New Space-Time Trellis Codes for Slow Fading Channels

Yi Hong
Institute for Telecommunications Research,
University of South Australia,
Mawson Lakes SA 5095, Australia
Email: Yi.Hong@unisa.edu.au

Albert Guillén i Fàbregas
Institute for Telecommunications Research,
University of South Australia,
Mawson Lakes SA 5095, Australia
Email: albert.guillen@unisa.edu.au

***Abstract*— New space-time trellis codes with 4-PSK and 8-PSK for two transmit antennas in slow fading channels are proposed in this paper. The codes are designed specifically to minimize the frame error probability. The performance of the proposed codes with various memory orders and receive antennas is evaluated by simulation. It is shown that the proposed codes outperform previously known codes [1].**

I. Introduction

Space-time trellis coding (STTC) techniques [1] have been proposed to achieve both diversity and coding gains on multi-input multi-output (MIMO) fading channels by combining multiple transmit antennas and coding with higher level modulation schemes. In [1], rank and determinant criteria (RDC) were proposed to maximize both diversity and coding gains of STTCs over slow fading channels. Several efforts have been dedicated to further maximize the coding gain using RDC [2, 3]. In [4], the determinant criterion was strengthened by analysis of the role of the Euclidean distance. In [5], STTCs are designed based on either RDC or EDC depending on the diversity gain of the system. Based on above criteria, some improved 4-PSK and 8-PSK STTCs are proposed through exhaustive computer search [5, 6].

A common feature of all above code design criteria is to minimize the worst case pairwise error probability (PWEP). To further improve code performance, the design criteria proposed in [7] attempt to minimize the worst case frame error probability, which is a function of the distance spectrum of the code. The distance spectrum is an enumeration of all the possible product measures (non-zero determinants) with their relative weights [8]. In [7], only a 4-state 4-PSK STTC was designed.

In this paper, we consider the design guidelines, which aim to minimize the truncated union bound on frame error rate (FER) by taking into account the first three terms. The PWEP terms depend on determinants and the associate weights for the systems with low and moderate diversity gains. This is similar to the approach taken in [9], where the PWEP terms depend on the Euclidean distances instead for moderate diversity gains. In our design, we construct two complete sets of 4-PSK and 8-PSK STTCs for two transmit antennas over slow fading channels. Through simulations, it is shown that the new codes outperform previously known codes.

The rest of the paper is organized as follows. Section II introduces the system model and code design criteria. Section III introduces STTC encoder structures for PSK. In Section IV, new 4-PSK and 8-PSK STTCs are presented together with simulation results. Finally, conclusions are drawn in Section V.

II. System Model and Code Design Criteria

The following notations are used in the paper: T denotes transpose and $\dagger$ denotes transpose conjugate. The symbol $\oplus_M$ and and $\ominus_M$ denotes modulo M addition and substraction. Superscripts I and Q denote the real and imaginary parts of a complex number. Let $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ denote the ring of the integers $\oplus_4$ and let $\mathbb{Z}_4[j]$ be the ring of Gaussian integers modulo 4, where each element $z \in \mathbb{Z}_4[j]$ has $\{z = z^I + jz^Q : z^I, z^Q \in \mathbb{Z}_4\}$ and $j^2 = -1$.

We consider a space-time coding system with n_T transmit and n_R receive antennas over slow fading channels. The received signal matrix $\mathbf{Y} \in \mathbb{C}^{n_R \times L}$ is given by

$$\mathbf{Y} = \sqrt{E_s}\mathbf{H}\mathbf{X} + \mathbf{N}, \tag{1}$$

where L is the frame length, $\mathbf{X} = [\mathbf{x}_1, ..., \mathbf{x}_t, ..., \mathbf{x}_L] \in \mathbb{C}^{n_T \times L}$ is the transmitted signal matrix, where $\mathbf{x}_t$=$[x_t^1, ..., x_t^{n_T}]^T$, $\mathbf{H} = [\mathbf{h}_1, ..., \mathbf{h}_{n_T}] \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, which is constant during a frame and varies from one frame to another independently. In (1), $\mathbf{N} \in \mathbb{C}^{n_T \times L}$ is a matrix of the complex white Gaussian noise samples i.i.d $\sim \mathcal{N}_{\mathbb{C}}(0, N_0)$, and $\frac{E_s}{N_0}$ is the average signal to noise ratio (SNR) per transmit antenna. The elements of $\mathbf{H}$ are assumed to be i.i.d circularly symmetric Gaussian random variables $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. The channel is assumed to be known at the receiver.

