Nearest Neighbour Decoding in Block-Fading Channels with Imperfect CSIR

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***Abstract*—This paper studies the outage performance of block-fading channels with imperfect channel state information at the receiver (CSIR). Using mismatched decoding error exponents, we prove the achievability of the generalised outage probability, the probability that the generalised mutual information is less than the rate. Then, using nearest neighbour decoding, we study the generalised outage probability in the limit of large signal-to-noise ratio (SNR). In particular, we study the SNR exponents and we show that the SNR exponent is given by the SNR exponent of the perfect CSIR case times the minimum of one and the channel estimation error SNR exponent.**

I. Introduction

Reliable communication over wireless channels is particularly challenging due to fading, i.e., the fluctuations in the received signal strength due to mobility and multiple path propagation [1]. The severity of fading depends on several factors such as the geography and the topography of scattering environment, mobile velocity, carrier frequency and transmitted signal bandwidth.

The design of efficient communication strategies largely depends on the nature of the signal and the targeted application. In particular, for applications where large delays are tolerable, the channel is considered to be *ergodic* and long interleaved codes of rate not exceeding the channel capacity can be used [2], [3]. On the other hand, for applications with stringent delay constraints long interleavers cannot be assumed, and the channel is considered *non-ergodic*. The block-fading channel [2]–[4] is a useful channel model for such slowly-varying scenarios, where a codeword spans only a finite number of degrees of freedom, or channel blocks. Frequency hopping schemes (e.g. Global System for Mobile Communications (GSM) and the Enhanced Data GSM Environment (EDGE)) and Orthogonal-Frequency Division Multiplexing (OFDM) can be accurately modelled as block-fading channels.

Reliable communication over block-fading channels has traditionally been studied under the assumption of perfect channel state information at the receiver (CSIR) [2], [4]–[8]. The block-fading channel is not information stable [9] and therefore it has zero channel capacity. Based on error exponent considerations, Malkämaki and Leib [6] showed that the outage probability, i.e., the probability that the mutual information is smaller than the target transmission rate [2], [4], is the natural fundamental limit of the channel. References [5]–[7] showed that optimal codes for the block-fading channel should be maximum-distance separable (MDS) on a blockwise basis, i.e., achieving the Singleton bound on the blockwise Hamming distance of the code with equality. They also proposed families of blockwise MDS codes based on convolutional and Reed-Solomon codes. Following the footsteps of [10], reference [7] proved that the outage probability as a function of the signal-to-noise ratio (SNR) decays linearly in a log-log scale with a slope given by the so called outage diversity or SNR exponent. This SNR exponent is precisely given by the Singleton bound [7] when codes constructed over discrete signal constellations are used; instead, Gaussian codebooks achieve the maximum diversity offered by the channel.

The perfect CSIR assumption implies that the channel estimator provides the receiver with a perfectly accurate channel estimate used for decoding. In this case, nearest neighbour decoding, which is used in most common wireless decoders, is optimal for minimising the word error probability under the assumption that all codewords are equally likely (i.e. a maximum-likelihood rule) [11]. In practice, however, most channel estimators incur channel estimation errors, making nearest neighbour decoding inherently suboptimal as there exists decoder with better error minimisation [12]. The natural question arising from the above discussion is the impact of this suboptimality on performance as compared to the perfect CSIR condition. In this paper, we study nearest neighbour decoding in block-fading channels with imperfect CSIR. We follow a mismatched decoding approach [13]–[16] to this problem and study the error probability of the random coding ensemble with error exponents [14], [17]. In particular, by using similar arguments to those used in [6], we prove the achievability of the *generalised outage probability*, i.e., the probability that the generalised mutual information (GMI) [13]–[16] is less than the target transmission rate. We furthermore study the SNR exponents or generalised outage diversity and find that the generalised outage diversity with imperfect CSIR is given by the simple formula

$$d_{\text{icsir}} = \min(1, d_e) \times d_{\text{csir}} \qquad (1)$$

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where d_{csir} is the perfect CSIR outage exponent and d_e is the channel estimation error diversity. Interestingly, this relationship holds for both Gaussian-input and discrete-input codebooks. Moreover, this result validates the use of perfect-CSIR code designs and provides guidelines for designing channel estimators that maximise the achievable diversity.

