Outage Analysis of the Hybrid Free-Space Optical and Radio-Frequency Channel

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***Abstract*—We study the hybrid free-space optical (FSO) and radio-frequency (RF) channel from an information theoretic perspective. Since both links operate at vastly different carrier frequencies, we model the hybrid channel as a pair of parallel channels. Moreover, since the FSO channel signals at a higher rate than the RF channel, we incorporate this key feature in the parallel channel model. Both channels experience fading due to scintillation, which is slow compared to typical signalling rates. Under this framework, we study the fundamental limits of the hybrid channel. In particular, we analyse the outage probability in the large signal-to-noise ratio (SNR) regime, and obtain the outage diversity or SNR exponent of the hybrid system. First we consider the case when only the receiver has perfect channel state information (CSIR case), and obtain the exponents for general scintillation distributions. These exponents relate key system design parameters to the asymptotic outage performance and illustrate the benefits of using hybrid systems with respect to independent FSO or RF links. We next consider the case when perfect CSI is known at both the receiver and transmitter, and derive the optimal power allocation strategy that minimises the outage probability subject to peak and average power constraints. The optimal solution involves non-convex optimisation, which is intractable in practical systems. We therefore propose a suboptimal algorithm that achieves significant power savings (on the order of tens of dBs) over uniform power allocation. We show that the suboptimal algorithm has the same diversity as the optimal power allocation strategy.**

***Index Terms*—Optical communication, radio-frequency communication, scintillation, outage probability, information theory, coded modulation, Singleton bound, power allocation.**

I. Introduction

In FREE-SPACE optical (FSO) communication an optical carrier is employed to convey information wirelessly. FSO systems have the potential to provide fiber-like data rates with the advantages of quick deployment times, high security and no frequency regulations. Unfortunately such links are highly susceptible to atmospheric effects. *Scintillation* induced by atmospheric turbulence causes random fluctuations in the received irradiance of the optical laser beam [1]. Numerous studies have shown that performance degradation caused by scintillation can be significantly reduced through the use of multiple-lasers and multiple-apertures, creating the well-known multiple-input multiple-output (MIMO) channel (see e.g. [2], [3], [4], [5], [6], [7], [8], [9], [10]). However, it is the large attenuating effects of cloud and fog that pose the most formidable challenge. Extreme low-visibility fog can cause signal attenuation on the order of hundreds of decibels per kilometre [11]. One method to improve the reliability in these circumstances is to introduce a radio frequency (RF) link to create a *hybrid FSO/RF* communication system [12], [13], [14], [15], [16], [11]. When the FSO link is blocked by cloud or fog, the RF link maintains reliable communications, albeit at a reduced data rate. Typically a millimetre wavelength carrier is selected for the RF link to achieve data rates comparable to that of the FSO link. At these wavelengths, the RF link is also subject to atmospheric effects, including rain and scintillation [17], [18], [19], [20], [21], but less affected by fog. The two channels are therefore complementary: the FSO signal is severely attenuated by fog, whereas the RF signal is not; and the RF signal is severely attenuated by rain, whereas the FSO is not. Both, however, are affected by scintillation.

Most works on the hybrid channel [12], [13], [14], [16], [11] consider the RF and FSO links as separate channels, i.e. the channels do not aid each other to compensate signal level fluctuations. In these works, the main purpose of the RF link is to act as a backup when the FSO link is down. In [15] a hybrid channel coding scheme is proposed that combines both FSO and RF channels and adapts the code rate to the channel conditions. Multilevel coding schemes have also been proposed in [22] for the hybrid channel.

Lacking so far in the literature on hybrid FSO/RF channels is the development of a suitable channel model and its theoretical analysis to determine the fundamental limits of communication. This is the central motivation for our paper. We propose a hybrid channel model based on the well known parallel channel [23], and take into account the differences in signalling rate, and the atmospheric fading effects present in both the FSO and RF links. These fading effects are slow compared to typical data rates, and as such, each channel is based on a *block-fading* channel model. Previously in [2], [3], using a block-fading channel model, we examined the *outage probability* of the multiple-input multiple-output (MIMO) FSO channel under the assumption of pulse-position modulation (PPM) for several well-known scintillation distributions, i.e. lognormal, exponential, gamma-gamma, lognormal-Rice and I-K distributed scintillation [1]. In particular, we examined the *SNR exponent* (or outage diversity) [24], [25], which

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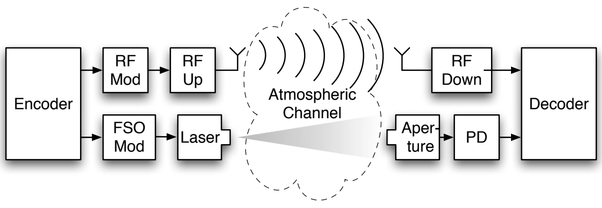

Fig. 1. Hybrid FSO/RF communication system.

describes the high SNR slope of the outage probability curve when plotted on a log-log scale. In this paper, we extend this analysis to include an RF link to create a hybrid FSO/RF channel. The message to be transmitted is encoded into parallel FSO and RF bit streams which are sent across the FSO and RF channels simultaneously. We examine the case when perfect CSI is known at the receiver only (CSIR case), then we consider the case when CSI is also known at the transmitter (CSIT case), and power allocation is employed to reduce the outage probability subject to power constraints. When CSI is not available at the transmitter, we calculate the SNR exponents of the hybrid channels for general scintillation distributions in each of the channels. On the other hand, when CSI is available at the transmitter, we derive the optimal power allocation algorithm subject to both peak and average power constraints. This optimal solution involves non-convex optimisation, which has prohibitive complexity in practical systems. We therefore propose a suboptimal solution and prove that it has the same SNR exponent as optimal power allocation.

The remainder of this paper is organised as follows. In Section II we present our channel model and assumptions. Section III reviews the required information-theoretic preliminaries. Section IV presents our main results for the CSIR-only case while Section V discusses power allocation and SNR exponents for the CSIT case. Section VI draws final concluding remarks. Proofs of our results can be found in the appendices.

II. Channel Model and Assumptions

A block diagram of the communication system of interest is shown in Fig. 1. A binary data sequence is binary encoded into parallel FSO and RF bit streams. The RF link modulates the encoded bits and up-converts the baseband signal to a millimetre wavelength RF carrier frequency. The FSO link employs intensity modulation and direct detection, i.e. information is modulated using only the irradiance of a laser beam. The RF and FSO signals are transmitted simultaneously through an atmospheric channel. The received RF signal is then downconverted to baseband and sent to the decoder. At the same time, the received irradiance is collected by an aperture, converted to an electrical signal via photodetection and sent to the decoder. The received signals are jointly decoded to recover the transmitted message.

In this paper we define a *hybrid channel symbol*, $(\boldsymbol{x}, \hat{x}) \in \mathcal{X}_{\text{fso}}^n \times \mathcal{X}_{\text{rf}}$, consisting of component FSO and RF symbols, which are transmitted in parallel with perfect synchronism and have the same symbol period T_s. The RF component symbol, denoted by $\hat{x}$, is drawn uniformly from an M-ary complex signal set $\mathcal{X}_{\text{rf}} \subset \mathbb{C}$ representing e.g. quadrature amplitude modulation (QAM) or phase shift keying (PSK). We assume the RF symbols are transmitted with unit average energy, i.e. $\mathbb{E}[|\hat{x}|^2] = 1$. The number of bits per RF symbol is $m \triangleq \log_2 M$ bits. Since the FSO link employs a much higher carrier frequency than the RF link, we assume the FSO component of the hybrid channel symbol consists of n symbols drawn uniformly from a constellation $\mathcal{X}_{\text{fso}}$. Most practical FSO systems employ a pulsed type of modulation, e.g. on-off keying (OOK), pulse-position modulation (PPM) or multi-PPM (MPPM). Hence we assume $\mathcal{X}_{\text{fso}}$ represents a pulse type modulation scheme, i.e. the FSO component of the hybrid symbol consists of n symbols, which are further composed of Q pulse intervals of duration T_p, where $T_s = nQT_p$. The signal set $\mathcal{X}_{\text{fso}} \subset (0,1)^Q$ is a set of Q length binary vectors, where a binary 1 at index i indicates a pulse of duration T_p in time slot i. Hence the FSO symbols $\boldsymbol{x} \in \mathcal{X}^n \subset (0,1)^{nQ}$ are $1 \times nQ$ binary vectors. We assume each T_p second 'on' pulse is normalised to have unit energy and denote the average FSO symbol energy by $\gamma = \mathbb{E}\left[\sum_{i=1}^{nQ} x_i\right]$. Let $q \triangleq \log_2(|\mathcal{X}_{\text{fso}}|)$, hence the total bits per hybrid channel symbol is $m + nq$ bits. Note that one could also consider pulse-amplitude modulated (PAM) symbols. In this case the same analysis and result that follow will apply, but for simplicity we assume on-off type pulses.

