MIMO ARQ Systems with Multi-Level Feedback

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***Abstract*— We consider improving the outage performance of incremental-redundancy automatic repeat request (INR-ARQ) transmission over the multiple-input multiple-output (MIMO) block-fading channel by allowing multi-bit receiver feedback. We show that multi-bit feedback offers significant gain in outage diversity when power adaptation is employed. A suboptimal feedback and power adaptation rule is proposed, illustrating the benefits provided by multi-bit feedback.**

I. Introduction

The block-fading channel [1] is a mathematical model for many practical communication scenarios. The channel consists of a finite number of blocks, where each is affected by an independent fading coefficient. The model approximates well the characteristics of slowly varying channels, including the Orthogonal Frequency Division Multiplexing (OFDM), the Global System for Mobile Communications (GSM) and the Enhanced Data GSM (EDGE) standards. Due to the finite number of fading blocks, the information rate supported by the channel is random, dependent on the instantaneous channel realization. When the instantaneous mutual information is less than the transmission rate, transmission is in outage. Then, for sufficiently long codewords, messages are erroneously decoded with probability one [2]. Therefore, adaptive transmission techniques, where the transmission rate/power is adjusted according to the instantaneous channel realizations, can provide performance gains for communications over the block-fading channel. INR-ARQ is a powerful and practical adaptive transmission technique based on receiver feedback (see [2] and references therein). Additionally, MIMO transmission has been employed as an efficient technique to improve the throughput and reliability of wireless communication systems. Therefore, INR-ARQ transmission over the MIMO channel is essential for high rate wireless communication systems.

In INR-ARQ, transmission starts with a high rate codeword, and additional redundancy bits are requested via the feedback link when the codeword is not successfully decoded. Transmission is in outage if the codeword is not decodable within the delay constraint of the system. Traditional INR-ARQ systems implement one-bit feedback from the receiver, indicating whether additional redundancy bits are required. However, due to the accumulative nature of INR-ARQ schemes, performance improvements are possible when additional information is provided through the feedback link regarding the status of the current transmission. Such multi-bit feedback is proposed in [3, 4] for INR-ARQ systems based on convolutional codes, and in [5] to improve the performance of INR-ARQ systems in a multi-layer broadcasting strategy. In this paper, we consider the performance of multi-bit feedback INR-ARQ transmission over the MIMO block-fading channel.

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An important performance measure for ARQ transmission in the MIMO block-fading channel is the rate-diversity-delay tradeoff. This tradeoff has been studied only for ARQ systems with one-bit feedback. Reference [6] characterizes the rate-diversity-delay tradeoff of INR-ARQ systems with Gaussian inputs, with both constant and adaptive transmit power. For discrete input constellations, the tradeoff for INR-ARQ systems with constant transmit power and power adaptation are correspondingly studied in [7] and [8], where power adaptation has been shown to offer significant gain in outage probability. In this paper, we extend the results of [8] to INR-ARQ systems with multi-bit feedback, where the quantized *accumulated mutual information* is fed back to the transmitter. We characterize the rate-diversity-delay tradeoff of multi-bit feedback INR-ARQ systems and show that multi-bit feedback and optimal power adaptation provide significant outage diversity gains for transmission over the block-fading channel. A suboptimal feedback and power adaptive rule is also proposed, illustrating the outage performance gain offered by multi-bit feedback.

II. Channel Model

Consider INR-ARQ transmission over the block-fading channel with N_t transmit and N_r receive antennas. Each ARQ round is transmitted over B additive white Gaussian noise (AWGN) blocks of J channel uses, where block b at ARQ round ℓ is affected by a flat fading channel matrix $\boldsymbol{H}_{\ell,b} \in \mathbb{C}^{N_r \times N_t}$. The baseband equivalent of the channel in the ℓ-th ARQ round is given by

$$\boldsymbol{Y}_\ell = \sqrt{\frac{P_\ell}{N_t}} \boldsymbol{H}_\ell \boldsymbol{X}_\ell + \boldsymbol{W}_\ell, \qquad (1)$$

where P_ℓ is the transmit power in round ℓ, $\boldsymbol{X}_\ell \in \mathbb{C}^{BN_t \times J}, \boldsymbol{Y}_\ell, \boldsymbol{W}_\ell \in \mathbb{C}^{BN_r \times J}$ are correspondingly the transmitted signal, the received signal, the additive noise and $\boldsymbol{H}_\ell$ is the block diagonal channel gain matrix at round ℓ with

$$\boldsymbol{H}_\ell = \text{diag}(\boldsymbol{H}_{\ell,1}, \ldots, \boldsymbol{H}_{\ell,B}).$$

