$ .
- (1.5 points) Determine the value of  $R_a$  that results in the same throughput in the M/M/1/1 system as in the M/M/2/2 system. Justify the results, and explain what happens with the delay.

```

function Exercise1()

% NIA
x1=1;
x2=2;
x3=3;
x4=4;

% Parameters
CW=8*x3;
Te=9E-6;
L=10000;
T1_a = 1*x2*1E-3; % T1 without the extender
T1_b = 0.3*x2*1E-3; % T1 with the extender

% 1)
Na = 2; % Number of contenders without the extender
Nb = 3; % Number of contenders with the extender
disp('Parameters and Number of Contenders');
disp([T1_a T1_b CW Na Nb]);

% 2)

% Transmission Probability
EB = CW/2; % Expected BO value (ok too if you calculate it as (CW-1)/2)
tau = 1/(EB+1);

% Slot probabilities
p0 = (1-tau)^Na;
p1 = Na*tau*(1-tau)^(Na-1);
pc = 1-p0-p1;

% Throughput
S_WiFi_a = L*p1/(p0*Te+(1-p0)*T1_a);
S_AP_a = S_WiFi_a/Na;

disp('Throughput Without Extender');
disp(S_AP_a);

% 3) We add the extender

% Slot probabilities
p0 = (1-tau)^Nb;
p1 = Nb*tau*(1-tau)^(Nb-1);
pc = 1-p0-p1;

% Throughput
S_WiFi_b = L*p1/(p0*Te+(1-p0)*T1_b);
S_AP_b = S_WiFi_b/(2*Na);

disp('Throughput: Without and With Extender');
disp([S_AP_a S_AP_b]);

% Adding the extender results in higher throughput since the extra amount of collisions (check pc)
% in both cases is compensated by the reduction of the transmission time thanks to the use of higher trans

end

function Exercise2()

```

```

%NIA
x1=2;
x2=3;
x3=4;
x4=2;

% Parameters
B=4*x2*1E6;
Ra=4*x3*1E6;
Rb=2*x3*1E6;
EL=1000*x4;

% Markov chains
% a) (0)--(1)
% b) (0)--(1)--(2)

% Parameters
lambda = B/(1000*x4); % Arrival rate
a_a = B/Ra; % Norm traffic load a)
a_b = B/Rb; % Norm traffic load b)

% 1)
% Case a)
pi0_a = 1/(1+a_a);
pi1_a = a_a/(1+a_a);
% Case b)
pi0_b = 1/(1+a_b+(a_b^2)/2);
pi1_b = a_b/(1+a_b+(a_b^2)/2);
pi2_b = ((a_b^2)/2)/(1+a_b+(a_b^2)/2);

disp('Stationary Distribution a');
disp([pi0_a pi1_a]);

disp('Stationary Distribution b');
disp([pi0_b pi1_b pi2_b]);

% 2)
% Throughput
Throughput_a = B*(1/(1+a_a)); % B*pi_0 = B(1-pi_1)
Throughput_b = B*((1+a_b)/(1+a_b+a_b^2/2));
disp('Throughput: a) | b)');
disp([Throughput_a Throughput_b]);

% Delay (using Little's relation)
EN_a = 1*pi1_a;
EN_b = 1*pi1_b + 2*pi2_b;
ED_a = EN_a / (lambda*(1-pi1_a));
ED_b = EN_b / (lambda*(1-pi2_b));
disp('Delay E[D]');
disp([ED_a ED_b]);

% It could be surprising that the second case more throughput and higher
% delay, whe we are used to the case that more throughput means more delay.
% However, in this case, the extra throughput and delay come from the lower
% blocking probability of b) In terms of throughput it is better to have
% two slow servers than one fast.

% 3) We need to compute a_b and R_b (new)

a_a_new = (B/Throughput_b)-1;
R_a_new = B/a_a_new;

```

```

disp('New Transmission Rate (R) of a');
disp(R_a_new);

% We compute the delay again
a_a = B/R_a_new; % Norm traffic load a) with the new rate
pi0_a = 1/(1+a_a);
pi1_a = a_a/(1+a_a);
EN_a = 1*pi1_a;
ED_a = EN_a / (lambda*(1-pi1_a));
disp('New Delay E[D]');
disp([ED_a ED_b]);

% To have the same throughput we need to increase a lot the transmission
% rate so we reduce significantly the blocking probability. Therefore, the delay it is also
% significantly reduced.
% Adding a single buffer position would result in a muhc lower rate.

```