The rest of the paper is organised as follows. Section II introduces the channel, imperfect CSIR and fading models. Section III reviews basic material on error exponents for the block-fading channel. Section IV shows the achievability of the generalised outage probability using mismatched decoding arguments. Section V presents our large-SNR analysis of the generalised outage probability and gives a sketch of the proof of (1). Section VI discusses the main findings, shows numerical evidence and provides important remarks. Finally, Section VII summarises the important points of the paper.

II. System Model

Consider transmission over a block-fading channel with B blocks corrupted by additive white Gaussian noise (AWGN) and affected by an i.i.d. flat-fading coefficient, $h_b \in \mathbb{C}, b = 1, \dots, B$. The input-output relationship of the channel is

$$\boldsymbol{y}_b = h_b \sqrt{\mathsf{snr}}\, \boldsymbol{x}_b + \boldsymbol{z}_b, \quad b = 1, \dots, B \tag{2}$$

where $\boldsymbol{y}_b, \boldsymbol{x}_b, \boldsymbol{z}_b \in \mathbb{C}^L$ are the received, transmitted and noise signal vectors corresponding to block b, and L denotes the block length. We assume that the entries of $\boldsymbol{z}_b$ are i.i.d. zero-mean unit-variance complex circularly symmetric Gaussians.

We consider coded modulation schemes $\mathcal{M} \subset \mathbb{C}^N$ of rate R and length $N = BL$, whose codewords are defined as $\boldsymbol{x} = \left[\boldsymbol{x}_1^T, \cdots, \boldsymbol{x}_B{}^T\right]^T \in \mathcal{X}^{BL}$, where $\mathcal{X}$ denotes the signal constellation. The codewords are assumed to satisfy the average input power constraint $\frac{1}{BL}\mathbb{E}[|\boldsymbol{X}|^2] \le 1$.

At the receiver side, in order to minimise the word error probability, the decoder requires perfect channel knowledge. Practical systems employ channel estimators that yield accurate yet imperfect channel estimates. We model the channel estimate as

$$\hat{h}_b = h_b + e_b, \quad b = 1, \dots, B \tag{3}$$

where $\hat{h}_b \in \mathbb{C}$ and $e_b \in \mathbb{C}$ are the noisy channel estimate and the channel estimation error, respectively. In particular, e_b is assumed to have an i.i.d. Gaussian distribution with zero mean and variance σ_e^2. We further assume that $\sigma_e^2 = \mathsf{snr}^{-d_e}$ with $d_e > 0$, i.e. the CSIR noise variance is decreasing with SNR. This model is widely used in pilot-based channel estimation employing maximum-likelihood (ML) or minimum mean-squared error (MMSE) estimators for which the error variance is proportional to the reciprocal of pilot SNR [18].

We consider a nearest neighbour decoder, which uses the imperfect channel estimate as if it were perfect, i.e., it calculates the following metric

$$Q_{Y|X,\hat{H}}(y|x, \hat{h}_b) = \frac{1}{\pi} e^{-|y - \hat{h}_b \sqrt{\mathsf{snr}} x|^2}. \tag{4}$$

Let $\gamma_b \triangleq |h_b|^2, \hat{\gamma}_b \triangleq |\hat{h}_b|^2, \xi_b \triangleq |e_b|^2$ be the channel, channel estimate and channel estimation power gains corresponding to block $b = 1, \dots, B$. Following the footsteps of [10], we define the corresponding SNR normalised gains $\alpha_b = -\frac{\log \gamma_b}{\log \mathsf{snr}}$, $\hat{\alpha}_b = -\frac{\log \hat{\gamma}_b}{\log \mathsf{snr}}$, $\theta_b = -\frac{\log \xi_b}{\log \mathsf{snr}}$. We then define the vectors $\boldsymbol{h} = [h_1, \dots, h_B]^T, \hat{\boldsymbol{h}} = [\hat{h}_1, \dots, \hat{h}_B]^T$, $\boldsymbol{\gamma} = [\gamma_1, \dots, \gamma_B]^T, \hat{\boldsymbol{\gamma}} = [\hat{\gamma}_1, \dots, \hat{\gamma}_B]^T, \boldsymbol{\xi} = [\xi_1, \dots, \xi_B]^T$ and $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_B]^T, \hat{\boldsymbol{\alpha}} = [\hat{\alpha}_1, \dots, \hat{\alpha}_B]^T, \boldsymbol{\theta} = [\theta_1, \dots, \theta_B]^T$.

