Analysis and Computation of the Outage Probability of Discrete-Input Block-Fading Channels

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***Abstract*— THIS PAPER IS ELIGIBLE FOR THE STUDENT PAPER AWARD. In this paper, we propose a tight lower bound to the outage probability of Nakagami-m block-fading channels. The approach permits an efficient method for numerical evaluation of the bound, providing an additional tool for system design. The optimal rate-diversity trade-off for the Nakagami-m block-fading channel is also derived and a tight upper bound is obtained for the optimal coding gain constant.**

I. Introduction

The block-fading channel [1], [2] is a useful channel model for a class of slowly-varying wireless communication channels. The model is particularly relevant for delay-constraint applications where channel usage is restricted to only include a finite number of distinct channel blocks, each subject to independent flat fading. Frequency-hopping schemes as encountered in the Global System for Mobile Communication (GSM) and the Enhanced Data GSM Environment (EDGE), respectively, as well as orthogonal frequency division multiplexing (OFDM) as encountered in more recently proposed wireless communication systems standards can conveniently be modelled as block-fading channels. The simplified model is mathematically tractable, while still capturing the essential features of the practical slowly-varying fading channels.

In a block-fading channel a codeword spans a finite number B of independent fading blocks. As the channel relies on particular realizations of the finite number of independent fading coefficients, the channel is non-ergodic and therefore not information stable [3], [4]. It follows that the Shannon capacity of this channel is zero since there is an irreducible probability that a given transmission rate R is not supported by a particular channel realization [1], [2]. This probability is named the information outage probability. For sufficiently large codes, the outage probability is the lowest achievable word error rate known.

Considerable efforts have been dedicated to describing the behavior of the word error probability and the outage probability for Rayleigh block-fading channels in the high signal-to-noise ratio (SNR) regime. In particular, analysis based on worst-case pairwise error probabilities shows that at high SNR the achievable word error probability of codes $\mathcal{C}$ of rate R constructed over a signal constellation $\mathcal{X}$ of size $|\mathcal{X}| = 2^M$ behaves as

$$\lim_{\mathrm{SNR}\to\infty} -\frac{\log P_e(\mathrm{SNR}, R)}{\log \mathrm{SNR}} = d_B(R) \tag{1}$$

where

$$d_B(R) = 1 + \left\lfloor B\left(1 - \frac{R}{M}\right)\right\rfloor \tag{2}$$

is the Singleton bound [5], [6], [7]. More recently, it has been shown [8] that the optimal SNR exponent

$$d^\star(R) \triangleq \sup_{\mathcal{C}} \lim_{\mathrm{SNR}\to\infty} -\frac{\log P_e(\mathrm{SNR}, R)}{\log \mathrm{SNR}} \tag{3}$$

is actually given by the Singleton bound (2). This establishes the Singleton bound as the optimal rate-diversity trade-off for transmission over the Rayleigh block-fading channel with discrete signal constellations.

While these results provide significant insight into code design, the analysis techniques do not provide explicit tools for the evaluation of the outage probability; a task which usually requires extensive numerical computations. To this end, an upper bound to the outage probability of Rayleigh and Rician block-fading channels is proposed in [9], [10]. In this paper, we propose a tight lower bound to the outage probability which can be efficiently evaluated for the general Nakagami-m block-fading channel [11]. We also show that the optimal rate-diversity trade-off for the Nakagami-m fading case is given by $d^\star(R) = md_B(R)$ for any $m > 0$, and we obtain an upper bound to the achievable coding gain for any coding scheme.

The remainder of the paper is organized as follows. In Section II, the system model is described for the Nakagami-m block-fading channel, while Section III defines the outage probability of this channel. In Section IV, we detail the proposed lower bound for the outage probability, as well as an efficient method for the evaluation of the bound. The asymptotic behavior of the outage probability is investigated in Section V, where the rate-diversity trade-off is extended to include the Nakagami-m fading statistics. Finally, conclusions are given in Section VI. Proofs are available in [12].

