Multidimensional Coded Modulation in Block-Fading Channels

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***Abstract*— We study the problem of constructing coded modulation schemes over multidimensional signal sets in Nakagami-m block-fading channels. In particular, we consider the optimal diversity reliability exponent of the error probability when the multidimensional constellation is obtained as the rotation of classical complex-plane signal constellations. We show that multidimensional rotations of full dimension achieve the optimal diversity reliability exponent, also achieved by Gaussian constellations. Multidimensional rotations of full dimension induce a large decoding complexity, and in some cases it might be beneficial to use multiple rotations of smaller dimension. We also study the diversity reliability exponent in this case, which yields the optimal rate-diversity-complexity tradeoff in block-fading channels with discrete inputs.**

I. Introduction

Rotated multidimensional constellations in fading channels were proposed in [1], [2] as a means of achieving high reliability with *uncoded* modulation in fading channels. Since, rotated constellations have been extensively studied, and have been shown to be an effective technique to achieve full-rate and full-diversity transmission in fading channels [3], [4], [5], [6]. Traditionally, rotated constellations have always been studied uncoded, with the exception of some recent works for the multiple-input multiple-output (MIMO) channel [7], [8].

In this work, we study the problem of constructing general coded modulation schemes over multidimensional signal sets, obtained by rotating classical complex-plane signal constellations, for block-fading channels with B fading blocks (or degrees of freedom) per codeword [9]. The block-fading channel is a useful model for transmission over slowly varying fading channels, such as orthogonal frequency division multiplexing (OFDM) or slow time-frequency-hopped systems such as GSM or EDGE.

Despite the elegance of full-diversity rotations of dimension B, they induce large decoding complexity since the set of candidate points for detection at a given time instant is exponential with B. In fact, when uncoded rotations are used, the sphere decoder [10] is usually employed to avoid exhaustive search over all candidate points. However, when coded modulation is used, the code itself can help to achieve full diversity. This means that sometimes rotations of smaller dimension $N < B$ might be sufficient. Also in the coded case, soft information should be provided to the decoder and this further complicates the problem. As a matter of fact, despite the recent advances in soft-output sphere decoding techniques [11], most of the proposed techniques still show some limitations in some cases, which might be undesirable in practice. Therefore, in practice, one might want to use rotations of dimension smaller than $N < B$, in order to establish the tradeoff between diversity, rate, constellation size and *complexity* induced by the rotations.

In this paper, we study the *reliability exponent*, namely, the optimal exponent of the error probability of such schemes with the signal-to-noise ratio (SNR), and illustrate the rate-diversity-complexity tradeoff for coded modulation schemes constructed over multidimensional signal sets.

II. System Model

We consider a single-input single-output block-fading channel with B fading blocks, and is defined as follows,

$$\mathbf{y}_b = \sqrt{\mathsf{SNR}}\, h_b\, \mathbf{x}_b + \mathbf{z}_b \qquad b = 1, \ldots, B \tag{1}$$

where $h_b \in \mathbb{C}$ is the b-th fading coefficient, $\mathbf{y}_b \in \mathbb{C}^L$ is the received signal vector corresponding to fading coefficient b, $\mathbf{x}_b \in \mathbb{C}^L$ is the portion of codeword allocated to block b and $\mathbf{z}_b \in \mathbb{C}^L$ is the vector of i.i.d. noise samples $\sim \mathcal{N}_{\mathbb{C}}(0,1)$. We assume that the transmitted signal is normalized in energy, i.e., $\mathbb{E}[|x|^2] = 1$. Hence, SNR is the average received SNR.

We assume that the fading coefficients are i.i.d. from block to block and from codeword to codeword, and that they are perfectly known at the receiver, i.e, perfect channel state information (CSI). Since the channel coefficients are perfectly known to the receiver, we assume that the phase of the fading has been corrected. We also assume that the magnitudes of the channel coefficients follow a Nakagami-m distribution

$$p_{|h|}(\xi) = \frac{2m^m \xi^{2m-1}}{\Gamma(m)} e^{-m\xi^2}$$

for $m > 0$ [2] where $\Gamma(\xi) \triangleq \int_0^{+\infty} t^{\xi-1} e^{-t} dt$ is the Gamma function [13]. By analizing Nakagami-m fading, we can recover the analysis for a large class of fading statistics, including Rayleigh fading by setting $m = 1$ and Rician fading with parameter K by setting $m = (K+1)^2/(2K+1)$ [14].