In INR-ARQ, the receiver attempts to decode at round ℓ based on the received signal collected in rounds $1,\ldots,\ell$. The entire channel after ℓ ARQ rounds is

$$\boldsymbol{Y}_{\overline{1,\ell}} = \boldsymbol{H}_{\overline{1,\ell}}\boldsymbol{X}_{\overline{1,\ell}} + \boldsymbol{W}_{\overline{1,\ell}}, \tag{2}$$

where, with $(\cdot)'$ denoting non-conjugate transpose,

$$\boldsymbol{Y}_{\overline{1,\ell}} = \left[\boldsymbol{Y}_1',\ldots,\boldsymbol{Y}_\ell'\right]'$$
$$\boldsymbol{X}_{\overline{1,\ell}} = \left[\boldsymbol{X}_1',\ldots,\boldsymbol{X}_\ell'\right]'$$
$$\boldsymbol{H}_{\overline{1,\ell}} = \mathrm{diag}\left(\sqrt{\frac{P_1}{N_t}}\boldsymbol{H}_1,\ldots,\sqrt{\frac{P_\ell}{N_t}}\boldsymbol{H}_\ell\right)$$
$$\boldsymbol{W}_{\overline{1,\ell}} = \left[\boldsymbol{W}_1',\ldots,\boldsymbol{W}_\ell'\right]'.$$

We consider transmission with input constellation $\mathcal{X}\subset\mathbb{C}$ of size 2^M, and assume that the constellation $\mathcal{X}$ has unit average energy, i.e., entries $x\in\mathcal{X}$ of $\boldsymbol{X}_\ell$ satisfy $\mathbb{E}\left[|x|^2\right]=1$. We further assume that the entries of $\boldsymbol{H}_{\ell,b}$ and $\boldsymbol{W}_{\ell,b}$ are independently drawn from a unit variance Gaussian complex distribution $\mathcal{N}_{\mathbb{C}}(0,1)$, and that $\boldsymbol{H}_{\ell,b}$ is available at the receiver. The average signal-to-noise ratio (SNR) at each receive antenna is then P_ℓ.

We consider ARQ transmission with a long-term power constraint, where the average transmit power is constrained to P, i.e.,

$$\mathbb{E}_{\boldsymbol{H}_{\overline{1,L}}}\left[\sum_{\ell=1}^{L} P_\ell\right] \le P, \tag{3}$$

where P_ℓ is adapted as a function of $\boldsymbol{H}_{\overline{1,\ell-1}}$.

III. Preliminaries

A. Accumulated Mutual Information

Assuming that the channel matrix at round ℓ is $\boldsymbol{H}_\ell$, the input-output mutual information of the MIMO channel in round ℓ is given by

$$I_\ell\left(\sqrt{\frac{P_\ell}{N_t}}\boldsymbol{H}_\ell\right) = \frac{1}{B}\sum_{b=1}^{B} I_{\mathcal{X}}\left(\sqrt{\frac{P_\ell}{N_t}}\boldsymbol{H}_{\ell,b}\right), \tag{4}$$

where $I_{\mathcal{X}}\left(\sqrt{\frac{P_\ell}{N_t}}\boldsymbol{H}_{\ell,b}\right)$ is the input-output mutual information [9], measured in bits per channel use (bpcu), of an AWGN MIMO channel with input constellation $\mathcal{X}$ and channel gain matrix $\sqrt{\frac{P_\ell}{N_t}}\boldsymbol{H}_{\ell,b}$. The average input-output mutual information after ℓ ARQ rounds is given by $\frac{1}{\ell}\sum_{l=1}^{\ell} I_l$ bpcu. Let

$$I_{\overline{1,\ell}} \triangleq \sum_{l=1}^{\ell} I_l \tag{5}$$

be the *accumulated mutual information* after ℓ ARQ rounds.

B. Multi-Level Feedback

We consider an INR-ARQ system with delay constraint L, i.e. the maximum number of ARQ rounds is L, where a feedback index $k\in\{0,\ldots,K-1\}$ is delivered after each transmission round through a zero-delay error-free channel. Power and rate adaptation are performed based on receiver feedback. The system model is illustrated by Figure 1.