Assume that a codeword $\mathbf{X}$ is transmitted, the maximum-likelihood receiver might decide erroneously in favor of another codeword $\hat{\mathbf{X}} = [\hat{\mathbf{x}}_1, ..., \hat{\mathbf{x}}_t, ..., \hat{\mathbf{x}}_L]$, where $\hat{\mathbf{x}}_t = [\hat{x}_t^1, ..., \hat{x}_t^{n_T}]^T$. Let r denote the rank of the codeword difference matrix $\mathbf{B}(\mathbf{X}, \hat{\mathbf{X}}) = \mathbf{X} - \hat{\mathbf{X}}$, and λ_k be the eigenvalues of the codeword distance matrix $\mathbf{A} = \mathbf{B}\mathbf{B}^\dagger$. Here we only consider the STTCs with full rank $r = n_T$. Defining d as the determinant of the codeword distance matrix $\mathbf{A}$, we have

[1]This work has been supported by the Australian Research Council (ARC) Grants DP0344856 and DP0558861.

0-7803-9392-9/06/$20.00 (c) 2006 IEEE

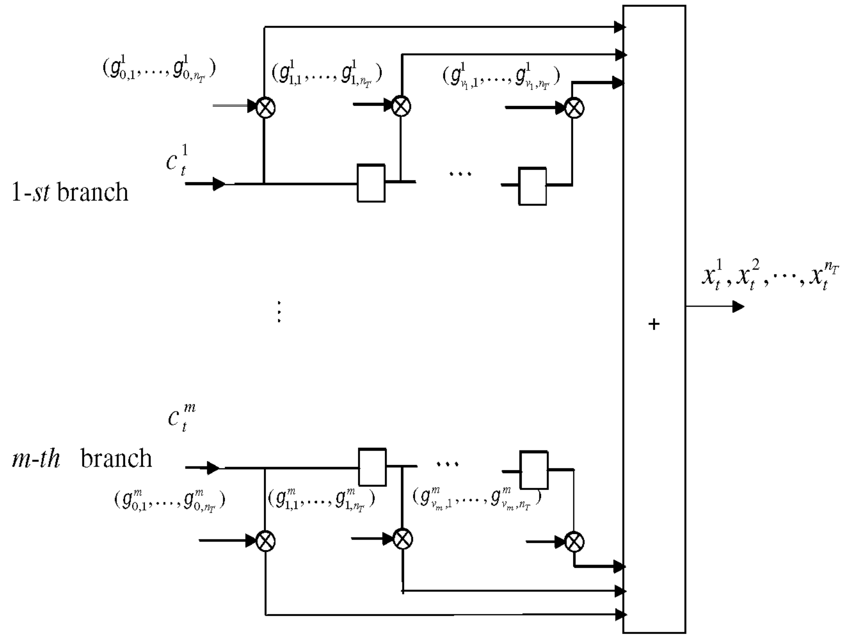

Fig. 1. M-PSK STTC encoder with n_T transmit antennas.

$d = \left(\prod_{k=1}^{r} \lambda_k\right)^{-n_R}$. Let $\mathcal{D}$ be a set of all possible determinants of the codeword distance matrix $\mathbf{A}$, namely product measures. Then the union bound on the FER yields [7]

$$P_{FER} \leq \left(\sum_{d\in\mathcal{D}} \bar{N}(d) d^{-n_R}\right) \left(\frac{E_s}{N_0}\right)^{-rn_R} \tag{2}$$

with

$$\bar{N}(d) = \frac{1}{m^v} \sum_{p=p_{min}}^{p_{max}} \frac{N(d,p)}{(2)^{mp}}, \tag{3}$$

where m is the number of bits per symbol for PSK, p_{min}=$\lfloor v/b+1 \rfloor$ is the minimum length of trellis paths for a simple error event [8], v is the memory order of the encoder, and $b = \log_2 M$ for M-PSK.

In (3), p_{max} is the maximum length of the trellis path that we take into account [5], $N(d,p)$ is the number of error events which have the determinant d over trellis paths with the length p, and $\bar{N}(d)$ is the associated weight [8].