[1]The work by A. Guillén i Fàbregas work has been partly supported by the Australian Research Council under ARC grant DP0558861.

[2]The literature usually considers $m \geq 0.5$ [12]. However, the distribution is well defined and reliable communication is possible for $0 < m < 0.5$.

We can express (1) in matrix form as

$$\mathbf{Y} = \sqrt{\mathsf{SNR}}\,\mathbf{H}\,\mathbf{X} + \mathbf{Z} \tag{2}$$

where $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_B]^T$, $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_B]^T = [\mathbf{X}_1, \ldots, \mathbf{X}_L]$, $\mathbf{Z} = [\mathbf{z}_1, \ldots, \mathbf{z}_B]^T \in \mathbb{C}^{B\times L}$ and $\mathbf{H} = \mathrm{diag}(h_1, \ldots, h_B) \in \mathbb{C}^{B\times B}$.

We consider that codewords $\mathbf{X}$ form a coded modulation scheme $\mathcal{X} \subset \mathbb{C}^{B\times L}$. In particular, we consider that $\mathcal{X}$ is obtained as the concatenation of a binary code $\mathcal{C} \in \mathbb{F}_2^n$ of rate r, a modulation over the signal constellation $\mathcal{S} \in \mathbb{C}$ with $M = \log_2 |\mathcal{S}|$, and K rotations $\mathbf{M}_k \in \mathbb{C}^{N\times N}$ with $KN = B$ (see Figure 1). In particular we have that at time $\ell = 1, \ldots, L$

$$\mathbf{x}_{\ell,k} = \mathbf{M}_k \mathbf{s}_{\ell,k} \tag{3}$$

where $\mathbf{s}_{\ell,k} = (s_{\ell,k,1}, \ldots, s_{\ell,k,N})^T \in \mathcal{S}^N$ is the vector of complex-plane signal constellation symbols that is rotated by the k-th rotation matrix, $\mathbf{x}_{\ell,k} = (x_{\ell,k,1}, \ldots, x_{\ell,k,N})^T$ is the portion of transmitted signal at time ℓ that has been rotated by the k-th rotation, and

$$\mathbf{x}_\ell = [\mathbf{x}_{\ell,1}^T, \ldots, \mathbf{x}_{\ell,K}^T]^T$$

is the transmitted signal at time ℓ. The rotation matrices are unitary, i.e., $\mathbf{M}_k \mathbf{M}_k^\dagger = \mathbf{I}$. We will be interested in *full-fiversity* rotations, namely, rotation matrices $\mathbf{M}$ for which $\forall \mathbf{s}, \mathbf{s}' \in \mathcal{S}^B, \mathbf{s} \neq \mathbf{s}'$

$$\mathbf{M}(\mathbf{s} - \mathbf{s}') \neq \mathbf{0} \tag{4}$$

componentwise. This implies that, if the vector $\mathbf{s} - \mathbf{s}'$ has only one entry different from zero, all the components of its rotation will be different from zero. The rate in bits per channel use of this scheme is $R = rM$. This general formulation includes the case where only one single rotation of dimension B is used and the non-rotated case.

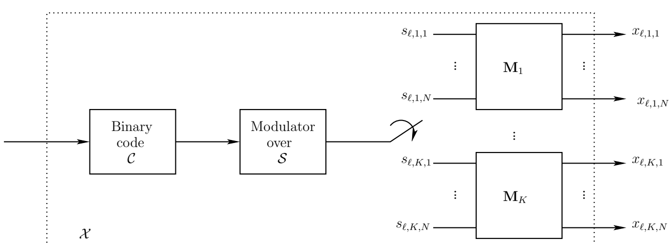


Fig. 1. Block diagram for coded modulation with K rotated constellations with rotation matrices $\mathbf{M}_1, \ldots, \mathbf{M}_K$.

Definition 1: The block-diversity of a coded modulation scheme $\mathcal{X} \subset \mathbb{C}^{B\times L}$ is defined as

$$\delta = \min_{\substack{\mathbf{X},\mathbf{X}'\in\mathcal{X}\\ \mathbf{X}'\neq\mathbf{X}}} |\{b \in (1, \ldots, B) \;|\; \mathbf{x}_b \neq \mathbf{x}_b'\}|. \tag{5}$$

In words, the block diversity is the minimum number of nonzero rows of $\mathbf{X} - \mathbf{X}'$ for any pair of codewords $\mathbf{X}' \neq \mathbf{X} \in \mathcal{X}$.