The following notation is used in the paper. Sets are denoted by calligraphic fonts with the complement denoted by superscript c. The exponential equality $g(\xi) \doteq \xi^d$ indicates

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that $\lim_{\xi\to\infty} \frac{\log g(\xi)}{\log \xi} = d$. The exponential inequalities $\dot{\leq}, \dot{\geq}$ are similarly defined, while $\lceil \xi \rceil$ ($\lfloor \xi \rfloor$) denotes the smallest (largest) integer greater (smaller) than ξ, and $\mathbb{A}^n_+ = \{\xi \in \mathbb{A}^n | \xi > 0\}$.

II. System Model

Consider transmission of codewords of length BL coded symbols over a block-fading channel with B blocks. Each block is an additive white Gaussian noise (AWGN) channel of L channel uses affected by the same flat fading coefficient. The complex baseband expression for the received signal is

$$\mathbf{y}_b = \sqrt{\mathrm{SNR}}\, h_b\, \mathbf{x}_b + \mathbf{z}_b, \quad b = 1, \ldots, B, \tag{4}$$

where $\mathbf{y}_b \in \mathbb{C}^L$ is the received signal in block b, $\mathbf{x}_b \in \mathbb{C}^L$ is the portion of the codeword assigned to block b, and $\mathbf{z}_b$ is a noise vector with independent, identically distributed (i.i.d.) circularly symmetric Gaussian entries $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$. We define $\mathbf{h} = (h_1, \ldots, h_B) \in \mathbb{C}^B$ as the vector of fading coefficients. The fading coefficients are i.i.d. from block to block and from codeword to codeword, and are assumed to be perfectly known to the receiver.

We consider a channel with a discrete input constellation set $\mathcal{X} \subset \mathbb{C}$ of cardinality 2^M. Without loss of generality, we assume that $\mathbb{E}[|x|^2] = 1$, where $x \in \mathcal{X}$, and that the fading coefficients are normalized such that $\mathbb{E}[|h_b|^2] = 1$. It follows that SNR is the average signal-to-noise ratio at the receiver end. Define $\gamma_b \triangleq |h_b|^2$ as the *fading power gain*. Then, the instantaneous received signal-to-noise ratio at block b is $\gamma_b\mathrm{SNR}$.

We consider the case where the fading coefficients follow the general Nakagami-m distribution [11], [13]. The probability density function (pdf) of $|h_b|$ is[2]

$$f_{|h_b|}(\xi) = \frac{2m^m \xi^{2m-1}}{\Gamma(m)} e^{-m\xi^2}, \tag{5}$$

where $\Gamma(a)$ is the Gamma function, $\Gamma(a) = \int_0^\infty t^{a-1}\, e^{-t} dt$. It follows that the fading power gain γ_b has the following pdf

$$f_{\gamma_b}(\xi) = \begin{cases} \frac{m^m \xi^{m-1}}{\Gamma(m)} e^{-m\xi}, & \xi \geq 0 \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

and cumulative distribution function (cdf)

$$F_{\gamma_b}(\xi) = \begin{cases} 1 - \frac{\Gamma(m, m\xi)}{\Gamma(m)}, & \xi \geq 0 \\ 0, & \text{otherwise}, \end{cases} \tag{7}$$

where $\Gamma(a, \xi)$ is the upper incomplete Gamma function, $\Gamma(a, \xi) = \int_\xi^\infty t^{a-1}\, e^{-t} dt$. The Nakagami-$m$ distribution represents a large class of fading statistics, including Rayleigh fading (by setting $m = 1$). The distribution also approximates Rician fading with parameter K (by setting $m = (K+1)^2/(2K+1)$) [13]. Therefore, the proposed analysis for systems with Nakagami-m fading is a generalization of previous results in the literature.

[2] Since the complex coefficients h_b are perfectly known to the receiver, we can assume phase coherent detection, and thus, only the amplitude is affected by the fading statistics.