Both FSO and RF channels are affected by scintillation [1], [17], [18], [19], which is a slow fading process compared to typical data rates.[1] We therefore model each component channel by a block-fading channel model, whereby the component channels are divided into a finite number of blocks of symbols, and each block experiences an i.i.d. fading realisation. The scintillation experienced by each component channel is also assumed to be independent.[2] Typically, the RF scintillation has a coherence time on the order of seconds [17], [18], [19], whereas the FSO scintillation is much faster, having a coherence time on the order of tens of milliseconds [1]. We therefore decompose the FSO and RF components of the codeword into A and B blocks of K and L symbols respectively, where $A \geq B$. Given that the coherence time of the RF scintillation is on the order of seconds, the most realistic scenario is $B = 1$. However, for generality we will assume B is an arbitrary positive integer. Note that the total number of symbols in each FSO/RF component codeword is the same, i.e. $AK = BL$. We assume that the number of symbols in each block tends to infinity, but the ratio remains a fixed constant, i.e.

$$\lim_{K,L\to\infty} \frac{L}{K} = \frac{A}{B}. \tag{1}$$

We assume both FSO/RF component channels are modelled by independent additive white Gaussian noise (AWGN) channels. Note that this assumption for the FSO channel may not be accurate under certain conditions. The photodetec-

[1]We use the terms scintillation and fading interchangeably to refer to fluctuations in power of the received FSO/RF signals.

[2]This will be true over short time intervals, but over longer time scales meteorological variations will result in correlated channel fades.

tor output electrical signal is proportional[3] to the incident optical irradiance waveform (instantaneous optical power). Under ideal photodetection, thermal noise is negligible and the received signal represents detected photons. The number of detected photons in a given time interval is governed by a Poisson process whose rate is proportional to the average irradiance over the time interval [26]. This is the well-known Poisson channel for which a number of works on FSO systems employ (see e.g. [10], [7], [27]). As the average irradiance increases due to increased background irradiation or high optical signal power, these statistics can be well approximated by Gaussian statistics. If avalanche photodetection is employed, then random photo-multiplication effects must be considered. A comprehensive comparison of the various photodetection models, Poisson, Webb, Gaussian (signal dependent and signal independent variations) is given in [28]. For simplicity and analytical convenience, we assume the combined shot and thermal noise can be considered as signal independent AWGN (an assumption also commonly used in the literature [26], [28], [9], [1], [29], [6], [4]). Under this assumption, the received signals can written as,

$$\boldsymbol{y}_a[k] = p_a \rho h_a \boldsymbol{x}_a[k] + \boldsymbol{z}_a[k] \tag{2}$$

$$\hat{y}_b[l] = \sqrt{\hat{p}_b \hat{\rho} \gamma \hat{h}_b} \hat{x}_b[l] + \hat{z}_b[l], \tag{3}$$

for $l = 1, \ldots, L$, $k = 1, \ldots, K$ $a = 1, \ldots, A$ and $b = 1, \ldots, B$, where: $\boldsymbol{y}_a[k] \in \mathbb{R}^{nQ}$ and $\hat{y}_b[l] \in \mathbb{C}$ are the noisy received symbols for the FSO and RF channels respectively; $\boldsymbol{x}_a[k] \in \mathcal{X}_{\text{fso}}^n$ and $\hat{x}_b[l] \in \mathcal{X}_{\text{rf}}$ denote the transmitted symbols; $\boldsymbol{z}_a[k] \in \mathbb{R}^{nQ}$ is a i.i.d. vector of zero mean unit variance real Gaussian noise, and $\hat{z}_b[l] \in \mathbb{C}$ is unit variance complex Gaussian noise ($\mathcal{CN}(0,1)$); $h_a > 0$ and $\hat{h}_b > 0$ are independent random power fluctuations due to scintillation, each i.i.d. drawn from distributions f_H and $f_{\hat{H}}$ respectively, with normalisation $\mathbb{E}[h_a] = \mathbb{E}[\hat{h}_b] = 1$; p_a and $\hat{p}_b$ denotes the power of block a and b for the FSO and RF channels respectively. The γ parameter in (3) ensures both FSO and RF symbols have the same energy. The parameters $0 < \rho, \hat{\rho} < 1$ in (2) and (3) model differences in the relative strengths of the two parallel channels, e.g. it reflects long-term fading effects due to rain, fog or cloud as well as other parameters such as aperture/antenna gains and propagation loss. When $\rho > \hat{\rho}$, the FSO channel is much stronger than the RF, e.g. modelling the effects of severe rain attenuation. On the other hand, if $\rho < \hat{\rho}$, then the RF channel is stronger than the FSO channel, e.g. modelling the effects of severe fog/cloud attenuation. Although in practise ρ and $\hat{\rho}$ are randomly varying with time (and are also most likely correlated random variables), we assume they remain unchanged over many codeword time intervals and therefore are fixed constants. As we shall see later, these parameters will not affect the asymptotic outage analysis that is to follow.

In this paper, we consider two CSI scenarios. First we will assume only the receiver has perfect CSI and the transmitter allocates power uniformly across all blocks (CSIR case). Then we will consider the case where perfect CSI is also known at the transmitter (CSIT case). The transmitter then performs power allocation to reduce the outage probability subject to power constraints. The transmit power for both channels is ultimately drawn from the same power resource. As such, we assume the long-term average power consumed by the hybrid system is constrained according to

$$\mathbb{E}\left[\langle \boldsymbol{p} \rangle\right] + \mathbb{E}\left[\langle \hat{\boldsymbol{p}} \rangle\right] \leq P_{\text{av}}, \tag{4}$$

where $\langle \boldsymbol{p} \rangle = \frac{1}{A}\sum_{a=1}^{A} p_a$ and $\langle \hat{\boldsymbol{p}} \rangle = \frac{1}{B}\sum_{b=1}^{B} \hat{p}_b$. Practical communication systems have limitations on the peak power that can be transmitted. Since the FSO and RF links will realistically have different peak power limitations, we assume each is subjected to its own individual peak (or short-term) power constraint, i.e.

$$\langle \boldsymbol{p} \rangle \leq P_{\text{peak}}^{\text{fso}} \quad \text{and} \quad \langle \hat{\boldsymbol{p}} \rangle \leq P_{\text{peak}}^{\text{rf}}, \tag{5}$$

where $P_{\text{peak}}^{\text{fso}} = \alpha_{\text{fso}} P_{\text{av}}$ and $P_{\text{peak}}^{\text{rf}} = \alpha_{\text{rf}} P_{\text{av}}$ for fixed peak-to-average power ratios α_{fso} and α_{rf}.

It is important to note the difference in signal scaling between the FSO and RF AWGN channels, (2) and (3) respectively. In standard RF channels, the amplitude of the received electrical signal is proportional to the square-root of the power of the transmitted electromagnetic signal [30]. For the FSO channel, the amplitude of the received electrical signal is directly proportional to the transmitted optical power, due to the photodetection process [26]. This FSO model is slightly different to [2], where the AWGN FSO channel model was written in terms of the received electrical power (proportional to the square of the optical power). In this paper, we write the FSO AWGN channel (2) and power constraints (4) and (5) with respect to the transmitted optical and RF power. This is necessary to ensure consistency between the power constraints over the FSO and RF channels. As we shall see later, this scaling will significantly affect the design of power allocation strategies when CSI is known at the transmitter.