Proposition 1: Given a coded modulation scheme $\mathcal{X} \subset \mathbb{C}^{B\times L}$, the block diversity is upperbounded by

$$\delta \leq N\left(1 + \left\lfloor \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rfloor\right). \tag{6}$$

Proof: The result follows from the straightforward application of the Singleton bound to the coded modulation $\mathcal{X}$ seen as a code of block-length K, over an alphabet of size 2^{MNL}. ■

We will say that a code is blockwise maximum-distance separable (MDS) if it attains the Singleton bound with equality.

III. Outage Probability

Strictly speaking, the channel defined in (1) is not information stable and has zero capacity for any finite B [15], since there is a non-zero probability that the transmitted message is detected in error. For sufficiently large L, the word error probability $P_e(\mathsf{SNR}, \mathcal{X})$ of any coding scheme $\mathcal{X} \subset \mathbb{C}^{B\times L}$ is lowerbounded by this limiting error probability, the *information outage probability* [9], [16], given by

$$P_e(\mathsf{SNR}, \mathcal{X}) \geq P_{\mathrm{out}}(\mathsf{SNR}, R) \triangleq \Pr(I(\mathsf{SNR}, \mathbf{H}) \leq R). \tag{7}$$

where $I(\mathsf{SNR}, \mathbf{H})$ is the input-output mutual information of the channel for a given fading realization $\mathbf{H}$. For a fixed $\mathbf{H}$, the outage probability is minimized when the entries of $\mathbf{X} \in \mathcal{X}$ are i.i.d. Gaussian $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. In this case [17]

$$I(\mathsf{SNR}, \mathbf{H}) = \frac{1}{B}\sum_{b=1}^{B} \log_2(1 + \mathsf{SNR}\gamma_b). \tag{8}$$

With the coded modulation scheme shown in Figure 1 is used (assuming uniform inputs), we can express the *instantaneous mutual information* for a given channel realization $\mathbf{H}$ as

$$I(\mathsf{SNR}, \mathbf{H}) = \frac{1}{K}\sum_{k=1}^{K} \frac{1}{N} I_k(\mathsf{SNR}, \mathbf{H}_k) = \frac{1}{B}\sum_{k=1}^{K} I_k(\mathsf{SNR}, \mathbf{H}_k)$$

where

$$I_k(\mathsf{SNR}, \mathbf{H}_k) = MN - \frac{1}{2^{MN}} \sum_{\mathbf{s}\in\mathcal{S}^N}$$

$$\mathbb{E}_{\mathbf{z}}\left[\log_2\left(1 + \sum_{\mathbf{s}'\neq\mathbf{s}} e^{-\|\sqrt{\mathsf{SNR}}\,\mathbf{H}_k\mathbf{M}_k(\mathbf{s}-\mathbf{s}')+\mathbf{z}\|^2 + \|\mathbf{z}\|^2}\right)\right] \tag{9}$$

denotes the mutual information of the $N \times N$ MIMO channel induced by the k-th rotation, and $\mathbf{H}_k = \mathrm{diag}(h_{(k-1)N+1}, \ldots, h_{kN}) \in \mathbb{C}^{N\times N}$ are the channel coefficients *used* by rotation k. Note that for small N, the expectation over the noise vector $\mathbf{z}$ in (9) can be efficiently computed using the Gauss-Hermite quadrature rules [13].

Note that concatenating a Gaussian random code with a rotation brings no benefit in terms of exponent nor mutual information. In fact, the output of the rotated Gaussian i.i.d. vector is also a Gaussian i.i.d. vector with identical distribution, provided that the rotation matrix is unitary. Therefore, the mutual information

$$I(\mathsf{SNR}, \mathbf{H}) = \frac{1}{B}\log_2 \det\left(\mathbf{I} + \mathsf{SNR}\,\mathbf{H}\mathbf{M}\mathbf{M}^\dagger\mathbf{H}^\dagger\right) \tag{10}$$

$$= \frac{1}{B}\sum_{b=1}^{B} \log_2(1 + \mathsf{SNR}\gamma_b). \tag{11}$$

is the same than without rotation, and so is therefore the corresponding diversity exponent. Rotations are usually seen as information lossless, when in fact they are simply not needed when combined with Gaussian inputs.