III. Mutual Information and Outage Probability

The instantaneous input-output mutual information of the block fading channel with a given channel realization $\mathbf{h}$ can be expressed as [1]

$$I(\mathrm{SNR}, \mathbf{h}) = \frac{1}{B} \sum_{b=1}^{B} I_{\mathrm{AWGN}}(\gamma_b \mathrm{SNR}),$$

where $I_{\mathrm{AWGN}}(\rho)$ is the input-output mutual information of an AWGN channel with SNR ρ. $I(\mathrm{SNR}, \mathbf{h})$ is the input-output mutual information of a set of B non-interfering parallel channels, each of which is used only for a fraction $\frac{1}{B}$ of the time. When the input signal set $\mathcal{X}$ is discrete, the mutual information $I_{\mathrm{AWGN}}(\rho)$ is given by

$$I_{\mathrm{AWGN}}(\rho) = M - \mathbb{E}_{X,Z}\left[\log_2\left(\sum_{x' \in \mathcal{X}} e^{-|\sqrt{\rho}(X - x') + Z|^2 + |Z|^2}\right)\right]. \tag{8}$$

The expectation over $Z \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ can be efficiently computed using the Gauss-Hermite quadrature rules [14].

Transmission at rate R bits per channel use is considered to be in outage whenever

$$\frac{1}{B} \sum_{b=1}^{B} I_{\mathrm{AWGN}}(\gamma_b \mathrm{SNR}) < R.$$

The corresponding outage probability is given by

$$P_{\mathrm{out}}(\mathrm{SNR}, R) = \Pr\left(\frac{1}{B} \sum_{b=1}^{B} I_{\mathrm{AWGN}}(\gamma_b \mathrm{SNR}) < R\right). \tag{9}$$

IV. Lower Bound to the Outage Probability

In general, when the channel has a discrete input constellation, evaluation of the outage probability in (9) is complicated since a closed form expression for $I_{\mathrm{AWGN}}(\rho)$ is not known. Typically, $P_{\mathrm{out}}(\mathrm{SNR}, R)$ is instead evaluated through simulation[3]. In this section, we propose a lower bound to the outage probability with discrete inputs, which can be efficiently computed.

The maximum input-output mutual information for a channel with input signal constellation $\mathcal{X}$ of size $|\mathcal{X}| = 2^M$ is always upper bounded by M. Furthermore, the input-output mutual information of the channel can also be upper bounded by that of the channel with Gaussian input. Therefore, for $b = 1, \ldots, B$, $I_{\mathrm{AWGN}}(\gamma_b \mathrm{SNR})$ is upper bounded by[4]

$$I^u_{\mathrm{AWGN}}(\gamma_b \mathrm{SNR}) \triangleq \min\{M, \log_2(1 + \gamma_b \mathrm{SNR})\} \tag{10}$$

$$= \begin{cases} \log_2(1 + \gamma_b \mathrm{SNR}), & b \in \mathcal{S}^c \\ M, & b \in \mathcal{S}, \end{cases} \tag{11}$$

where $\mathcal{S} = \left\{b \in \{1, 2, \ldots, B\} : \gamma_b > \frac{2^M - 1}{\mathrm{SNR}}\right\}$ and $\mathcal{S}^c$ denotes its complement.

[3] Even if the inputs to the channel are Gaussian, for which $I_{\mathrm{AWGN}}(\gamma_b \mathrm{SNR}) = \log_2(1 + \gamma_b \mathrm{SNR})$, a closed form expression for the outage probability is not known.

[4] Superscripts u and ℓ will denote upper and lower bounds respectively.

Let $|\mathcal{S}|$ be the cardinality of $\mathcal{S}$. Since $\gamma_b,\ b=1,\dots,B$, are independent random variables, $|\mathcal{S}|$ is a binomially distributed random variable with success rate $p \triangleq \Pr\left(\gamma_b > \frac{2^M-1}{\mathrm{SNR}}\right)$. Hence,

$$\Pr(|\mathcal{S}|=t) = \binom{B}{t} p^t (1-p)^{B-t}, \quad t=1,2,\dots,B \quad (12)$$

where

$$p = \frac{\Gamma\left(m, m\frac{2^M-1}{\mathrm{SNR}}\right)}{\Gamma(m)}. \quad (13)$$

Using the upper bound of mutual information in (10) and (11), we lower bound $P_{\mathrm{out}}(\mathrm{SNR}, R)$ as

$$P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R) \triangleq \Pr\left(\frac{1}{B}\sum_{b=1}^{B} I_{\mathrm{AWGN}}^{u}(\gamma_b \mathrm{SNR}) < R\right) \quad (14)$$

$$= \Pr\left(|\mathcal{S}|M + \sum_{b\in\mathcal{S}^c} \log_2(1+\gamma_b\mathrm{SNR}) < BR\right). \quad (15)$$