We consider a general family of fading distributions for which the p.d.f. of the normalised fading gain α_b is given by

$$p_\alpha(\alpha_b) \doteq \exp\left(-\varsigma\, \alpha_b \log \mathsf{snr}\right), \quad \text{for } \alpha_b \ge 0 \tag{5}$$

for large SNR, where $\doteq$ is the exponential equality [10] and ς is the channel related parameter which characterises the diversity associated to a single fading coefficient, i.e., the large-SNR slope of the error probability. This generalisation subsumes a number of important fading distributions such as Rayleigh ($\varsigma = 1$), Rician with parameter K $\left(\varsigma \approx \frac{(K+1)^2}{2K+1}\right)$ and Nakagami-m ($\varsigma = m$) [8] for RF wireless, and lognormal-Rice, I-K ($\varsigma = \frac{1}{2}$) or gamma-gamma ($\varsigma = \frac{1}{2}\min(a, b)$ where a, b are the parameters of the individual gamma distributions) for optical wireless [19].

III. Preliminaries

It has been shown in [6] that the error probability for the ensemble of random codes of rate $R - \epsilon$ and a fixed channel realisation $\boldsymbol{h}$, input distribution $P_X(x)$ and perfect CSIR is given by

$$\bar{P}_e(\boldsymbol{h}) \le 2^{-NE_r(R-\epsilon, \boldsymbol{h})} \tag{6}$$

where

$$E_r(R - \epsilon, \boldsymbol{h}) = \sup_{0 \le \rho \le 1} \frac{1}{B} \sum_{b=1}^{B} E_0(\rho, h_b) - \rho(R - \epsilon) \tag{7}$$

is the error exponent for channel realisation $\boldsymbol{h}$ and

$$E_0(\rho, h_b) = -\log_2 \mathbb{E}\left[\left(\mathbb{E}\left[\frac{P_{Y|X,H}(Y|X', h_b)}{P_{Y|X,H}(Y|X, h_b)}\middle| X, Y\right]^{\frac{1}{1+\rho}}\right)^{\rho}\right] \tag{8}$$

is the Gallager function for a given fading realisation h_b [11]. Note that the inner expectation is taken over X', while the outer expectation is taken over X, Y for a fixed $H = h_b$. Then, the average error probability for the ensemble of random codes is

$$\bar{P}_e \le \inf_{\epsilon > 0} \mathbb{E}\left[2^{-NE_r(R-\epsilon, \boldsymbol{H})}\right]. \tag{9}$$

Basic error exponent results show that $E_r(R - \epsilon, \boldsymbol{h})$ is positive only when $R - \epsilon < I(\boldsymbol{h})$, where $I(\boldsymbol{h})$ is the input-output mutual information, and zero otherwise. The instantaneous mutual information for block-fading channels is easily expressed as

$$I(\boldsymbol{h}) = \frac{1}{B} \sum_{b=1}^{B} I^{\text{awgn}}(\mathsf{snr}\gamma_b) \tag{10}$$

where $I^{\text{awgn}}(\eta)$ is the mutual information of an AWGN channel with SNR η. Then, for large N we obtain that

$$\bar{P}_e \leq \inf_{\epsilon>0} \Pr\{I(\boldsymbol{h}) \leq R-\epsilon\} = \Pr\{I(\boldsymbol{h}) < R\} \triangleq P_{\text{out}}(R) \tag{11}$$

which is the information outage probability [4]. This result shows the achievability of the outage probability [6]. Furthermore, using Arimoto's converse [20] it is possible to show that the outage probability is the lowest possible error probability of any coding scheme [6]. The above results introduce the outage probability as the natural fundamental limit for block-fading channels.