III. Information Theoretic Preliminaries

The channels described by (2) and (3) under the quasi-static assumption are not information stable [31] and therefore, their individual channel capacities in the strict Shannon sense are zero. For these channels, it is more appropriate to analyse the information outage probability, which lower bounds the codeword error probability of any coding scheme [32], [33]. Define the vector channel realisations $\boldsymbol{h} = (h_1, \ldots, h_A)$ and $\hat{\boldsymbol{h}} = (\hat{h}_1, \ldots, \hat{h}_B)$, and vector power allocations $\boldsymbol{p} = (p_1, \ldots, p_A)$ and $\hat{\boldsymbol{p}} = (\hat{p}_1, \ldots, \hat{p}_B)$. The total instantaneous mutual information in bits per channel use is therefore [23]

$$I_{\text{tot}}(\boldsymbol{p}, \hat{\boldsymbol{p}}, \boldsymbol{h}, \hat{\boldsymbol{h}}) = \frac{n}{A}\sum_{a=1}^{A} I_{\mathcal{X}_{\text{fso}}}^{\text{awgn}}(h_a^2 \rho^2 p_a^2) + \frac{1}{B}\sum_{b=1}^{B} I_{\mathcal{X}_{\text{rf}}}^{\text{awgn}}(\hat{h}_b \hat{\rho} \gamma \hat{p}_b), \tag{6}$$

where $I_{\mathcal{X}}^{\text{awgn}}(u) \in (0, \log_2|\mathcal{X}|)$ denotes the input-output mutual information of the AWGN channel with input constellation $\mathcal{X}$ and SNR u. Note that the achievable rate (6) implicitly assumes joint encoding and decoding across FSO and RF channels.

[3]The proportionality constant is referred to as the *responsivity* (measured in Watts per Ampere), and is dependent on the optical wavelength and photodetector material.

TABLE I
SNR EXPONENTS FOR SOME TYPICAL SCINTILLATION DISTRIBUTIONS.

Distribution	PDF $f(h)$	SNR Exponent
Exponential	$\exp(-h)$	$d^{(1)} = 1$
Lognormal	$\frac{1}{h\sigma\sqrt{2\pi}} e^{-(\log h-\mu)^2/(2\sigma^2)}$	$d^{(2)} = \frac{1}{4\log(1+\sigma^2)}$
Gamma-gamma	$\frac{2(ab)^{\frac{a+b}{2}}}{\Gamma(a)\Gamma(b)} h^{\frac{a+b}{2}-1} \mathrm{K}_{a-b}(2\sqrt{abh})$	$d^{(1)} = \min(a,b)$

The information outage probability of the hybrid system is given by

$$P_{\text{out}}(P_{\text{av}}, R) \triangleq \Pr\left\{ I_{\text{tot}}(\boldsymbol{p}, \hat{\boldsymbol{p}}, \boldsymbol{h}, \hat{\boldsymbol{h}}) < R \right\}, \tag{7}$$

where R is the target rate of the system in bits per hybrid channel use. We define the overall binary code rate of the hybrid system as $R_c \triangleq R/(m+nq)$, $0 < R_c < 1$. In addition, we define $0 < \delta < 1$ as the ratio of FSO bits to total transmitted bits, i.e. $\delta \triangleq \frac{nq}{m+nq}$.

In this paper we study the *SNR exponent* of the system, defined as

$$d^{(k)} \triangleq \lim_{P_{\text{av}}\to\infty} -\frac{\log P_{\text{out}}(P_{\text{av}}, R)}{(\log P_{\text{av}})^k}, \tag{8}$$

where $k = 1, 2$. Note that by including the integer k, (8) is more general than the SNR exponent normally defined in RF systems [24], [25]. This modified definition is required to allow for scintillation cases where the resulting outage probability curve will not converge to a constant slope when plotted on a log-log scale, but does when plotted on a $\log - \log^2$ scale (most notably under weak turbulence conditions where the scintillation is log-normal distributed [2]).

The SNR exponent will depend on the distribution of the fading coefficients. Rather than assuming a specific distribution, we characterise the fading via the component channel's single block transmission SNR exponent, defined as

$$d_{\text{fso}}^{(i)} \triangleq \lim_{u\to\infty} -\frac{\log \Pr\{I_{\text{fso}}^{\text{awgn}}(h^2 u^2) < R_{\text{fso}}\}}{(\log u)^i} \tag{9}$$

$$d_{\text{rf}}^{(j)} \triangleq \lim_{u\to\infty} -\frac{\log \Pr\{I_{\text{rf}}^{\text{awgn}}(\hat{h} u) < R_{\text{rf}}\}}{(\log u)^j}, \tag{10}$$

for given component channel rate constraints R_{fso} and R_{rf}, where $i, j \in \{1, 2\}$. In Table I we list some single block transmission SNR exponents (derived in [2]) for some typical scintillation distributions [1].[4] Note that the exponents derived in [2] defined the SNR exponents in terms of the received electrical SNR. In this paper, we define the FSO exponent in terms of the transmitted optical power. This results in a factor of 2 in the exponents listed in Table I compared to those given in [2].

IV. ASYMPTOTIC OUTAGE ANALYSIS: CSIR CASE

First let us assume that perfect CSI is known only at the receiver (CSIR case). The transmitter allocates power uniformly across all blocks, i.e. $p_1 = \ldots = p_A = \hat{p}_1 = \ldots = \hat{p}_B = p = P_{\text{av}}$.

[4]Note that if MIMO FSO with transmit repetition and equal gain combining is employed, then the exponents listed in Table I are simply multiplied by $N_t N_r$, where N_t and N_r denote the number of lasers and apertures respectively [2].

Theorem 4.1: Define component channel SNR exponents $d_{\text{fso}}^{(i)}$ and $d_{\text{rf}}^{(j)}$ as in (9) and (10) respectively. Suppose $\rho, \hat{\rho} > 0$ and $i = j = k$. Then the SNR exponent is given by

$$d^{(k)} = \inf_{\mathcal{K}(\delta, R_c)} \left\{ d_{\text{fso}}^{(k)} \kappa_1 + d_{\text{rf}}^{(k)} \kappa_2 \right\}, \tag{11}$$

where

$$\mathcal{K}(\delta, R_c) \triangleq \Big\{ \kappa_1, \kappa_2 \in \mathbb{Z} : \delta \frac{\kappa_1}{A} + (1-\delta)\frac{\kappa_2}{B} > 1 - R_c, \\ 0 \le \kappa_1 \le A, 0 \le \kappa_2 \le B \Big\}. \tag{12}$$

Proof: See Appendix A. ■

From Theorem 4.1, we see that the overall SNR exponent depends on R_c, δ, A, B and the individual SNR exponents d_{fso} and d_{rf} in a non-trivial way. Although in general, the solution to (11) can be straightforwardly determined numerically, it is difficult to obtain insight as to how the various system parameters influence the overall SNR exponent. However, for the most basic and interesting scenario, $A = B = 1$, the solution to (11) reduces to a simple intuitive form.

Corollary 4.1: Suppose $A = B = 1$. The solution to (11) is divided into two cases as follows.