Defining

$$\eta(p_{max}) = \frac{1}{m^v} \sum_{d\in\mathcal{D}} \left\{ \sum_{p=p_{min}}^{p_{max}} \frac{N(d,p)}{(2)^{mp}} \right\} d^{-n_R}, \tag{4}$$

the code design criteria for STTCs are formulated as follows:

1) Diversity gain: The codeword distance matrix $\mathbf{A}$ has to be full rank for maximizing the diversity gain;
2) Coding gain: To optimize the coding gain, the term $\eta(p_{max})$ has to be minimized over all the possible error events in the trellis diagram.

III. M-PSK STTC ENCODER

An M-PSK STTC encoder with memory order v and n_T transmit antennas is shown in Fig. 1 (a). The M-PSK STTC encoder consists of an m-branch shift register with total memory order v. At time t, m binary inputs c_t^i, $i = 1, 2, \ldots, m$, are fed into the m branches. The memory order of the i-th branch, v_i, is given by

$$v_i = \left\lfloor \frac{v+i-1}{b} \right\rfloor, \tag{5}$$

where $\lfloor x \rfloor$ denotes the maximum integer not larger than x.

The m streams of input bits are simultaneously passed through their respective shift register branches and multiplied by the coefficient vectors,

$$\mathbf{g}^1 = [(g_{0,1}^1, g_{0,2}^1, \ldots, g_{0,n_T}^1), \ldots, (g_{v_1,1}^1, g_{v_1,2}^1, \ldots, g_{v_1,n_T}^1)]$$
$$\vdots$$
$$\mathbf{g}^m = [(g_{0,1}^m, g_{0,2}^m, \ldots, g_{0,n_T}^m), \ldots, (g_{v_m,1}^m, g_{v_m,2}^m, \ldots, g_{v_m,n_T}^m)],$$

where $g_{q_i,k}^i \in \{0, 1, \ldots, M-1\}$, $i = 1, 2, ..., m$, $q_i = 0, 1, 2, ..., v_i$, $k = 1, 2, ..., n_T$. The encoder output w_t^k, $t = 1, ..., L$, $k = 1, ..., n_T$, can be computed as

$$w_t^k = \left(\sum_{i=1}^{m} \sum_{q_i=1}^{v_i} \left(g_{q_i,k}^i c_{t-q_i}^k\right)\right) \mod M. \tag{6}$$

These outputs are mapping into x_t^k in an M-PSK constellation. The STTC encoder can also be described in generator polynomial format. The binary input stream $\mathbf{c}^i$ can be represented