Since $\gamma_b,\ b=1,\dots,B$ are i.i.d. random variables, $\sum_{b\in\mathcal{S}^c}\log_2(1+\gamma_b\mathrm{SNR})$ is the summation of $|\mathcal{S}^c| = B - |\mathcal{S}|$ i.i.d. random variables. Each random variable inside the summation is given by $\log_2(1+\gamma_b\mathrm{SNR})$ conditioned on $b\in\mathcal{S}^c$, or equivalently on the event $\mathcal{E}$, where $\mathcal{E}$ is defined as

$$\mathcal{E} \triangleq \left\{\gamma_b : \gamma_b \le \frac{2^M-1}{\mathrm{SNR}}\right\}. \quad (16)$$

Denote A_b as the random variable $\log_2(1+\gamma_b\mathrm{SNR})$ conditioned on $\mathcal{E}$. The distribution of A_b is given by the following proposition.

Proposition 1: Assume γ_b is a random variable whose distribution is given by (6). Denote A_b as the random variable $\log_2(1+\gamma_b\mathrm{SNR})$ conditioned on the event $\mathcal{E}$ given in (16). The distribution of A_b is then given by

$$f_{A_b}(\xi) = \begin{cases} \dfrac{f_{\gamma_b}\left(\frac{2^\xi-1}{\mathrm{SNR}}\right)}{F_{\gamma_b}\left(\frac{2^M-1}{\mathrm{SNR}}\right)} \dfrac{2^\xi\log(2)}{\mathrm{SNR}}, & 0\le\xi\le M \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

$\diamondsuit$

Therefore, each random variable inside the summation in (15) follows the distribution described by (17). Denoting $A_k, k=1,\dots,|\mathcal{S}^c|$, as the $B-|\mathcal{S}|$ independent random variables that follow the distribution given in (17), we can write (15) as

$$P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R) = \Pr\left(|\mathcal{S}|M + \sum_{k=1}^{B-|\mathcal{S}|} A_k < BR\right). \quad (18)$$

By conditioning on $|\mathcal{S}|$, we can express $P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R)$ as

$$\begin{aligned} &P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R) \\ &= \sum_{t=0}^{B} \Pr\left(\sum_{k=1}^{B-|\mathcal{S}|} A_k < BR - |\mathcal{S}|M \,\middle|\, |\mathcal{S}|=t\right)\Pr(|\mathcal{S}|=t) \\ &= \sum_{t=0}^{B} \Pr\left(\sum_{k=1}^{B-t} A_k < BR - tM\right)\Pr(|\mathcal{S}|=t). \end{aligned} \quad (19)$$

From the distribution in (17), note that $\Pr(A_k \le 0) = 0$. Therefore, for any t such that $BR - tM \le 0$, or equivalently for all $t \ge \lceil \frac{BR}{M} \rceil$, the corresponding probability is zero. Hence, we can rewrite (19) as

$$\begin{aligned} &P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R) = \\ &\sum_{t=0}^{\lceil \frac{BR}{M}\rceil - 1} \Pr\left(\sum_{k=1}^{B-t} A_k < BR - tM\right)\Pr(|\mathcal{S}|=t). \end{aligned} \quad (20)$$

If we now define the random variable $Y_t \triangleq \sum_{k=1}^{B-t} A_k$, we can write

$$P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R) = \sum_{t=0}^{\lceil \frac{BR}{M}\rceil - 1} F_{Y_t}(BR - tM)\binom{B}{t}p^t(1-p)^{B-t}, \quad (21)$$

where $F_{Y_t}(\xi)$ is the cdf of Y_t.