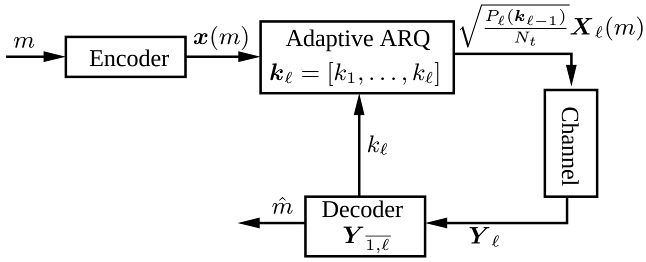


Fig. 1. The INR-ARQ system with multi-bit feedback.

1) The transmitter: Consider a code book $\mathcal{C}$ of rate $\frac{R_M}{L}$, $R_M\in(0,M)$ bits per coded symbol, that maps a message $m\in\{0,\ldots,2^{R_M N_t BJ}\}$ to a codeword $\boldsymbol{x}(m)\in\mathcal{X}^{N_t BJL}$. At transmission round ℓ, $N_t BJ$ of the coded symbols are formatted into $\boldsymbol{X}_\ell(m)\in\mathcal{X}^{BN_t\times J}$ and transmitted via the channel in (1) with power $P_\ell(\boldsymbol{k}_{\ell-1})$, where $\boldsymbol{k}_{\ell-1}=[k_1,\ldots,k_{\ell-1}]$ is the vector of feedback indices collected from rounds $1,\ldots,\ell-1$. The realized code rate after ℓ ARQ rounds is $\frac{R_M N_t}{\ell}$ bpcu; and the realized rate of a single ARQ round is $R\triangleq R_M N_t$. After ℓ transmission rounds, if feedback $k_\ell=K-1$ (denoting positive acknowledgment (ACK)) is received, the transmission is successful and the transmission of the next message starts. Otherwise, the transmitter continues with new transmission rounds until feedback index $K-1$ is received or until L transmission rounds have elapsed.

2) The receiver: Upon receiving round ℓ, the receiver attempts to decode the transmitted message from the received signals collected from rounds 1 to ℓ. The receiver employs a decoder with error detection capability as described in [2]. The decoder outputs $\hat{m}\in\{1,\ldots,2^{RBJ}\}$ if $\hat{m}$ is the unique message such that $\boldsymbol{X}_{\overline{1,\ell}}(\hat{m})$ and $\boldsymbol{Y}_{\overline{1,\ell}}$ are jointly typical [9]; and an ACK is delivered to the transmitter via feedback index $k_\ell=K-1$. Otherwise, a quantization of the *accumulated mutual information* $I_{\overline{1,\ell}}$ is delivered via feedback index k_ℓ satisfying $I_{\overline{1,\ell}}\in\left[\,\overline{I}([\boldsymbol{k}_{\ell-1},k_\ell]),\overline{I}([\boldsymbol{k}_{\ell-1},k_\ell+1])\right)$, with predefined quantization thresholds $\overline{I}(\boldsymbol{k}_\ell),\boldsymbol{k}_\ell\in\{0,\ldots,K-2\}^\ell$, and $\overline{I}([\boldsymbol{k}_{\ell-1},K-1])=\infty$ for $\ell=1,\ldots,L-1$. An example of the feedback thresholds for the first two rounds of an ARQ system with $K=4$ is illustrated in Figure 2. Noting that $I_{\overline{1,\ell+1}}\ge I_{\overline{1,\ell}}$, the feedback thresholds in round $\ell+1$ should be designed such that $\overline{I}(\boldsymbol{k}_\ell)=\overline{I}([\boldsymbol{k}_\ell,0])<\ldots<\overline{I}([\boldsymbol{k}_\ell,K-2])$. Thus, the set of quantization thresholds is completely defined by $\overline{I}(\boldsymbol{k}_{L-1})$ for all practical purposes.

3) Power constraint: The probability of having feedback vector $\boldsymbol{k}_\ell$ at round ℓ, denoted as $q(\boldsymbol{k}_\ell)$, is recursively given by

$$q([\boldsymbol{k}_{\ell-1},k]) = \Pr\left\{k_\ell=k|\boldsymbol{k}_{\ell-1}\right\}q(\boldsymbol{k}_{\ell-1}), \tag{6}$$
$$\Pr\left\{k_\ell=k\middle|\boldsymbol{k}_{\ell-1}\right\} =$$
$$\Pr\left\{I_{\overline{1,\ell-1}}+I_\ell\in\left[\,\overline{I}([\boldsymbol{k}_{\ell-1},k]),\overline{I}([\boldsymbol{k}_{\ell-1},k+1])\right)\middle|\boldsymbol{k}_{\ell-1}\right\},$$

where I_ℓ is given by (4) with $P_\ell=P_\ell(\boldsymbol{k}_{\ell-1})$. Then, the power constraint in (3) can be written as

$$P_1+\sum_{\ell=2}^{L}\sum_{\boldsymbol{k}_{\ell-1}\in\{0,\ldots,K-1\}^{\ell-1}} q(\boldsymbol{k}_{\ell-1})P_\ell(\boldsymbol{k}_{\ell-1}) \le P. \tag{7}$$