IV. Generalised Outage Probability

We now turn our attention to the mismatched decoding case, i.e., when the decoder has only available the channel estimate $\hat{\boldsymbol{h}}$. In this situation, the decoder treats the channel estimate $\hat{\boldsymbol{h}}$ as if it was the true channel. By following the same steps outlined in Section III, we can upperbound the error probability of the ensemble of random codes as [14], [17]

$$\bar{P}_e(\hat{\boldsymbol{h}}) \leq 2^{-NE_r^Q(R-\epsilon,\hat{\boldsymbol{h}})} \tag{12}$$

where now the mismatched decoding error exponent is

$$E_r^Q(R-\epsilon,\hat{\boldsymbol{h}}) = \sup_{\substack{s\geq 0\\ 0\leq\rho\leq 1}} \frac{1}{B}\sum_{b=1}^{B} E_0^Q(s,\rho,\hat{h}_b) - \rho(R-\epsilon) \tag{13}$$

and

$$E_0^Q(s,\rho,\hat{h}_b) = -\log_2 \mathbb{E}\left[\left(\mathbb{E}\left[\frac{Q_{Y|X,\hat{H}}(Y|X',\hat{h}_b)}{Q_{Y|X,\hat{H}}(Y|X,\hat{h}_b)}\,\middle|\, X,Y\right]^s\right)^\rho\right] \tag{14}$$

is the generalised Gallager function for a given fading realisation h_b and its estimate $\hat{h}_b$ [14] (a full derivation is included in [17]). Using Hölder's inequality it can be easily verified that $E_0^Q(s,\rho,h_b)$ is a concave function of ρ for $0\leq\rho\leq 1$. The maximum slope of $E_0^Q(s,\rho,h_b)$ with respect to ρ occurs at $\rho=0$. The maximisation over s results in a maximum slope equal to the GMI [13], [14]

$$I^{\text{gmi}}(\hat{\boldsymbol{h}}) = \frac{1}{B}\sum_{b=1}^{B} I_b^{\text{gmi}}(\hat{h}_b) \tag{15}$$

where

$$I_b^{\text{gmi}}(\hat{h}_b) = \sup_{s>0} \mathbb{E}\left[\log_2 \frac{Q^s_{Y|X,\hat{H}}(Y|X,\hat{h}_b)}{\mathbb{E}\left[Q^s_{Y|X,\hat{H}}(Y|X',\hat{h}_b)\,\middle|\, Y\right]}\right] \tag{16}$$

The above analysis shows that the exponent $E_r^Q(R,\hat{\boldsymbol{h}})$ is only positive whenever $R-\epsilon < I^{\text{gmi}}(\hat{\boldsymbol{h}})$, and zero otherwise, proving the achievability of $I^{\text{gmi}}(\hat{\boldsymbol{h}})$. Then, the average error probability over the ensemble of random codes is then

$$\bar{P}_e \leq \inf_{\epsilon>0} \mathbb{E}\left[2^{-NE_r^Q(R-\epsilon,\hat{\boldsymbol{H}})}\right], \tag{17}$$

which, for large N becomes

$$\begin{aligned}\bar{P}_e &\leq \inf_{\epsilon>0} \Pr\{I^{\text{gmi}}(\hat{\boldsymbol{h}}) \leq R-\epsilon\}\\ &= \Pr\{I^{\text{gmi}}(\hat{\boldsymbol{h}}) < R\} \triangleq P_{\text{g-out}}(R),\end{aligned} \tag{18}$$

the generalised outage probability. The above analysis shows the achievability of $P_{\text{g-out}}(R)$. Unfortunately, there are no generally tight converse results for mismatched decoding [13] which implies that one might be able to find codes whose error probability for large N might be lower than $P_{\text{g-out}}(R)$. However, as shown in [15], [21], there is a converse for the GMI with i.i.d. codebooks, i.e., the no rate larger than the GMI can be transmitted with vanishing error probability for i.i.d. codebooks. Hence, the generalised outage probability is the fundamental limit for i.i.d. codebooks. Furthermore, due to the data-processing inequality for error exponents, i.e., $E_r^Q(R-\epsilon,\hat{\boldsymbol{h}}) \leq E_r(R-\epsilon,\boldsymbol{h})$ [14], [17], we obtain that $I^{\text{gmi}}(\hat{\boldsymbol{h}}) \leq I(\boldsymbol{h})$ [13], and hence $P_{\text{g-out}}(R) \geq P_{\text{out}}(R)$.