Since $A_k, k=1,\dots,B-t$ are independent random variables, the pdf of Y_t can be evaluated by performing $B-t$ convolutions of $f_{A_b}(\xi)$. Numerically, this convolution can be efficiently computed in the frequency domain using fast Fourier transform (FFT) techniques [15]. With this method, we can efficiently evaluate the cdf of Y_t, $F_{Y_t}(\xi)$, and therefore we can also efficiently evaluate $P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R)$ in (21). Numerical results for Nakagami-m block-fading channels with $B = 4, M = 4$ and $m = 2$ or $m = 0.5$ are given in Figure 1. The transmission rates considered are $R = 1, 2, 3$ bits per channel use, which correspond to Singleton bounds $d_B(R) = 4, 3, 2$, respectively. The figure shows the simulation and analytical curves of the lower bound to the outage probability of the channel based on (14) and (21), respectively, together with the 16-QAM outage simulation curve based on (9). We observe

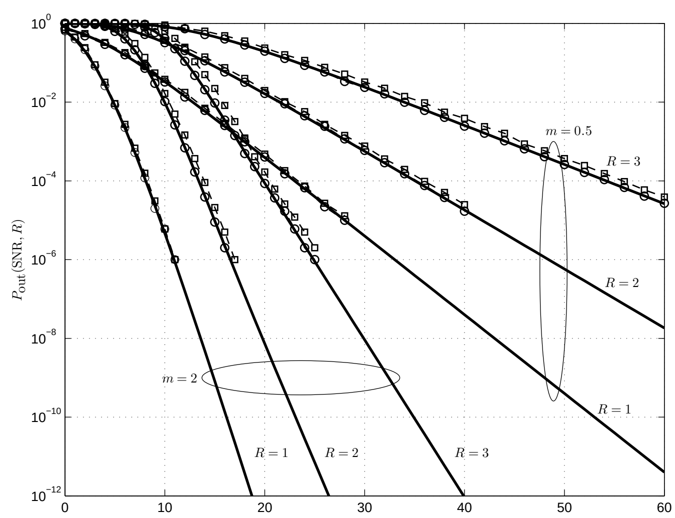

Fig. 1. Outage probability of Nakagami-m block-fading channels with $B = 4, M = 4, m = 0.5$ and $m = 2$. The thick solid lines correspond to the lower bound (21), thin dashed lines with circles denote the simulation of (14) and thin dashed lines with squares denote the simulation of (9) with 16-QAM modulation.

that the analytical curves coincide with the corresponding lower bound simulation curves. The analytical curves give a tight lower bound to the 16-QAM outage curve. Note that the bound is very tight for the important case of $R = 1$, which, from the Singleton bound expression in (2), is the largest rate that can be achieved with full diversity. Figure 2 provides a plot of the outage probability of the same channels as a function of the code rate R at $\mathrm{SNR} = 10\mathrm{dB}$, illustrating the validity of the bound over a wide range of transmission rates.

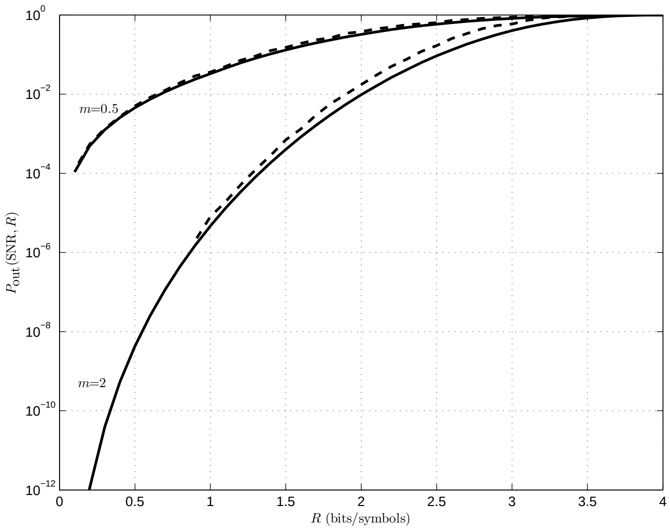

Fig. 2. Outage probability for the of Nakagami-m block-fading channels with $B = 4, M = 4, m = 0.5$ and $m = 2$ at $\mathrm{SNR} = 10\mathrm{dB}$. The solid lines correspond to the lower bound (21). The dashed lines denote the simulation of (9) with 16-QAM modulation.