1) If $\delta \le \frac{1}{2}$, then

$$d^{(k)} = \begin{cases} d_{\text{fso}}^{(k)} + d_{\text{rf}}^{(k)} & 0 < R_c \le \delta \\ d_{\text{rf}}^{(k)} & \delta < R_c \le 1-\delta \\ \min(d_{\text{fso}}^{(k)}, d_{\text{rf}}^{(k)}) & 1-\delta < R_c < 1. \end{cases} \tag{13}$$

2) If $\delta \ge \frac{1}{2}$, then

$$d^{(k)} = \begin{cases} d_{\text{fso}}^{(k)} + d_{\text{rf}}^{(k)} & 0 < R_c \le 1-\delta \\ d_{\text{fso}}^{(k)} & 1-\delta < R_c \le \delta \\ \min(d_{\text{fso}}^{(k)}, d_{\text{rf}}^{(k)}) & \delta < R_c < 1. \end{cases} \tag{14}$$

Proof: See Appendix B. ■

From Corollary 4.1 we can see directly how the hybrid system parameters will affect the asymptotic slope of the outage probability curve for the single block transmission case. In particular, we see that the asymptotic slope is affected by the SNR exponent of the individual component channels (which is in turn dependent on the scintillation distribution), the size of the FSO/RF signal set constellation, and the overall binary code rate of the system. In most practical systems, $\delta \ge \frac{1}{2}$, i.e. in a hybrid symbol period, the number of transmitted FSO bits will be greater than the number of RF transmitted bits. From (14), we see that the highest diversity is achieved if the binary code rate R_c is set to be less than $1-\delta = m/(m+nq)$, i.e. the total information rate is less than the maximum information rate of the stand-alone RF channel. If $1-\delta < R_c \le \delta$, the exponent is the same as a single FSO link, i.e. the additional coding over an RF link will not improve the asymptotic slope of the outage probability curve. For high binary code rates, $\delta < R_c < 1$, the asymptotic performance is dominated by the worst of the two exponents. Note that code rates above δ are not achievable with a stand-alone FSO link.

Although we have concentrated on the single block case, given the short coherence time of the optical channel compared to the RF channel, the cases of $A = 2, 3$ and $B = 1$ are also of practical interest and are readily evaluated from Theorem 4.1. In particular, these provide significant SNR exponent

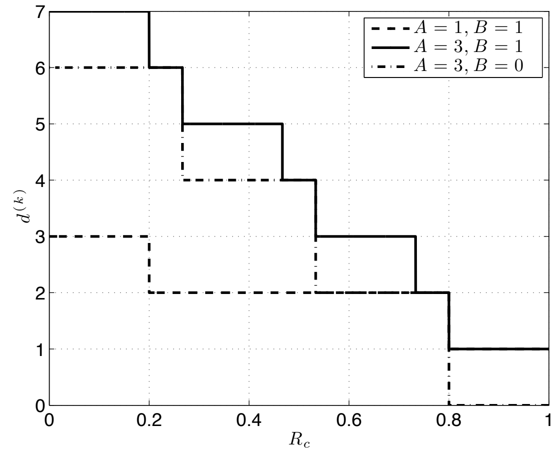

Fig. 2. Hybrid channel SNR exponent (11) with $d_{\rm fso}^{(k)}=2$, $d_{\rm rf}^{(k)}=1$ and $\delta=0.8$: $A=1$, $B=1$ system (dashed line); $A=3$, $B=1$ system (solid line); and a stand-alone FSO link with three blocks, i.e. $A=3$, $B=0$ (dot-dashed line).

improvements for lower rates, compared to the single block case. This is illustrated in Fig. 2, which plots the hybrid channel SNR exponent with, $d_{\rm fso}^{(k)}=2$, $d_{\rm rf}^{(k)}=1$ and $\delta=0.8$ for a: $A=1$, $B=1$ system (dashed line); $A=3$, $B=1$ system (solid line); and a stand-alone FSO link with three blocks[5], i.e. $A=3$, $B=0$ (dot-dashed line). By coding over more FSO blocks, vast improvements in the SNR exponent can be seen, particularly as the code rate decreases. In addition, we see that the $A=3$, $B=1$ exponent exceeds that of the stand-alone FSO system by $d_{\rm rf}^{(k)}$ for most code rates.

Theorem 4.2: Define component channel SNR exponents $d_{\rm fso}^{(i)}$ and $d_{\rm rf}^{(j)}$ as in (9) and (10) respectively. Suppose $i>j$ then the SNR exponent is

$$d^{(i)}=d_{\rm fso}^{(i)}\left(1+\left\lfloor\frac{A}{\delta}(\delta-R_c)\right\rfloor\right) \qquad 0<R_c\le\delta \quad (15)$$

$$d^{(j)}=d_{\rm rf}^{(j)}\left(1+\left\lfloor\frac{B}{1-\delta}(1-R_c))\right\rfloor\right) \qquad \delta<R<1. \quad (16)$$

Otherwise, if $i<j$ then the SNR exponent is

$$d^{(j)}=d_{\rm rf}^{(j)}\left(1+\left\lfloor\frac{B}{1-\delta}(1-\delta-R_c)\right\rfloor\right) \qquad 0<R_c\le 1-\delta \quad (17)$$

$$d^{(i)}=d_{\rm fso}^{(i)}\left(1+\left\lfloor\frac{A}{\delta}(1-R_c)\right\rfloor\right) \qquad 1-\delta<R_c<1. \quad (18)$$

Proof: See Appendix C. ∎

Theorem 4.2 shows how the overall performance of the hybrid channel will be affected when one of the component channels has an asymptotic outage probability that decays with SNR much faster than the other. In particular, we see that the overall SNR exponent will be dominated by the worst of the two component channel SNR exponents unless the binary code rate is below a certain threshold dependent on the ratio of FSO bits to total transmitted bits (δ). Note that these exponents are related to how outage-approaching codes should be designed [34].

To demonstrate the implications of our asymptotic results, we conducted a number of Monte Carlo simulations. This involved computing the mutual information curves $I_{\mathcal{X}_{\rm fso}}(u)$ and $I_{\mathcal{X}_{\rm rf}}(u)$ for a given constellation. These curves are then used as look-up-tables to determine the total mutual information (6) for a given set of vector fading realisations $\boldsymbol{h}$ and $\hat{\boldsymbol{h}}$, which are generated randomly according to one of the distributions listed in Table I. The simulation results are shown in Fig. 3. In this example we chose $A=B=1$, $n=4$ (hence $\gamma=4$), 2PPM for the FSO channel and 16QAM for the RF channel. Therefore $m=nq=4$ bits, and a maximum of 8 bits per hybrid channel symbol can be transmitted ($\delta=0.5$). For simplicity we have chosen $\hat{\rho}=1-\rho$. The dot-dashed and dashed curves illustrate the case when only the FSO and RF channels are available, i.e. when $\rho=1$ and $\rho=0$ respectively, and must support $R=3$ bits per symbol. The solid and dot marked curves show the hybrid channel outage performance to support a $R_c=3/8$ and $R_c=6/8$ binary code rate respectively when $\rho=0.5$. In Fig. 3(a), both FSO and RF channels experience exponential scintillation, corresponding to very strong turbulence conditions. In this case, $d_{\rm fso}^{(1)}=d_{\rm rf}^{(1)}=1$ and hence Corollary 4.1 applies. As expected the dashed and dot-dashed curves have the same slope since they both have the same SNR exponent. When $R_c=3/8<\delta$, we see the slope of the outage curve becomes steeper when both channels are available (solid curve), in fact the SNR exponent is now $d^{(1)}=2$. However, when $R_c=6/8>\delta$ (dot-marked curve), from (14) the exponent is $d^{(1)}=1$, and hence the curve has the same slope as the FSO and RF only cases previously described. Fig. 3(b) shows the case when the FSO channel now experiences gamma-gamma scintillation with $a=2$ and $b=3$, therefore $d_{\rm fso}^{(1)}=2$. When $R_c=3/8$ the system benefits from both channels, the overall exponent increases to $d^{(1)}=3$. However, as predicted by our asymptotic results, if $R_c=6/8$ the overall exponent is dominated by the RF component and the exponent is $d^{(1)}=1$. Fig 3(c) shows the case when the FSO and RF channels experience exponential and lognormal scintillation (with $\mu=-\log(2)/2$ and $\sigma^2=\log(2)$) respectively. From Table I, $d_{\rm fso}^{(1)}=1$ and $d_{\rm rf}^{(2)}=\frac{1}{4\sigma^2}$. Hence Theorem 4.2 applies in this case. In particular, for the $R_c=3/8$ system, from (17), the hybrid channel exponent is dominated by the RF channel exponent and hence the hybrid outage curve (solid) has the same asymptotic slope as the RF outage curve (dashed). If the required rate is increased to $R_c=6/8$, the exponent is now given by (18), hence the exponent of the FSO channel dominates overall hybrid channel performance. This behaviour can be seen in Fig. 3(c), where the dot marked curve (corresponding to the $R_c=6/8$ hybrid case) has the same slope as the FSO channel (dot-dashed curve). Fig. 3(d) illustrates the case when both channels experience lognormal scintillation corresponding to weak turbulence conditions. In this case, $d_{\rm fso}^{(2)}=d_{\rm rf}^{(2)}=\frac{1}{4\sigma^2}$, and we see that when $R_c=3/8$ the hybrid channel benefits from both component channel exponents.