TABLE I
4-PSK STTCs

v	code	Generator Coefficients $\mathbf{g}^1,\mathbf{g}^2$	Determinants d_1, d_2, d_3	Weight $\bar{N}(d_i)$, $i=1,2,3$	$\eta(p_{max})$ $n_R=2$	SNR (dB) FER=10^{-4} $n_R=1$	SNR (dB) FER=10^{-4} $n_R=2$
2	[TSC]	[(0,2),(2,0)], [(0,1),(1,0)]	(4, 12, 16)	(2, 4, 1)	1.6e-1	31	18.88
	[YCVF]	[(0,2),(1,0)], [(2,2),(0,1)]	(8, 12, 20)	(3,2.12,3)	--	31	--
		[(0,2),(1,2)], [(2,3),(2,0)]	(4, 8, 12)	(0.25, 2, 0.5)	5.0e-2	--	18.3
	[JL]	[(1,2),(2,2)], [(2,1),(0,2)]	(8, 12, 16)	(1.5, 2.12, 2)	4.2e-2	30.96	18.15
	New	[(0,2),(2,1)], [(2,2),(3,2)]	(8, 12, 16)	(1.5, 2.12, 1)	4.2e-2	30.95	17.96
3	[TSC]	[(0,2),(2,0)], [(0,1),(1,0),(2,2)]	(12, 16, 20)	(2, 1, 1)	2.0e-2	30.2	17.5
	[YCVF]	[(0,2),(2,0)], [(2,1),(1,2),(0,2)]	(16, 20, 28)	(1,5,1)	--	29.91	--
		[(2,2),(2,1)], [(2,0),(1,2),(0,2)]	(8, 12, 16)	(0.5, 1, 1)	1.9e-2	--	17.35
	[LP]	[(0,2),(2,1)], [(2,0),(1,3),(0,2)]	(16, 20, 28)	(1, 3.5, 0.7)	--	29.8	--
		[(0,2),(2,2)], [(2,1),(1,1),(0,2)]	(12, 16, 20)	(0.25, 1, 1)	8e-3	--	16.84
	New	[(1,2),(2,0)], [(2,0),(3,1),(0,2)]	(16, 20, 28)	(1, 0.35, 1)	6e-3	29.69	16.6
4	[TSC]	[(0,2),(2,0),(0,2)], [(0,1),(1,2),(2,0)]	(12, 20, 28)	(1, 1.5, 3.88)	1.6e-2	29	16.75
	[YCVF]	[(0,2),(1,2),(2,2)], [(2,0),(1,1),(0,2)]	(32, 36, 44)	(2.25, 2.35, 3)	--	28.5	--
		[(1,2),(1,3),(3,2)], [(2,0),(2,2),(2,0)]	(8, 16, 24)	(0.25, 0.063, 2.56)	1.3e-2	--	16.6
	New	[(2,2),(2,1),(2,0)], [(0,2),(3,2),(2,2)]	(32, 36, 44)	(2, 2.53, 3.12)	5.5e-3	28.4	15.75
5	[YCVF]	[(2,0),(2,3),(0,2)],[(2,2),(1,0),(1,2),(2,2)]	(36, 40, 44)	(0.25, 1.53, 1.88)	--	28.2	--
		[(0,2),(2,3),(1,2)],[(2,2),(1,2),(2,3),(2,0)]	(20, 24, 28)	(0.25, 0.25, 0.5)	1.7e-3	--	15.85
	New	[(0,2),(2,1),(2,0)], [(2,2),(1,0),(1,2),(2,2)]	(36, 40, 44)	(0.006, 1.25, 1.38)	1.5e-3	28	15.5
6	[YCVF]	[(1,2),(2,2),(0,3)(2,0)],[(2,0),(2,0),(1,3),(0,2)]	(40, 48, 52)	(0.01, 0.3, 0.4)	--	26.8	--
		[(0,2),(3,1),(3,3)(3,2)],[(2,2),(2,2),(0,0),(2,0)]	(32, 40, 44)	(0.4, 0.5, 0.1)	7.5e-4	--	15.5
	New	[(2,2),(0,1),(2,0)(0,2)],[(0,2),(1,0),(1,2),(2,2)]	(48, 52, 56)	(0.3, 0.125, 1.5)	6.5e-4	26.6	15.25

as

$$\mathbf{c}^i(D) = c_0^i + c_1^i D + c_2^i D^2 + \cdots + c_t^i D^t + \cdots , \quad (7)$$

where D represents a unit delay operator. The generator matrix for antenna k can be represented as

$$\mathbf{G}_k(D) = \begin{bmatrix} \mathbf{G}_k^1(D) \\ \mathbf{G}_k^2(D) \\ \vdots \\ \mathbf{G}_k^m(D) \end{bmatrix}, \quad (8)$$

where

$$\mathbf{G}_k^i(D) = g_{0,k}^i + g_{1,k}^i D + \cdots + g_{v_i,k}^i D^{v_i}, \quad (9)$$

is the $i-$th branch generator polynomial for the transmit antenna k. The coded symbol sequence transmitted from antenna k is given by

$$\mathbf{w}^k(D) = \left(\sum_{i=1}^{m} \mathbf{c}^i(D) \mathbf{G}_k^i(D) \right) \mod M. \quad (10)$$

IV. NEW STTCS AND SIMULATION RESULTS

In this Section, we present new 4-PSK and 8-PSK STTCs for two transmit antennas over slow fading channels. The signal-to-noise ratio (SNR) per receive antenna is defined as SNR=$n_T E_s/N_0$. For PSK signal sets, we assume that each frame consists of 130 symbols out of each transmit antenna. This corresponds to 260 information bits. The coding gain of 4-PSK and 8-PSK STTCs are optimized by taking into account the first three distance spectra in the truncated union bound [8]. The coding gain only provides an estimate of performance due to the fact that the length p_{max} considered in the distance spectrum is significantly less than the frame length. Therefore new 4-PSK and 8-PSK STTCs are chosen based on both the coding gain and further simulation results.

In the code design for 4-PSK and 8-PSK signal sets, generator coefficients are determined through exhaustive search. Since the encoder structure cannot guarantee geometrical uniformity of the code, the search was based on all possible pairwise error events. In order to reduce the complexity of the code search, we use the determinants of the known codes in [5] as the bench marks. The complexity of the code construction is the same as that for previously known codes [5, 9].