We also observe from Figure 1 that the slope of each curve is $md_B(R)$, representing the SNR exponent of the outage probability. In the following section, we rigorously prove that the optimal SNR-exponent over the channel is

$$d^{\star}(R) = md_B(R). \tag{22}$$

In proving this result, we characterize not only the SNR-exponent but also the asymptotic coding gain.

V. Asymptotic Behavior

Using (21) and analysis techniques from [8], we obtain the following result for the asymptotic diversity of Nakagami-m block-fading channels, for all $m > 0$.

Proposition 2: Assume transmission over the block-fading channel as defined in (4) with input signal constellation size 2^M. Assume further that the fading power gain γ_b is a random variable whose distribution is given by (6). In this case, the lower bound on $P_{\mathrm{out}}(\mathrm{SNR}, R)$ given in (21) can asymptotically be expressed as

$$P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R) \doteq \mathcal{K}_{\ell}\mathrm{SNR}^{-md_B(R)}, \tag{23}$$

where $d_B(R)$ is the Singleton bound given in (2). Furthermore, $\mathcal{K}_{\ell}$ is a constant independent of SNR given by

$$\begin{aligned}\mathcal{K}_{\ell} =& F_{\overline{Y}_{B-d_B(R)}}\Big(BR - (B - d_B(R))M\Big) \\ & \binom{B}{B - d_B(R)} \frac{(m(2^M - 1))^{md_B(R)}}{(m\Gamma(m))^{d_B(R)}},\end{aligned} \tag{24}$$

where $F_{\overline{Y}_t}(\xi) = \lim_{\mathrm{SNR}\to\infty} F_{Y_t}(\xi)$. $\diamondsuit$

This proposition not only shows that the SNR exponent of the outage probability is upper bounded by $md_B(R)$ but also gives the asymptotic constant $\mathcal{K}_{\ell}$ of $P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R)$. This is indeed useful for code design since it gives an upper bound for the best performance achievable by any coding scheme. At the same time, together with the expression of $P_{\mathrm{out}}^{\ell}(\mathrm{SNR}, R)$ given in (21), it gives a more specific characterization of the outage probability, indicating the word error probability (or SNR) region where asymptotic analysis is valid.

The lower bound to the outage probability and the asymptotic term given in (23) are illustrated in Figure 3 . The same set of parameters as in Figure 1 has been chosen, namely $B = 4, M = 4$ and $R = 1, 2, 3$.

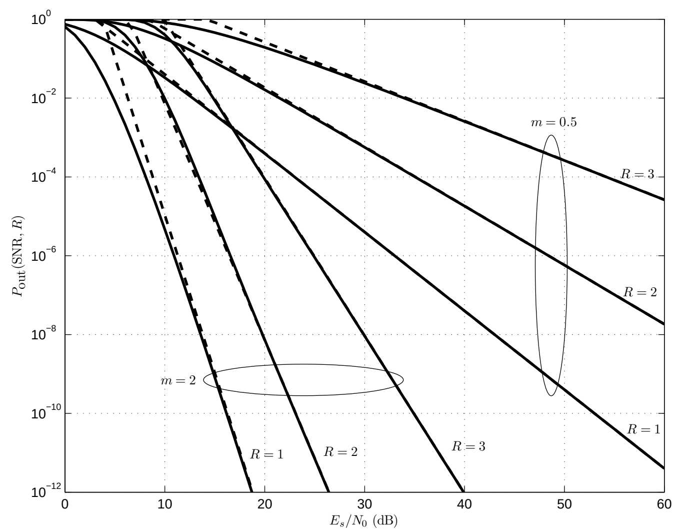

Fig. 3. Outage probability of Nakagami-m block-fading channels with $B = 4, M = 4, m = 0.5$ and $m = 2$. The solid lines correspond to the lower bound (21) and the dashed lines to its asymptotic expression given in (23) using $\mathcal{K}_{\ell}$ in (24).

So far, we have shown that $d^{\star}(R) \leq md_B(R)$. To prove the optimality of the SNR-exponent $md_B(R)$, we need to prove the achievability result given in the next proposition.