Figure 2 shows the mutual information with Gaussian inputs, unrotated 16-QAM (identity rotation) and rotated[3] 16-QAM in a block-fading channel with $B = 4$ blocks and $h_1 = 1.5$ and $h_2 = h_3 = h_4 = 0.1$. This choice of the channel coefficients is particularly interesting since 3 out of the 4 components are in a deep fade. Rotations of dimension N yield vanishing (with SNR) error probability whenever there are up tp $N-1$ deeply faded blocks. As we observe, the mutual information corresponding to the rotated 16-QAM follows the Gaussian input in a larger SNR support than the unrotated 16-QAM. For example, at $\mathsf{SNR} = 25$dB, the Krüskemper rotation gains 1 bit of information with respect to unrotated 16-QAM. Combining 2 cyclotomic rotations of dimension $N = 2$ brings also significant information gains with respect to unrotated 16-QAM. As we shall see, this effect brings substantial exponent benefits with respect to the unrotated case. We also appreciate some difference between optimal Krüskemper and the mixed (2×2) rotations, especially at low rates. As a matter of fact, rotations provide only mutual information advantages at high rates. At low rates, unrotated transmission performs almost as well with much less decoding complexity.

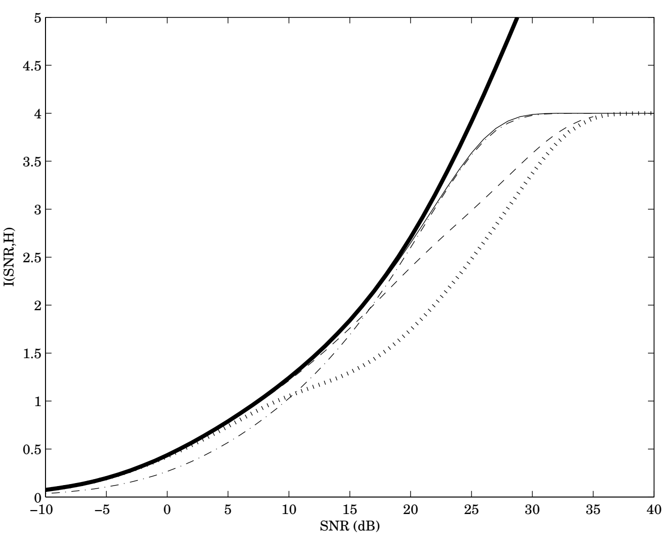


Fig. 2. Instantaneous mutual information $I(\mathsf{SNR}, \mathbf{H})$ (bits/channel use) in a block-fading channel with $B = 4$ blocks and $h_1 = 1.5$ and $h_2 = h_3 = h_4 = 0.1$ with Gaussian inputs (thick solid) and rotated 16-QAM inputs with the optimal Krüskemper (thin solid), mixed (thin dash-dotted), 2 independent 2-dimensional cyclotomic rotations (thin dashed) and no rotations (thick dotted).

IV. Optimal Reliability

We define the *diversity reliability exponent* of a given coded modulation scheme $\mathcal{X}$ as

$$d_{\mathcal{X}} = \lim_{\mathsf{SNR}\to+\infty} -\frac{\log P_e(\mathsf{SNR}, \mathcal{X})}{\log \mathsf{SNR}} \tag{12}$$

[3]Rotation matrices are extracted from [18]. This reference reports rotation matrices using the row convention used in [19]. In this paper, we use a column convention for lattice generator matrices, and therefore, matrices from [18] are transposed.

and the optimal diversity reliability exponent is

$$d^\star \triangleq \sup_{\mathcal{X}} d_{\mathcal{X}} = \sup_{\mathcal{X}} \lim_{\mathsf{SNR}\to+\infty} -\frac{\log P_e(\mathsf{SNR}, \mathcal{X})}{\log \mathsf{SNR}}. \tag{13}$$

When no particular structure is imposed on the coded modulation scheme $\mathcal{X}$, we have the following result.

Lemma 1: The diversity reliability exponent $d_{\mathcal{X}}$ of any coded modulation scheme $\mathcal{X}$ subject to the power constraint $\frac{1}{BL}\mathbb{E}[\|\mathbf{X}\|^2] \le 1$ is upperbounded by

$$d_{\mathcal{X}} \le d^\star = mB. \tag{14}$$

The optimal diversity reliability exponent can be achieved by random Gaussian codes of rate $R > 0$ with entries $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. The optimal exponent $d^\star$ can also be achieved by random coded modulation schemes $\mathcal{X}$ of rate R consisting of a random coded modulation scheme over a discrete signal constellation $\mathcal{S}$ of size $|\mathcal{S}| = 2^M$ concatenated with a full-diversity rotation of dimension B, whenever $0 \le \frac{R}{M} < 1$.