Tables I and II list the new and known 4-PSK and 8-PSK STTCs with bandwidth efficiency 2 bits/sec/Hz and 3 bits/sec/Hz, respectively. All these codes have full rank of $r = 2$. The codes are described by memory order (v), generator coefficients ($\mathbf{g}^1, \mathbf{g}^2$), the first three minimum determinants ($d_1, d_2, d_3 \in d$), the associated weights ($\bar{N}(d_1), \bar{N}(d_2), \bar{N}(d_3)$), the term ($\eta(p_{max})$) with $p_{max} = 7$. Finally, SNRs at a FER of 10^{-4} with $n_R = 1, 2$ are given. Given $n_T = 2$, some known codes in Tables I and II were specifically designed for very low diversity gain, say $n_T n_R < 4$. We only report the corresponding SNRs. We can see that the proposed codes have the lowest SNRs required to achieve the FER of 10^{-4}, compared to previously known codes.

Figs. 2 and 3 compare the performance of the 4 and 8 state 4-PSK STTCs, respectively, for the system with two receive antennas. We can see that the proposed codes outperform the best previously known codes by 0.19 dB and 0.24dB, at the FER of 10^{-4}, respectively. The code performance is around 4.6 dB and 3.4 dB away from the outage probability for this block length at the FER of 10^{-4}. When the memory order increases from 3 to 4, the proposed 16-state 4-PSK STTC significantly outperforms the best known codes by 0.85 dB at

TABLE II
8-PSK STTCS

v	code	Generator Coefficients $\mathbf{g}^1, \mathbf{g}^2$	Determinants d_1, d_2, d_3	Weight $\bar{N}(d_i)$, $i=1,2,3$	$\eta(p_{max})$ n_R=2	SNR (dB) FER=10^{-4} n_R=1	SNR (dB) FER=10^{-4} n_R=2
3	[TSC]	[(0,4),(4,0)], [(0,2),(2,0)], [(0,1),(5,0)]	(2, 3.37, 4)	(0.5, 0.02, 1)	0.1893	34.45	24.8
	[YCVF]	[(0,2),(2,0)], [(0,4),(4,0)], [(0,5),(1,4)]	(4, 4.34, 6.4)	(1,0.03,0.25)	--	34.1	--
		[(2,1),(3,4)], [(4,6),(2,0)], [(0,4),(4,0)]	(2,3.37,4)	(0.5,0.00165,1)	0.1876	--	20.8
	New	[(3,4),(4,1)], [(4,0),(0,4)], [(2,0),(0,6)]	(4, 4.34, 6.7)	(1,0.03,0.5)	0.07	34.0	20.45
4	[TSC]	[(0,4),(4,4)], [(0,2),(2,2)], [(0,1),(5,1),(1,5)]	(3.5,4,4.34)	(0.0156, 1, 0.0625)	0.8171	33.95	20.4
	[YCVF]	[(0,4),(4,0)], [(0,2),(2,0)], [(2,0),(6,5),(1,4)]	(4, 4.34, 5.4)	(1,0.006,0.004)	--	33.9	--
		[(2,4),(3,7)], [(4,0),(6,6)], [(7,2),(0,7),(4,4)]	(0.86, 0.9, 1.1)	(0.002, 0.008, 0.0007)	0.0132	--	20.6
	New	[(0,6),(6,0)], [(7,4),(4,0)], [(4,5),(2,1),(2,4)]	(4, 4.34, 5.86)	(0.5, 0.0009, 0.001)	0.0032	33.5	20.0
5	[TSC]	[(0,4),(4,4)],[(0,2),(2,2),(2,2)], [(0,1),(5,1),(3,7)]	(3.51,4,4.34)	(0.008, 0.25, 0.03)	0.0179	33.7	19.75
	[YCVF]	[(0,4),(4,4)],[(0,2),(2,2),(0,0)], [(3,5),(2,0),(4,0)]	(7.029,7.5,8.2)	(0.05, 0.06255, 0.094)	--	33.6	--
		[(0,4),(4,4)],[(0,2),(2,3),(2,2)], [(3,0),(2,2),(3,7)]	(2.69,3.86,4.34)	(0.25,0.0005,0.002)	0.0347	--	19.5
	New	[(4,0),(2,6)],[(7,5),(6,6),(2,2)], [(7,3),(4,6),(4,4)]	(7.5,8.0,10.8)	(0.0625, 0.0156,0.094)	0.0022	33.5	19.4