V. SNR Exponents

We now characterise the behaviour of the generalised outage probability. In particular, we study the generalised outage diversity, defined as the asymptotic slope of the generalised outage probability in curve on a log-log scale for large SNR. This definition can be viewed as the extension of the SNR exponent definition for perfect CSIR case [10]. Define the perfect and imperfect CSIR SNR exponents as

$$d_{\text{csir}} \triangleq \lim_{\mathsf{snr}\to\infty} -\frac{\log P_{\text{out}}(R)}{\log \mathsf{snr}}, \tag{19}$$

$$d_{\text{icsir}} \triangleq \lim_{\mathsf{snr}\to\infty} -\frac{\log P_{\text{g-out}}(R)}{\log \mathsf{snr}}. \tag{20}$$

From the relationship between mutual information and GMI outlined above we easily obtain that $d_{\text{icsir}} \leq d_{\text{csir}}$. We have the following result.

Theorem 1: Consider the channel, imperfect CSIR and fading models described by (2), (3) and (5), respectively. The generalised outage diversity using nearest neighour decoding (4) with imperfect CSIR is given by

$$d_{\text{icsir}} = \min(1, d_e) \times d_{\text{csir}} \tag{21}$$

where

$$d_{\text{csir}} = \begin{cases} \varsigma B & \text{for Gaussian inputs}\\ \varsigma\, d_B(R) & \text{for discrete inputs,}\end{cases} \tag{22}$$

is the perfect CSIR outage diversity and

$$d_B(R) = 1 + \left\lfloor B\left(1 - \frac{R}{\log_2|\mathcal{X}|}\right)\right\rfloor \tag{23}$$

is the Singleton bound, achieved by random codes.

Proof: We here give a sketch of the proof of the above result. We follow standard steps to derive the outage diversity. Firstly, the generalised outage probability is derived by taking into consideration the fading distribution and the channel estimation error. Secondly, the asymptotic behaviour in the limit of large SNR of that outage is characterised and the set of SNR normalised fading values that yield a generalised outage is derived. Thirdly, large deviation analysis is used to derive the generalised outage diversity.

Let $\mathcal{O}$ be the large SNR generalised outage set. Since $\boldsymbol{\gamma}$ and $\boldsymbol{\xi}$ are independent, the generalised outage probability can be obtained by integrating over the outage set the product p.d.f.s of $\boldsymbol{\gamma}$ and $\boldsymbol{\xi}$ or equivalently $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$.

$$P_{\text{g-out}}(R) = \int_{\mathcal{O}} p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) p_{\boldsymbol{\theta}}(\boldsymbol{\theta}) d\boldsymbol{\alpha} d\boldsymbol{\theta}. \tag{24}$$

The asymptotic behaviour of the distribution of α is given in (5). The channel estimation error power ξ follows an exponential distribution with the parameter snr^{d_e}. After changing the variable to θ, the asymptotic p.d.f of θ is given by

$$p_\theta(\theta_b) \doteq \mathsf{snr}^{d_e - \theta_b} \quad \text{for } \theta \geq d_e. \tag{25}$$

Taking this into account, the generalised outage probability can be shown to be

$$P_{\text{g-out}}(R) \doteq \int_{\mathcal{O} \cap \{\boldsymbol{\alpha} \succeq 0, \boldsymbol{\theta} \succeq d_e\}} \overbrace{\mathsf{snr}^{-\varsigma \sum_{b=1}^{B} \alpha_b}}^{\text{fading}} \overbrace{\mathsf{snr}^{\sum_{b=1}^{B}(d_e - \theta_b)}}^{\text{estimation error}} d\boldsymbol{\alpha} d\boldsymbol{\theta} \tag{26}$$

where $\succeq$ denotes componentwise inequality. Note the separate effect that the fading distribution and the channel estimation error have on the exponent. Then, by inserting (26) to (20) and applying Varadhan's lemma [22] we obtain

$$d_{\text{icsir}} = \inf_{\mathcal{O} \cap \{\boldsymbol{\alpha} \succeq 0, \boldsymbol{\theta} \succeq d_e\}} \left\{ \varsigma \sum_{b=1}^{B} \alpha_b + \sum_{b=1}^{B} (\theta_b - d_e) \right\} \tag{27}$$