Note that we have added the achievability with random coded modulation ensemble over the B-dimensional rotated constellation to illustrate that a coding scheme with discrete inputs can also achieve the optimal exponent. This result which is based on a *divide and conquer* approach, should be rather intuitive: the rotation of dimension B takes care of achieving full diversity while the coding gain is then left to the outer coded modulation scheme over $\mathcal{S}$. When no rotations are used, the optimal diversity reliability exponent for $m = 1$ is given by the Singleton bound [20]

$$d^\star = 1 + \left\lfloor B\left(1 - \frac{R}{M}\right)\right\rfloor. \tag{15}$$

As shown in Figure 3 the advantage of rotations is clear: they can achieve optimal diversity reliability exponent for all the range of rates. Instead, when no rotations are used, the largest rate such that optimal diversity reliability exponent is achieved is $R = \frac{M}{B}$.

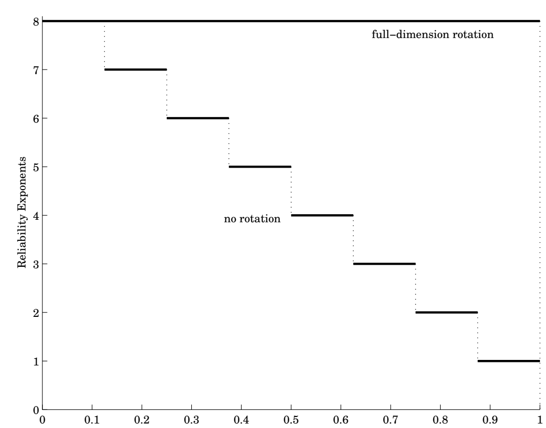


Fig. 3. Diversity reliability exponents for $B = 8$ and $m = 1$. Optimal exponent (14) and Singleton bound (15).

As outlined in the Introduction, full-diversity rotations induce large decoding complexity, since the size of set of candidate points at a given time instant is 2^{MB}. We are therefore interested in characterizing the optimal diversity reliability exponent when rotations of smaller size $N < B$ are employed. We have the following results

Proposition 2: The diversity reliability exponent for the coded modulation schemes based on K rotations of dimension N, in a Nakagami-m block-fading channel with $B = KN$ blocks is upperbounded by

$$d_{\mathcal{X}} \le mN\left(1 + \left\lfloor \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rfloor\right). \tag{16}$$

Proposition 3: The diversity reliability exponent in a Nakagami-m block-fading channel with $B = KN$ of random coded modulation schemes based on K rotations of dimension N of length L satisfying $\lim_{\mathsf{SNR}\to\infty} \frac{L(\mathsf{SNR})}{\mathsf{SNR}} = \lambda$, is lower-bounded by

$$d_{\mathcal{X}} \ge \lambda BM \log 2\left(1 - \frac{R}{M}\right) \tag{17}$$

when $0 \le \lambda NM \log 2 < m$ and by

$$d_{\mathcal{X}} \ge \min\left\{ mN\left\lceil \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rceil, mN\left\lfloor \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rfloor + \lambda M \log 2\left(B\left(1 - \frac{R}{M}\right) - N\left\lfloor \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rfloor\right)\right\} \tag{18}$$

otherwise.

Theorem 1: The optimal diversity reliability exponent for the coded modulation schemes based on K rotations of dimension N, in a Nakagami-m block-fading channel with $B = KN$ blocks is given by

$$d^{\star}_{\mathcal{X}} = mN\left(1 + \left\lfloor \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rfloor\right) \tag{19}$$

whenever $\frac{B}{N}\left(1 - \frac{R}{M}\right)$ is not an integer.