[5]Note that for the stand-alone FSO system with A blocks, the exponent is given by $d^{(k)}=d_{\rm fso}^{(k)}(1+\lfloor A(1-R_c/\delta)\rfloor)$ for $0<R_c<\delta$, since we have defined R_c with respect to the hybrid system.

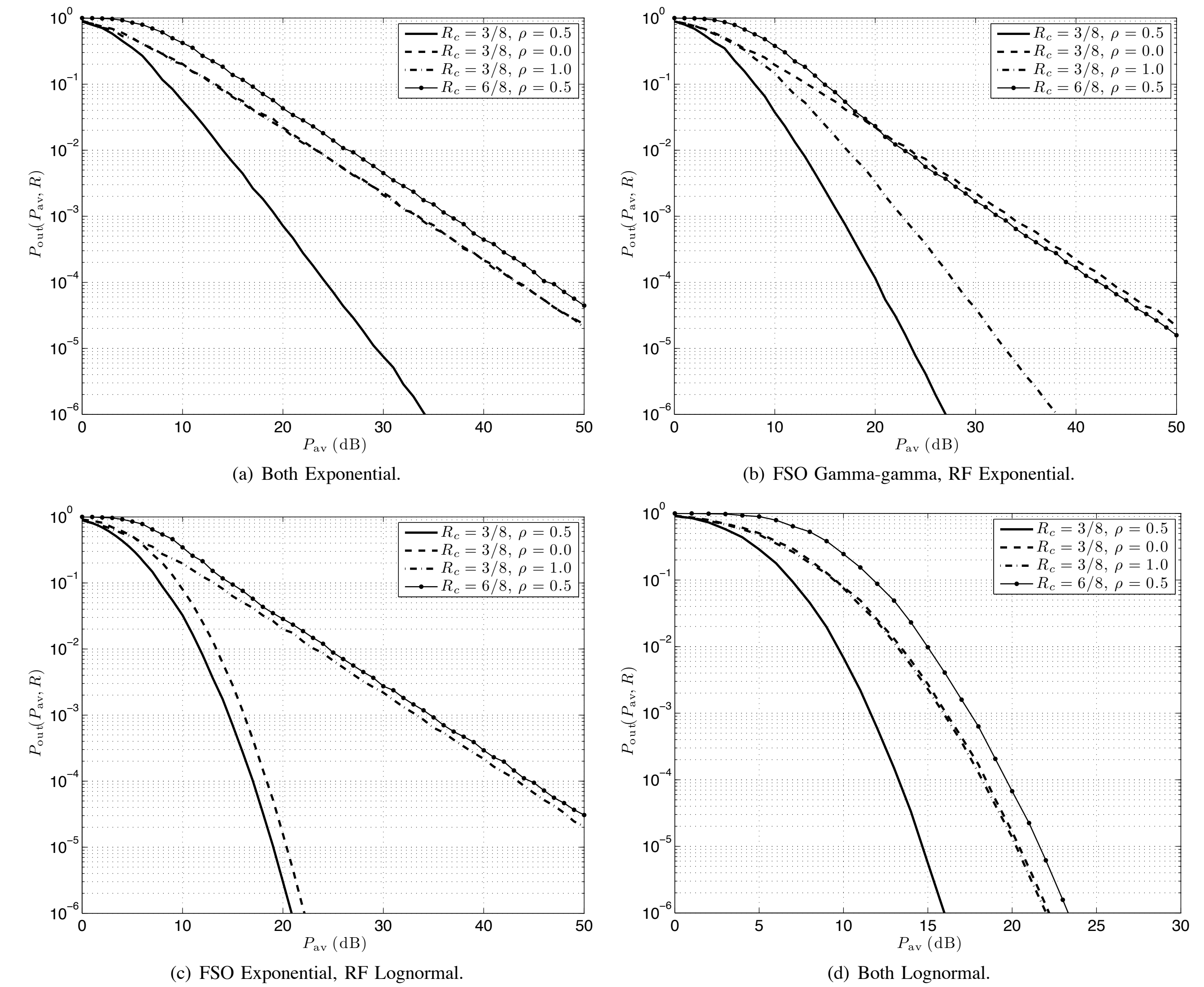

Fig. 3. Outage performance of the hybrid FSO/RF channel with $A = B = 1$, $n = 4$ ($\gamma = 4$), 2PPM FSO and 16QAM RF: solid, dashed and dot-dashed curves correspond to hybrid channel with $R_c = 3/8$ for $\rho = 0.5, 0, 1$ respectively (where $\hat{\rho} = 1 - \rho$). The dot-marked curves show the hybrid outage performance with $R_c = 6/8$ and $\rho = 0.5$.

V. Asymptotic Outage Analysis: CSIT Case

In this section we assume both the transmitter and receiver have perfect knowledge of the FSO and RF fading coefficients. Due to the slow nature of the scintillation processes, this is a realistic assumption, which, as we shall see, induces significant power gains. In this case the transmitter adapts the power (subject to power constraints) to compensate for channel fluctuations to significantly reduce the outage probability.

A. Power Allocation Strategies

To find the optimal power allocation strategy, we require the solution to the following minimisation problem.

$$\begin{cases} \text{Minimise:} & \Pr\left\{ I_{\rm tot}(\boldsymbol{p}, \hat{\boldsymbol{p}}, \boldsymbol{h}, \hat{\boldsymbol{h}}) < R \right\} \\ \text{Subject to:} & \mathbb{E}[\langle \boldsymbol{p} \rangle] + \mathbb{E}[\langle \hat{\boldsymbol{p}} \rangle] \le P_{\rm av}, \\ & \langle \boldsymbol{p} \rangle \le P_{\rm peak}^{\rm fso}, \ \ \langle \hat{\boldsymbol{p}} \rangle \le P_{\rm peak}^{\rm rf}. \end{cases} \tag{19}$$

Theorem 5.1: The solution to problem (19) is given by

$$(\wp^*, \hat{\wp}^*) = \begin{cases} (\wp, \hat{\wp}) & \langle \wp \rangle + \langle \hat{\wp} \rangle \le s^* \\ (\mathbf{0}, \mathbf{0}) & \text{otherwise}, \end{cases} \tag{20}$$

where $(\wp, \hat{\wp})$ is the solution to the following minimisation problem

$$\begin{cases} \text{minimise} & \langle \boldsymbol{p} \rangle + \langle \hat{\boldsymbol{p}} \rangle \\ \text{subject to} & I_{\rm tot}(\boldsymbol{p}, \hat{\boldsymbol{p}}, \boldsymbol{h}, \hat{\boldsymbol{h}}) \ge R \\ & \langle \boldsymbol{p} \rangle \le P_{\rm peak}^{\rm fso}, \langle \hat{\boldsymbol{p}} \rangle \le P_{\rm peak}^{\rm rf} \\ & \boldsymbol{p}, \hat{\boldsymbol{p}} \succeq \mathbf{0}. \end{cases} \tag{21}$$

In (20), s^* is a threshold determined by

$$s^* = \sup\left\{ s : \mathbb{E}_{(\boldsymbol{h}, \hat{\boldsymbol{h}}) \in \mathcal{R}(s)} \left[\langle \wp \rangle + \langle \hat{\wp} \rangle \right] \le P_{\rm av} \right\}, \tag{22}$$

where

$$\mathcal{R}(s) \triangleq \left\{ (\boldsymbol{h}, \hat{\boldsymbol{h}}) \in \mathbb{R}^{A+B} : \langle \wp \rangle + \langle \hat{\wp} \rangle \le s \right\}. \tag{23}$$

Proof: The proof follows similar arguments described in [35], [36]. Essentially, (21) gives the minimum set of power allocations that satisfy the peak power constraints such that total mutual information is greater than the rate requirement. The inclusion of threshold s^* ensures that the long-term power constraint is also satisfied. The larger s^* is, the smaller the outage probability will be. Since the solution $(\wp, \hat{\wp})$ from (21) gives the minimum sum power to satisfy the rate constraint, then this solution also gives the maximum s^*, which in turn minimises the outage probability. ∎

Unfortunately, (21) does not lend itself to a simple solution, since in general $I_{\mathcal{X}}^{\text{awgn}}(p^2)$ is not a concave function in p for all $p > 0$. Thus (21) is a non-convex optimisation problem [37]. Instead of solving (21), we propose a suboptimal algorithm that, as we shall see, exhibits the same asymptotic behaviour as the optimal solution. In this direction, first consider the following lemmas. The proofs follow straightforwardly via the Karush-Kahn-Tucker conditions [37] and using the relationship between mutual information and the minimum mean-square error (MMSE) for Gaussian channels [38], as done in [39].