The large-SNR generalised outage set denoted by $\mathcal{O}$ depends on the required data rate R and the input probability distribution $P_X(x)$. To determine $\mathcal{O}$, the GMI is evaluated for large SNR. It seems that from (27), the SNR exponent d_{icsir} does not depend on the phases of $\boldsymbol{h}$ and $\hat{\boldsymbol{h}}$. However, it can be shown that these phases affect the generalised outage set $\mathcal{O}$. These details are omitted because of space limitation.

■

VI. Discussion

The model used for deriving the generalised outage diversity highlights the essential aspects of the wireless communications system design. The results as outlined in Theorem 1 show diversity order achieved by nearest neighbour decoding in the absence of perfect CSIR. The following remarks are in order:

1) The results show the role of channel estimation error SNR exponent d_e for determining the generalised outage exponent. By having channel estimators with $d_e \geq 1$, we are essentially able to achieve the same SNR exponent of the perfect CSIR case for the same setup. If $d_e < 1$, the SNR exponent achieved scales linearly with d_e and approaching zero for $d_e \downarrow 0$. Fig. 1 illustrates this effect in a discrete-input block-fading channel with $B = 4$ and $\varsigma = 1$ (Rayleigh fading).

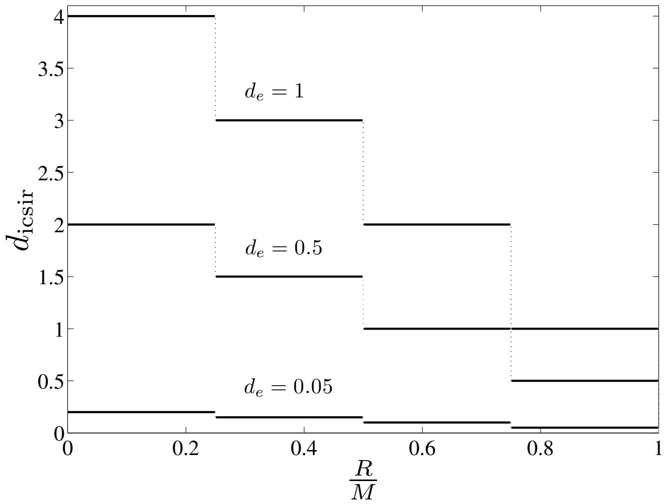

Fig. 1. Generalised outage SNR exponent for discrete-input block-fading channel, $B = 4$, $\varsigma = 1$ (Rayleigh fading).

2) The term $\min(1, d_e)$ comes out naturally from the relationship between the mutual information and the GMI. It highlights the importance of having channel estimators that can achieve error diversity $d_e \geq 1$. This ensures that the limiting term in $P_{\text{g-out}}(R)$, i.e. $\boldsymbol{\theta} \succeq d_e \geq 1$, does not decay exponentially to zero. In the limit of large SNR, the error level is most likely to be much less than the reciprocal of the SNR level. This is consistent with the results of [16].
3) The role of error diversity d_e is governed by the channel estimation model. With ML estimator, it can be shown that d_e is proportional to the pilot power [18]. Larger pilot power implies larger d_e. Hence, the price for obtaining high diversity is in the power of pilot which does not contain any information data. Nevertheless, with bounding condition $\min(1, d_e)$ we may design the pilot power so that $d_e = 1$ to obtain the highest possible diversity.
4) The outage diversity proven by Theorem 1 is valid for general fading models described by (5).