Proof: Proposition 2 shows that

$$d_{\mathcal{X}} \le mN\left(1 + \left\lfloor \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rfloor\right). \tag{20}$$

Letting $\lambda \to \infty$ in Proposition 3 shows that

$$d_{\mathcal{X}} \ge mN\left\lceil \frac{B}{N}\left(1 - \frac{R}{M}\right)\right\rceil. \tag{21}$$

Noting that $\lceil x \rceil = \lfloor x \rfloor + 1$ whenever x is not an integer leads the desired result. ■

As we observe, Theorem 1 shows that the optimal exponent is given by m times the Singleton bound of (6), proving its optimality and separating the the roles of the channel distribution (through m) and of the code construction. When $N = 1$, there is no rotation, and the inputs to the channel are directly signal constellation points drawn from $\mathcal{S}$. In this case, we have that the optimal reliability exponent is given by the Singleton bound [21], [20]

$$d^{\star}_{\mathcal{X}} = m\left(1 + \left\lfloor B\left(1 - \frac{R}{M}\right)\right\rfloor\right) \tag{22}$$

for any $R \le M$, and the optimal codes are *blockwise* MDS in a channel with B blocks. For $N > 1$ Theorem 1 suggests that the optimal coding scheme is to use a coded modulation scheme constructed over $\mathcal{S}$ which is MDS in a block-fading channel with $K = \frac{B}{N}$ blocks concatenated with rotations of dimension N. In this case the MDS constraint on the code is relaxed, since it has to be MDS for a smaller number of blocks, at an expense of a decoding complexity increase. Theorem 1 implicitly introduces an equivalent channel model, namely, a block-fading channel with $K = \frac{B}{N}$, where each block has diversity mN. When $K = 1$, $N = B$, there is only one single rotation of *full* dimension, Theorem 1 generalizes Lemma 1. The optimal coding scheme here does not need to be MDS. Therefore, Theorem 1 generalizes and proves the optimality of the *modified Singleton bound* introduced in [7].

Figure 4 shows the reliability exponents in the case of $B = 8$, $m = 0.5$ and $N = 1, 2, 4$. The figure confirms the intuition behind such designs that the rotations should increase the reliability exponent. For example, for $\frac{R}{M} = \frac{1}{2}$, we have that with classical complex-plane inputs the reliability exponent is $d^{\star}_{\mathcal{X}} = m5$, while for rotations with $N = 2$ the exponent is $d^{\star}_{\mathcal{X}} = m6$ and for $N = m4$ the exponent is $d^{\star}_{\mathcal{X}} = m8$, full diversity. This approach can be seen as a divide-and-conquer approach, namely, the task of achieving diversity is split between both, the code $\mathcal{C}$ and the rotations. Figure 5 shows the diversity upper bound as well as the random coding lower bounds given in Propositions 2 and 3, respectively. As we see, if λ is increased, both bounds coincide in a larger support. Eventually, for $\lambda \to \infty$ they coincide wherever they are continuous.

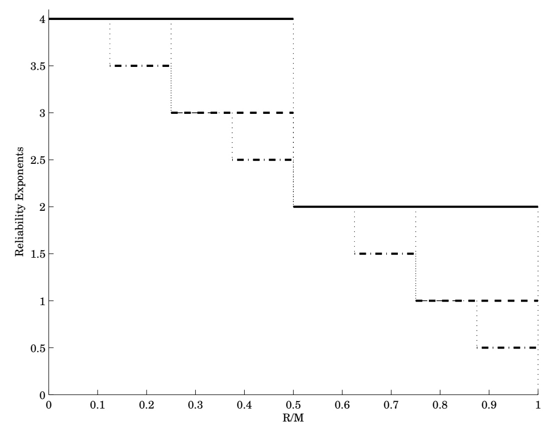


Fig. 4. Reliability exponents for $B = 8$, $m = 0.5$ and rotations of dimensions $N = 1$ (dash-dotted), $N = 2$ (dashed) and $N = 4$ (solid).

To illustrate the performance benefits of rotations, Figure 6 shows $P_{\rm out}(\mathsf{SNR}, R)$ as a function of $\frac{E_{\rm b}}{N_0}$ in a block-fading channel with $m = 1$ and $B = 4$ for $R = 2$, with Gaussian inputs (solid), discrete inputs (dotted), rotated discrete inputs with two cyclotomic rotations with $N = 2$ (dash-dotted) and rotated discrete inputs with one Krüskemper rotation with $N = 4$ (dashed). Gaussian inputs achieve the optimal exponent, namely $d^{\star} = B = 4$, while unrotated inputs have $d^{\star}_{\mathcal{X}} = 3$ [20]. As we observe from the curves, using two rotations of dimension $N = 2$, not only allows to recover the largest possible exponent (in agreement with Theorem 1) but also brings a large gain in terms of gain. Using one rotation of dimension $N = 4$ incurs much larger complexity and does not bring any benefits in terms of exponent nor gain.