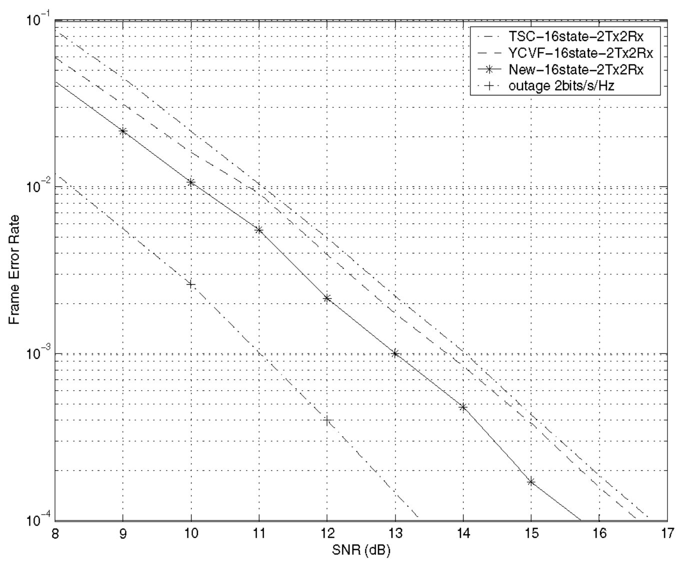

Fig. 2. Performance of 16-state 4-PSK STTCs, 2Tx 2Rx

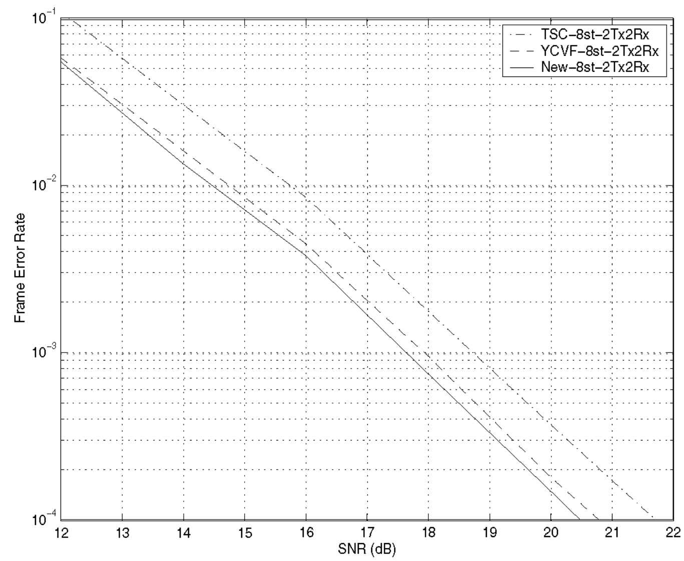

Fig. 3. Performance of 8-state 8-PSK STTCs, 2Tx 2Rx

the FER of 10^{-4}, which is shown in Fig. 4. The performance of the proposed 16-state code is around 2.4 dB away from the outage probability.

Figs. 5 and 6 compare the performance of 8 and 16 state 8-PSK STTCs with two receive antennas over slow fading channels. It is shown that the new codes outperform the best previously known codes by 0.35 dB and 0.4 dB at the FER of 10^{-4}.

Fig. 7 plots the performance of 16 state 8-PSK STTCs with one receive antenna only. It is shown that the new code outperforms the best previously known codes by 0.4 dB at the FER of 10^{-4}.

V. CONCLUSIONS

Three complete sets of 4-PSK and 8-PSK STTCs for two transmit antennas over slow fading channels are proposed. In order to minimize the frame error probability, the new codes are constructed by 1) guaranteeing the codeword distance matrix to be full rank, and 2) minimizing the term $(\eta(p_{max}))$ which dominated the truncated union bound. New codes are found based on exhaustive search. Through simulations, it is shown that the proposed codes outperform previously known codes for systems with low and moderate diversity gains.

REFERENCES

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. on Inform. Theory*, March 1998.

[2] S. Baro, G. Bauch, and A. Hansmann, "Improved codes for space-time trellis coded modulation," *IEEE Commun. Lett.*, vol. 4, pp. 20-22, Jan. 2000.