Figs. 2 and 3 illustrate the generalised outage probability for Gaussian and binary phase-shift keying (BPSK) inputs, respectively, over a block-fading channel. The following parameters are specified: $B = 4$, Rayleigh fading and $R = 2$ bits/channel use for Gaussian input and $B = 2$, Rayleigh fading and $R = 0.5$ bits/channel use for BPSK input. Various values of the channel estimation error diversity d_e are used for comparison with the perfect CSIR outage probability. As predicted by Theorem 1, the slope becomes steeper with increasing d_e, eventually becoming parallel to the perfect CSIR outage curve for $d_e \geq 1$.

VII. Conclusions

We have studied the generalised outage probability for nearest neighbour decoding with imperfect CSIR. In particular,

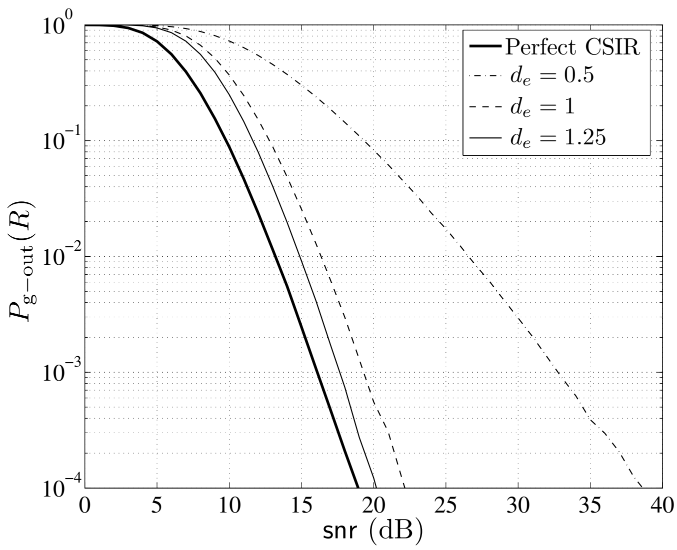

Fig. 2. Generalised outage probability $P_{\text{g-out}}(R)$ as a function of snr for Gaussian-input block-fading channel, $B = 4$, $\varsigma = 1$ (Rayleigh fading) and $R = 2$ bits/channel use.

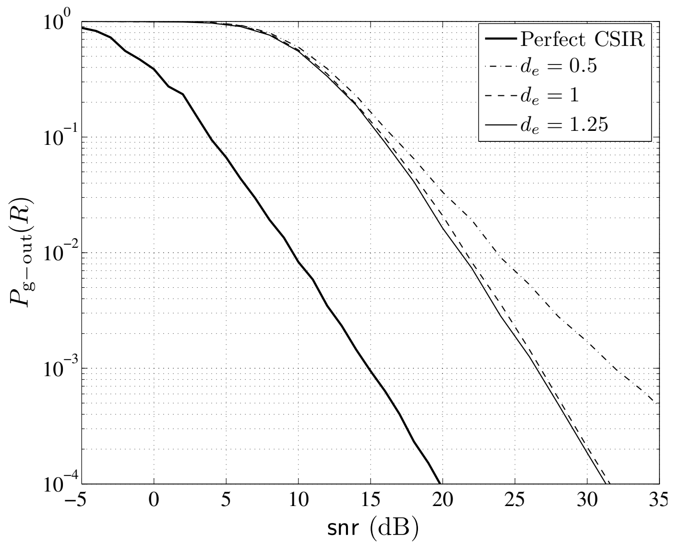

Fig. 3. Generalised outage probability $P_{\text{g-out}}(R)$ as a function of snr for BPSK-input block-fading channel, $B = 2$, $\varsigma = 1$ (Rayleigh fading) and $R = 0.5$ bits/channel use.

following the footsteps of [6], we have proved the achievability of the generalised outage probability using error exponents for mismatched decoding. Due to the data-processing inequality for error exponents and mismatched decoders, the generalised outage probability is larger than the outage probability of the perfect CSIR case. We have further analysed the generalised outage probability in the large-SNR regime and we have derived the SNR exponents for both Gaussian and discrete inputs. We have shown that for both inputs, the SNR exponent is given by the perfect CSIR SNR exponent scaled by the minimum of channel estimation error diversity and one. Therefore, in order to achieve the highest possible SNR exponent, the channel estimator scheme should be designed in such a way so as to make the estimation error diversity equal to or larger than one.

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