Lemma 5.1: Define the minimisation problem,

$$\begin{cases} \text{minimise} & \langle \boldsymbol{p}^2 \rangle + \langle \hat{\boldsymbol{p}}^2 \rangle \\ \text{subject to} & I_{\text{tot}}(\boldsymbol{p}, \hat{\boldsymbol{p}}, \boldsymbol{h}, \hat{\boldsymbol{h}}) \geq R \\ & \boldsymbol{p}, \hat{\boldsymbol{p}} \succeq \boldsymbol{0}, \end{cases} \tag{24}$$

where $\langle \boldsymbol{p}^2 \rangle \triangleq \frac{1}{A} \sum_{a=1}^{A} p_a^2$. The solution to (24) is

$$p_a^* = \sqrt{\Upsilon_{\mathcal{X}_{\text{fso}}}\left(h_a^2 \rho^2, \frac{1}{n\lambda}\right)} \quad \text{and} \quad \hat{p}_b^* = \Psi_{\mathcal{X}_{\text{rf}}}\left(\hat{\rho} \hat{h}_b \gamma, \lambda\right), \tag{25}$$

where $\Upsilon_{\mathcal{X}}(u,t) \triangleq \frac{1}{u} \mathsf{mmse}_{\mathcal{X}}^{-1}\left(\min\left\{\mathsf{mmse}_{\mathcal{X}}(0), \frac{t}{u}\right\}\right)$, $\Psi_{\mathcal{X}}(u,t)$ is the solution x to the equation $\mathsf{mmse}_{\mathcal{X}}(xu) = \frac{2x}{tu}$, $\mathsf{mmse}_{\mathcal{X}}(p)$ denotes the MMSE of a Gaussian channel with discrete input constellation $\mathcal{X}$, $\mathsf{mmse}_{\mathcal{X}}^{-1}(u)$ is the inverse MMSE function, and λ is chosen such that $I_{\text{tot}}(\boldsymbol{p}, \hat{\boldsymbol{p}}, \boldsymbol{h}, \hat{\boldsymbol{h}}) = R$.

Lemma 5.2: Define the minimisation problem

$$\begin{cases} \text{minimise} & \langle \boldsymbol{p}^2 \rangle + \langle \hat{\boldsymbol{p}}^2 \rangle \\ \text{subject to} & I_{\text{tot}}(\boldsymbol{p}, \hat{\boldsymbol{p}}, \boldsymbol{h}, \hat{\boldsymbol{h}}) \geq R \\ & \sqrt{\langle \boldsymbol{p}^2 \rangle} \leq P_{\text{peak}}^{\text{fso}}, \sqrt{\langle \hat{\boldsymbol{p}}^2 \rangle} \leq P_{\text{peak}}^{\text{rf}} \\ & \boldsymbol{p}, \hat{\boldsymbol{p}} \succeq \boldsymbol{0}. \end{cases} \tag{26}$$

Let $\boldsymbol{p}^*$ and $\hat{\boldsymbol{p}}^*$ be the solution to (24) in Lemma 5.1, and $\boldsymbol{\wp}$ and $\hat{\boldsymbol{\wp}}$ be the solution to (26). The solution to (26) is separated into four cases depending on $\boldsymbol{p}^*$ and $\hat{\boldsymbol{p}}^*$.

1) If $\boldsymbol{p}^*$ and $\hat{\boldsymbol{p}}^*$ satisfy the constraints in (26). Then $\boldsymbol{\wp} = \boldsymbol{p}^*$ and $\hat{\boldsymbol{\wp}} = \hat{\boldsymbol{p}}^*$.
2) If $\sqrt{\langle (\boldsymbol{p}^*)^2 \rangle} \leq P_{\text{peak}}^{\text{fso}}$ and $\sqrt{\langle (\hat{\boldsymbol{p}}^*)^2 \rangle} > P_{\text{peak}}^{\text{rf}}$. Then

$$\wp_a = \sqrt{\Upsilon_{\mathcal{X}_{\text{fso}}}\left(h_a^2 \rho^2, \frac{1}{n\lambda_1}\right)} \tag{27}$$

$$\hat{\wp}_b = \Psi_{\mathcal{X}_{\text{rf}}}\left(\hat{h}_b \gamma \hat{\rho}, \lambda_2\right), \tag{28}$$

where λ_2 is chosen such that $\sqrt{\langle \hat{\boldsymbol{\wp}}^2 \rangle} = P_{\text{peak}}^{\text{rf}}$ and λ_1 is chosen such that

$$\frac{n}{A} \sum_{a=1}^{A} I_{\mathcal{X}_{\text{fso}}}^{\text{awgn}}(\rho^2 h_a^2 \wp_a^2) = R - \frac{1}{B} \sum_{b=1}^{B} I_{\mathcal{X}_{\text{rf}}}^{\text{awgn}}(\hat{\wp}_b \hat{h}_b \hat{\rho} \gamma).$$

If $\sqrt{\langle \boldsymbol{\wp}^2 \rangle} > P_{\text{peak}}^{\text{fso}}$ then the solution to (26) is infeasible.
3) If $\sqrt{\langle (\boldsymbol{p}^*)^2 \rangle} > P_{\text{peak}}^{\text{fso}}$ and $\sqrt{\langle (\hat{\boldsymbol{p}}^*)^2 \rangle} \leq P_{\text{peak}}^{\text{rf}}$. Then the solution to (26) is the same as the previous case, with the roles of rf and fso interchanged.
4) If $\sqrt{\langle (\boldsymbol{p}^*)^2 \rangle} > P_{\text{peak}}^{\text{fso}}$ and $\sqrt{\langle (\hat{\boldsymbol{p}}^*)^2 \rangle} > P_{\text{peak}}^{\text{rf}}$, then the solution to (26) is infeasible.

Comparing (21) with (26), we see that (26) is minimising the sum of the mean-square power of the FSO and RF channels, subject to individual short-term root mean-square (RMS) power constraints. By applying Jensen's inequality [23] to these constraints, we see that a solution to (26) will also satisfy the constraints in (21) and hence can be considered a suboptimal solution to (21). Therefore to find a suboptimal solution to the original minimisation problem (19) we use the solutions in Lemma 5.2 for $(\wp, \hat{\wp})$ instead of solving (21).

B. Asymptotic Analysis

The asymptotic outage performance of optimal power allocation for discrete-input block-fading AWGN channels was analysed by Nguyen *et al.* in [35], [40]. In particular, from [35, Prop. 3], if the peak-to-average power ratios α_{fso} and α_{rf} are finite, then the SNR exponent will be the same as the CSIR case given in Theorems 4.1 and 4.2. When there are no peak-to-average power constraints then the SNR exponent of the optimal power allocation strategy is [40, Th. 2]

$$d_{\text{csit}}^{(1)} = \begin{cases} \infty & d_{\text{csir}}^{(1)} > 1 \\ \frac{d_{\text{csir}}^{(1)}}{1 - d_{\text{csir}}^{(1)}} & d_{\text{csir}}^{(1)} < 1, \end{cases} \tag{29}$$

where $d_{\text{csir}}^{(1)}$ is the SNR exponent for the CSIR case. We now prove that the SNR exponent of our suboptimal power allocation strategy in Section V-A is the same as (29).