Proposition 3: Assume transmission with random codes of rate R and block length $L(\mathrm{SNR})$ satisfying

$$\lim_{\mathrm{SNR}\to\infty} \frac{L(\mathrm{SNR})}{\log(\mathrm{SNR})} = \lambda \tag{25}$$

over a block-fading channel as defined in (4) with input signal constellation size 2^M. Further assume that the fading power gain γ_b is a random variable whose distribution is given by (6). In this case, the random coding SNR-exponent is lower bounded by

$$d^{(r)}(R) \geq \lambda BM \log(2) \left(1 - \frac{R}{M}\right) \qquad (26)$$

when $\lambda < \frac{m}{M \log(2)}$, and by

$$\begin{aligned} d^{(r)}(R) &\geq m(d_B(R) - 1) \\ &+ \min\left\{m, \lambda M \log(2) \left(B\left(1 - \frac{R}{M}\right) - d_B(R) + 1\right)\right\} \end{aligned} \qquad (27)$$

when $\lambda \geq \frac{m}{M \log(2)}$. ♢

The preceding propositions lead to the following theorem.

Theorem 1: Assume transmission over a block-fading channel as defined in (4) with input constellation size 2^M. Further assume that the fading power gain γ_b is a random variable whose distribution is given by (6). In this case, the optimal SNR-exponent is given by

$$d^\star(R) = md_B(R) \qquad (28)$$

for all R, M where $B\left(1 - \frac{R}{M}\right)$ is not an integer. ♢

Theorem 1 can be proved using techniques proposed in [8]. However, in this contribution Propositions 2 and 3 provide additional information. In particular, Proposition 2 provides an upper bound on the coding gain $\mathcal{K}_\ell$, and Proposition 3 provides an extension for the SNR-exponent of random codes with finite block length in [8] to a more general fading distribution.

The diversity of random codes for block-fading channels with $B = 4, M = 4, m = 0.5$ is illustrated in Figure 4. Random codes with block length satisfying $\lambda = \frac{2m}{M \log(2)}$ and $\lambda = \frac{m}{2M \log(2)}$ are considered, where λ is defined in (25). We observe that the SNR-exponent is always upper bounded by $md_B(R)$. Except for points of discontinuity of $d_B(R)$, the upper bound can be achieved by increasing λ since $d^{(r)}(R)$ will coincide $md_B(R)$ over larger ranges of R.

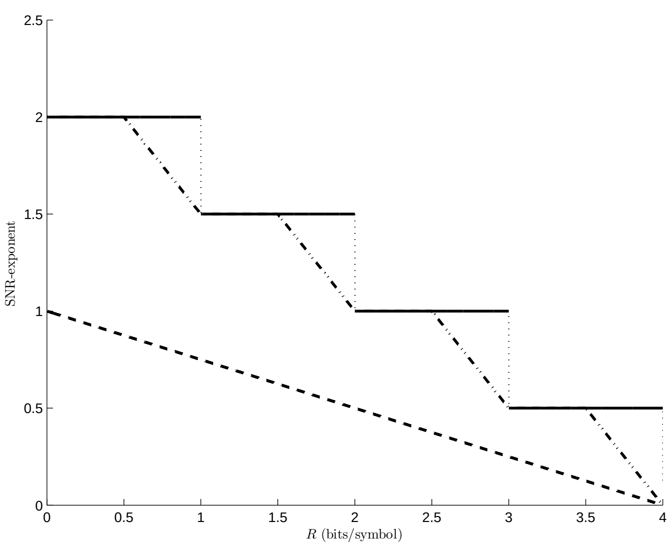

Fig. 4. Optimal and random coding SNR-exponent for Nakagami-m block-fading channels with $m = 0.5, B = 4, M = 4$. The solid line corresponds to $md_B(R)$, dashed-dotted line and dashed line denote the random coding exponent with $\lambda M \log(2) = 2m$ and $\lambda M \log(2) = \frac{m}{2}$, respectively.

VI. Conclusion

In this paper, we have proposed a tight lower bound to the outage probability of block-fading channels with Nakagami-m fading statistics. The lower bound can be computed efficiently and is therefore useful for system evaluations. We also show the optimal rate-diversity trade-off for Nakagami-m block-fading channels and obtain an upper bound for the achievable coding gain, which is useful for code design.

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