[3] R. S. Blum, "New analytical tools for designing space-time convolutional codes," *Conf. Inform. Sci. Syst*, Princeton, NJ, 2000.

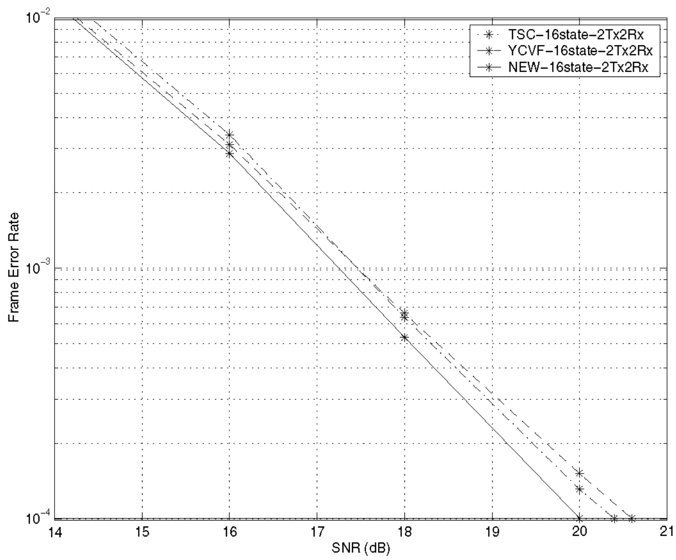

Fig. 4. Performance of 16-state 8-PSK STTCs, 2Tx 2Rx

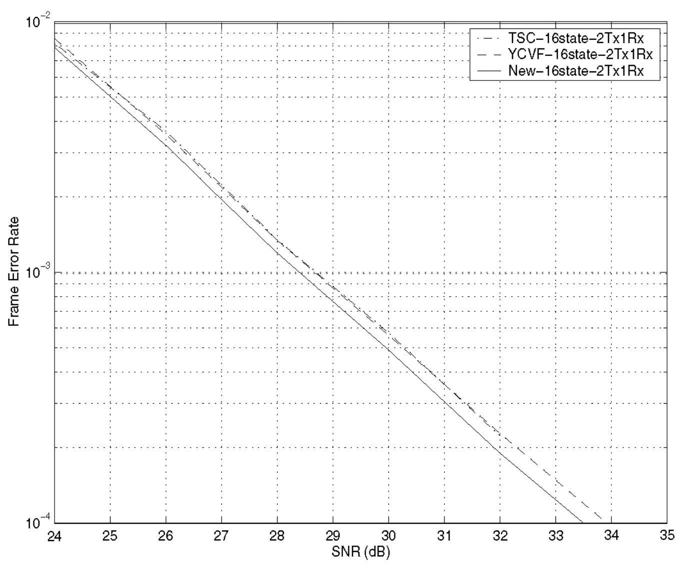

Fig. 5. Performance of 16-state 8-PSK STTCs, 2Tx 1Rx

[4] D. M. Ionescu, K. K. Mukkavilli and Z. Yuan, "Improved 8- and 16-state space-time codes for 4-PSK with two transmit antennas," *IEEE Commun. Lett.*, vol. 5, pp. 301-303, Mar. 2003.

[5] J. Yuan, Z. Chen, B. Vucetic, and W. Firmanto, "Performance and design of space-time coding in fading channels," *IEEE Trans. on Commun.*, vol. 51, no. 12, pp. 1991-1996, Dec. 2003.

[6] Y. Hong, J. Yuan, Z. Chen, and B. Vucetic, "Space-time Turbo Trellis codes for two, three and four transmit antennas," *IEEE Trans on Veh. Technol.*, vol. 53, no. 2, pp. 318-328, Mar. 2004.

[7] Young Seok Jung and Jae Hong Lee, "New measure of coding gain for space-time trellis codes," *IEEE International Symposium on Information Theory*, p. 198, Washington DC, July 2001.

[8] D. Aktas and M. P. Fitz, "The distance spectrum of Space-time trellis coded modulations in Quasi-static Rayleigh Fading channels," *IEEE Trans. on Inform. Theory*, vol. 49, no. 12, 2003, pp. 3335-3344.

[9] Chen Liao and V. K. Prabhu, "Improved code design criteria for space-time trellis codes over quasi-static flat fading channels," *IEEE International Workshop on Signal Processing Advances in Wireless Communications*, New York, USA, June 2005.