Theorem 5.2: Suppose $\alpha_{\text{fso}}, \alpha_{\text{rf}} \to \infty$. Then the SNR exponent of the suboptimal power allocation scheme described by (19) with (26) is given by (29).

Proof: A sketch of proof proceeds as follows. Let $(\boldsymbol{p}^*, \hat{\boldsymbol{p}}^*)$ solve (24). Now consider the following power allocation rule

$$(\boldsymbol{\wp}^\dagger, \hat{\boldsymbol{\wp}}^\dagger) = \begin{cases} (\boldsymbol{p}^*, \hat{\boldsymbol{p}}^*) & \sqrt{\langle (\boldsymbol{p}^*)^2 \rangle + \langle (\hat{\boldsymbol{p}}^*)^2 \rangle} \leq s^\dagger \\ (\boldsymbol{0}, \boldsymbol{0}) & \text{otherwise}, \end{cases} \tag{30}$$

where $s^\dagger$ is a threshold such that $\mathbb{E}_{(\boldsymbol{h}, \hat{\boldsymbol{h}}) \in \mathcal{R}(s)}\left[\sqrt{\langle \boldsymbol{p}^2 \rangle + \langle \hat{\boldsymbol{p}}^2 \rangle}\right] = P_{\text{av}}$ and

$$\mathcal{R}(s) \triangleq \left\{(\boldsymbol{h}, \hat{\boldsymbol{h}}) \in \mathbb{R}^{A+B} : \sqrt{\langle \boldsymbol{p}^2 \rangle + \langle \hat{\boldsymbol{p}}^2 \rangle} \leq s\right\}. \tag{31}$$

Using similar arguments as [40] it can be proved that the hybrid system with power allocation rule (30) has diversity (29). This diversity lower bounds the diversity achieved by the suboptimal algorithm (19) with (26), since (30) satisfies more stringent power constraints. Hence we conclude the suboptimal power allocation algorithm also has diversity (29). ∎

The implications of (29) are described as follows. When $d_{\text{csit}}^{(1)} = \infty$, then the outage probability curve will be vertical at a certain threshold of average power, i.e. the hybrid system is able to maintain a constant level of instantaneous input-output mutual information. The threshold at which this occurs is referred to as the *delay-limited capacity* of the system [41]. Note that if $d_{\text{csir}}^{(1)} = 1$ in (29) then $d_{\text{csit}}^{(1)} = \infty$, however, the outage curve will not go vertical, nor will it converge to a constant slope when plotted on a $\log$-$\log$ scale [35]. When the peak-to-average power ratios are finite, the peak power constraints introduce an error floor with a slope equal to the CSIR case. The height of the error floor is dependent on α_{fso} and α_{rf} [35].

To demonstrate the benefits of power allocation and its asymptotic behaviour, we simulated the suboptimal hybrid power allocation strategy for a $A = B = 1$, $n = 4$, 2PPM FSO and 16QAM RF system with $\rho = \hat{\rho} = 0.5$ and peak-to-average power ratios $\alpha_{\text{fso}} = \alpha_{\text{rf}} = \alpha$. Exponential distributed fading was applied to both channels. Fig. 4(a) shows the hybrid outage performance with our suboptimal power allocation strategy compared to uniform power allocation (cross marked curve) when $R_c = 3/8$. Since, from Corollary 4.1, $d_{\text{csir}}^{(1)} = 2$, then from (29), the SNR exponent is $d_{\text{csit}}^{(1)} = \infty$, i.e. when there are no peak power constraints, the curve will go vertical at a certain average power threshold. This can be seen in Fig. 4(a) (thick solid curve), for $P_{\text{av}} > 8$ dB outages are completely removed. We see that there is a power saving of more than 20 dB compared to uniform power allocation to achieve 10^{-5} outage probability. When peak power constraints are introduced, as expected, we see that an error floor is introduced with the same slope as the CSIR case. The floor shifts down in probability as the peak-to-average power ratio increases. Fig. 4(b) shows the case when $R_c = 6/8$. Since $d_{\text{csir}}^{(1)} = 1$, when there are no peak power constraints, the outage curve will no longer go vertical (thick solid curve). As expected we see an error floor is introduced when the peak-to-average power ratio is finite. Nonetheless, we still see significant power savings compared to uniform power allocation.

VI. Conclusions

We proposed a simple hybrid FSO/RF channel model based on parallel block fading channels. This hybrid model takes into account differences in signalling rates and fading effects typically experienced by the component channels involved. Under this framework, we examined the information theoretic limits of the hybrid channel. In particular, we studied its asymptotic high SNR outage performance by analysing the outage diversity or SNR exponents. When CSI is only available at the receiver, in the general case, the exponent is not available in closed form. Instead, we derived simple expressions from which it can be computed numerically. For the special case when transmission consists of single FSO and RF blocks, we derived the SNR exponent in closed form in terms of each component channel's SNR exponent, the ratio component channel bits to total bits, and the overall binary code rate of the system. The highest outage diversity is achieved if the hybrid binary code rate is set less than the minimum of the two component channel's maximum binary code rates. At the other extreme, if the hybrid binary code rate is set larger than the maximum of the two component channel's maximum binary code rates, then the SNR exponent is dominated by the worst of the two component channel's SNR exponents. When CSI is also available at the transmitter, we derived the optimal power allocation scheme that minimises the outage probability subject to peak and average power constraints. Due to the power scaling of the FSO channel, this requires the solution to a non-convex optimisation problem, which is intractable in practical systems. We proposed a suboptimal power allocation strategy, which is much simpler to implement and has the same SNR exponent as the optimal power allocation. Our results indicate that significant power savings (on the order of tens of dBs) are achievable using the suboptimal algorithm compared to uniform power allocation. Future extensions to the work presented in this paper include: asymptotic analysis with a joint distribution on parameters ρ and $\hat{\rho}$; independent non-identically distributed fading; adaptive coding and modulation; practical outage-approaching code design; and lower complexity power adaptation strategies.

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Appendix A
Proof of Theorem 4.1

Under uniform power allocation, $p_1 = \ldots = p_A = \hat{p}_1 = \ldots = \hat{p}_B = p = P_{\text{av}}$. Define the normalised fading coefficients

$$\zeta_a \triangleq -\frac{\log(h_a)}{\log p}, \qquad \xi_b \triangleq -\frac{\log(\hat{h}_b)}{\log p}. \tag{32}$$

Assume the FSO and RF component channels have SNR exponents $d_{\text{fso}}^{(k)}$ and $d_{\text{rf}}^{(k)}$ as defined in (9) and (10) respectively. This implies[6]

$$f_\zeta(\zeta) \doteq \exp\left(-d_{\text{fso}}^{(k)}\zeta(\log p)^k\right)$$
$$f_\xi(\xi) \doteq \exp\left(-d_{\text{rf}}^{(k)}\xi(\log p)^k\right),$$

and hence

$$f(\boldsymbol{\zeta},\boldsymbol{\xi}) \doteq \exp\left(-(\log p)^k\left(d_{\text{fso}}^{(k)}\sum_{a=1}^{A}\zeta_a + d_{\text{rf}}^{(k)}\sum_{b=1}^{B}\xi_b\right)\right). \tag{33}$$