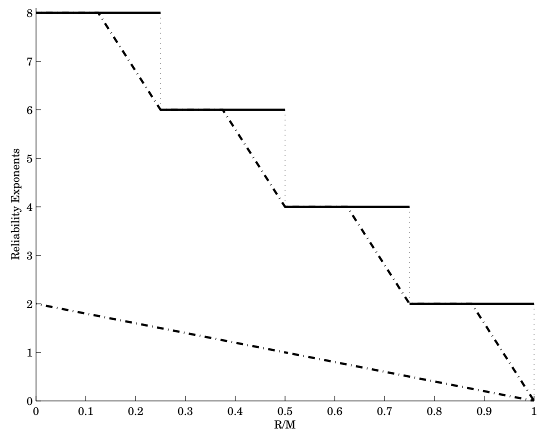
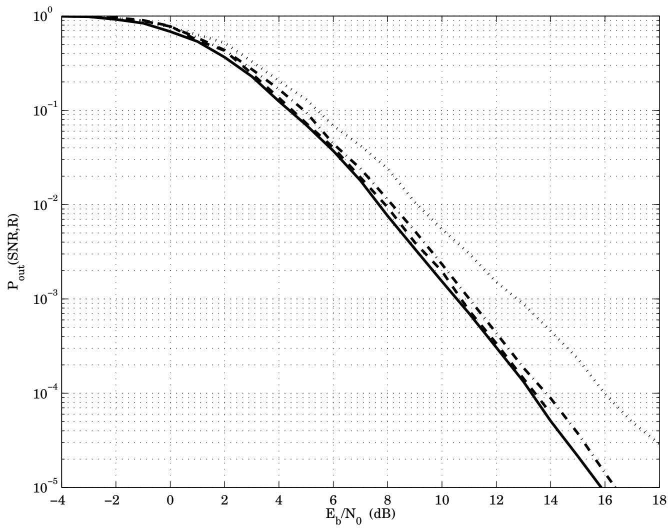
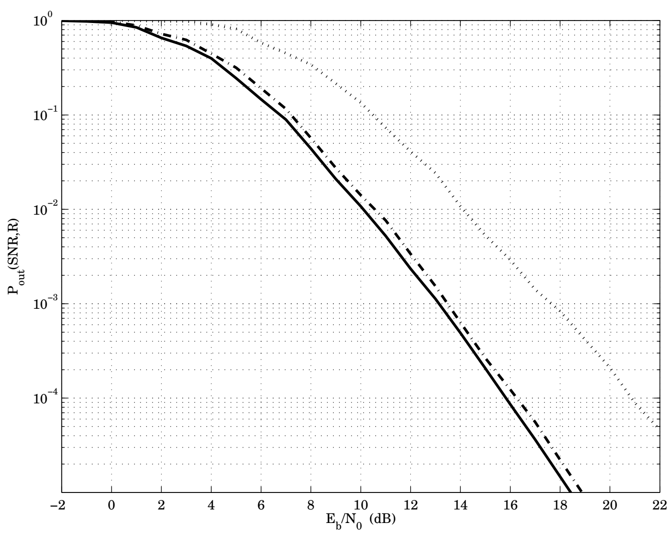


Fig. 5. Reliability exponents for $B = 8$, $m = 1$ and rotations of dimensions $N = 2$. The random coding exponents for $\lambda M \log 2 = \frac{m}{2N}$ (lower dash-dotted curve) and $\lambda M \log 2 = \frac{4m}{N}$ (upper dash-dotted curve) are also shown.



(a) QPSK, $R = 1$ bits per channel use.



(b) 16-QAM, $R = 2$ bits per channel use.

Fig. 6. Outage probability for $R = 1, 2$ in a block-fading channel with $B = 4$, $m = 1$, with Gaussian, rotated and unrotated inputs.

V. Conclusions

We have studied coded modulation schemes over Nakagami-m block-fading channels with discrete input signal constellations. In particular, we have derived the optimal diversity reliability exponent for multidimensional signal constellations obtained from the rotation of classical complex-plane constellations, and we have shown that there is a tradeoff between the transmission rate, optimal achievable diversity, dimension of the rotations and size of the complex-plane signal constellation given by a modified form of the Singleton bound. Since using rotated constellations induces an increase in decoding complexity, the Singleton bound establishes the optimal rate-diversiy-complexity tradeoff.

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