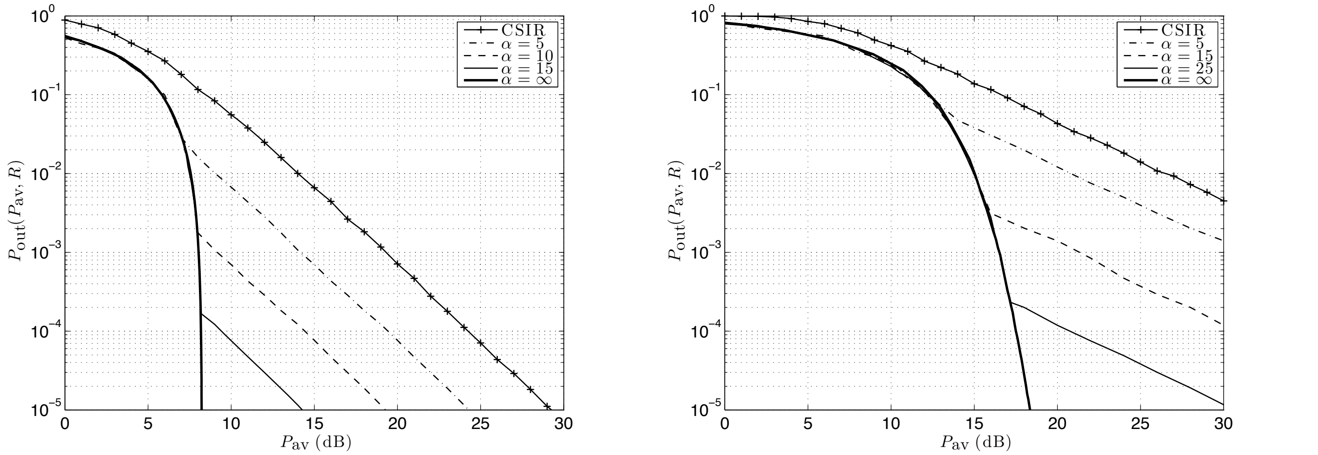
In the large SNR limit, the total instantaneous mutual information becomes

$$\lim_{p\to\infty} I_{\text{tot}}(\boldsymbol{p},\hat{\boldsymbol{p}},\boldsymbol{h},\hat{\boldsymbol{h}}) = \frac{nq}{A}\sum_{a=1}^{A}(1-\mathbb{1}\{\zeta_a>1\}) + \frac{m}{B}\sum_{b=1}^{B}(1-\mathbb{1}\{\xi_b>1\}), \tag{34}$$

where $\mathbb{1}\{\cdot\}$ denotes the indicator function. Let

$$\mathcal{O} \triangleq \left\{\boldsymbol{\zeta},\boldsymbol{\xi} : \frac{nq}{A}\sum_{a=1}^{A}(1-\mathbb{1}\{\zeta_a>1\}) + \frac{m}{B}\sum_{b=1}^{B}(1-\mathbb{1}\{\xi_b>1\}) < R\right\} \tag{35}$$

[6]Note that $g(z) \doteq \exp(-d\log z)$ indicates that $\lim_{z\to\infty} -\frac{\log g(z)}{\log z} = d$.



(a) $R_c = 3/8$. (b) $R_c = 6/8$.

Fig. 4. Outage performance of the hybrid FSO/RF channel with CSIT (solid) and uniform power allocation (dashed). System parameters included: $\rho = \hat{\rho} = 0.5$, $A = B = 1$, $n = 4$, 2PPM FSO and 16QAM RF with peak and average power constraints, and peak-to-average power ratios $\alpha_{\rm fso} = \alpha_{\rm rf} = \alpha$ in decibels. Exponential distributed fading on both channels.

be the outage set. Hence the outage probability is

$$P_{\rm out}(p, R) = \int_{\mathcal{O}} f(\boldsymbol{\zeta}, \boldsymbol{\xi}) d\boldsymbol{\zeta} d\boldsymbol{\xi}. \tag{36}$$

From the SNR exponent definition (8), using (33) and Varadhan's lemma [42] we have

$$d^{(k)} = \inf_{\mathcal{O}} \left(d_{\rm fso}^{(k)} \sum_{a=1}^{A} \zeta_a + d_{\rm rf}^{(k)} \sum_{b=1}^{B} \xi_b \right). \tag{37}$$

Let $0 \le \kappa_1 \le A$ and $0 \le \kappa_2 \le B$ be the number elements of $\boldsymbol{\zeta}$ and $\boldsymbol{\xi}$ set to one respectively. Dividing both sides of the inequality in (35) by $m + nq$ we obtain the set $\mathcal{K}(\delta, R_c)$ as defined in (12), and the infimum (37) is now (11), as given in the theorem.

APPENDIX B
PROOF OF COROLLARY 4.1

Proof: From Theorem 4.1 we must find the infimum (11) over the set

$$\mathcal{K}(\delta, R_c) \triangleq \{\kappa_1, \kappa_2 \in (0,1) : \delta\kappa_1 + (1-\delta)\kappa_2 > 1 - R_c\}. \tag{38}$$

Possible solutions to κ_1 and κ_2 include: (a) $\kappa_1 = \kappa_2 = 1$, $d^{(k)} = d_{\rm fso}^{(k)} + d_{\rm rf}^{(k)}$, $0 < R_c < 1$; (b) $\kappa_1 = 1$, $\kappa_2 = 0$, $d^{(k)} = d_{\rm fso}^{(k)}$, $1 - \delta < R_c < 1$; (c) $\kappa_1 = 0$, $\kappa_2 = 1$, $d^{(k)} = d_{\rm rf}^{(k)}$, $\delta < R_c < 1$. Now suppose $\delta \le \frac{1}{2}$, then $\delta \le 1 - \delta$, hence: for $0 < R_c \le \delta$ only solution (a) is possible, therefore $d^{(k)} = d_{\rm fso}^{(k)} + d_{\rm rf}^{(k)}$; for $\delta < R_c \le 1 - \delta$ solutions (a) and (c) are valid, hence taking the infimum results in $d^{(k)} = d_{\rm rf}^{(k)}$; for $1 - \delta < R_c < 1$, solutions (a), (b) and (c) are valid, and hence the resulting infimum is $d^{(k)} = \min(d_{\rm fso}^{(k)}, d_{\rm rf}^{(k)})$. The case when $\delta \ge \frac{1}{2}$ follows similar arguments. ■

APPENDIX C
PROOF OF THEOREM 4.2

The proof follows the same arguments as the proof of Theorem 4.1. First we define the normalised fading coefficients ζ_a and ξ_b as in (32). From (9) and (10) we have

$$f(\boldsymbol{\zeta}, \boldsymbol{\xi}) \doteq \exp\left(-(\log p)^i d_{\rm fso}^{(i)} \sum_{a=1}^{A} \zeta_a - (\log p)^j d_{\rm rf}^{(j)} \sum_{b=1}^{B} \xi_b \right). \tag{39}$$

From the SNR exponent definition (8), using (33) and Varadhan's lemma [42] we have

$$d^{(i)} = \inf_{\mathcal{K}(\delta, R_c)} \left(d_{\rm fso}^{(k)} \kappa_1 + (\log p)^{j-i} d_{\rm rf}^{(j)} \kappa_2 \right), \tag{40}$$

$$d^{(j)} = \inf_{\mathcal{K}(\delta, R_c)} \left((\log p)^{i-j} d_{\rm fso}^{(k)} \kappa_1 + d_{\rm rf}^{(j)} \kappa_2 \right), \tag{41}$$

where the outage set $\mathcal{K}$ is defined as in (12). Now, if $i > j$, then from (41) we need to set κ_1 as small as possible, whilst satisfying (12). If we set $\kappa_1 = 0$ then we require $\kappa_2 = 1 + \lfloor \frac{B}{1-\delta}(1 - R_c) \rfloor$. Since $\kappa_2 \le B$, then we can only set $\kappa_1 = 0$ if $R_c > \delta$. In this case the exponent is $d^{(j)} = d_{\rm rf}^{(j)}(1 + \lfloor \frac{B}{1-\delta}(1 - R_c) \rfloor$. If $0 < R_c \le \delta$ then we must choose $\kappa_1 > 0$. In this case we set $\kappa_2 = B$ and hence to satisfy (12) we require $\kappa_1 = 1 + \lfloor \frac{A}{\delta}(\delta - R_c) \rfloor$. Since $i > j$ the SNR exponent of the FSO channel dominates, and the overall SNR exponent becomes $d^{(i)} = 1 + \lfloor \frac{A}{\delta}(\delta - R_c) \rfloor$ as given in the statement of the theorem. The case when $i < j$ follows the same arguments.

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