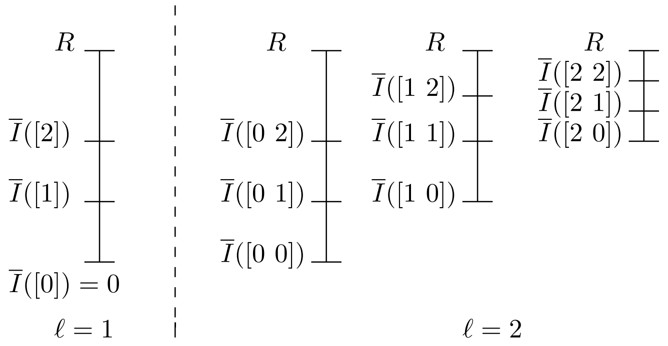


Fig. 2. An example of feedback thresholds.

C. Information Outage

After ℓ ARQ rounds, the input-output mutual information is $\frac{I_{\overline{1,\ell}}}{\ell}$ and the realized code rate is $\frac{R_M N_t}{\ell} = \frac{R}{\ell}$. The transmission is in outage at round ℓ if $I_{\overline{1,\ell}} < R$. The probability of having an outage at round ℓ is then given by

$$p(\ell) \triangleq \Pr\left\{I_{\overline{1,\ell}} < R\right\}. \qquad (8)$$

With an optimal coding scheme, and in the limit $J \to \infty$, the codeword is correctly decoded if $I_{\overline{1,\ell}} > R$, otherwise, an error is detected [2]. Therefore, the outage probability is an achievable lower bound of the word error probability.

IV. Asymptotic Analysis

Consider a power adaptation rule $P_\ell = P_\ell(\boldsymbol{k}_{\ell-1})$ satisfying the power constraint in (7). We prove that asymptotic to P, the outage probability at round ℓ behaves like

$$p(\ell) \doteq P^{-d_\ell(R)}, \qquad (9)$$

where $d_\ell(R)$ is the outage diversity at round ℓ and the exponential equality ($\doteq$) indicates [10]

$$d_\ell(R) = \lim_{P\to\infty} \frac{-\log(p(\ell))}{\log(P)}. \qquad (10)$$

We analyze the optimal rate-diversity-delay tradeoff $d_\ell(R)$ of ARQ systems with K levels feedback and prove that the optimal outage diversity is achievable.

Firstly, the input-output mutual information within each ARQ round asymptotically behaves as follows [8].

Proposition 1: Let I_ℓ be the realized mutual information in round ℓ as defined in (4). For large P_ℓ, we have that

$$\Pr\{I_\ell < I\} \doteq P_\ell^{-d(I)}, \qquad (11)$$

where $d(I)$ is bounded by $d^\ddagger(I) \le d(I) \le d^\dagger(I)$, and

$$d^\dagger(I) \triangleq N_r\left(1 + \left\lfloor B\left(N_t - \frac{I}{M}\right)\right\rfloor\right) \qquad (12)$$

$$d^\ddagger(I) \triangleq N_r\left\lceil B\left(N_t - \frac{I}{M}\right)\right\rceil. \qquad (13)$$

Furthermore, $d^\ddagger(I)$ is the SNR-exponent of random codes with rate I, where the coded symbols are uniformly drawn from $\mathcal{X}$.

From Proposition 1, the outage diversity of the MIMO block-fading channel satisfies $d(I) = d^\dagger(I)$ when $\frac{BI}{M}$ is not an integer. The result is useful for the asymptotic analysis of ARQ systems with multi-bit feedback. The optimal rate-diversity-delay tradeoff is given by the following Theorem.

Theorem 1: Consider INR-ARQ transmission over the MIMO block-fading channel in (1) using constellation $\mathcal{X}$ of size 2^M and the transmission scheme described in Section III-B, where a codeword is considered successfully delivered at round ℓ if $I_{\overline{1,\ell}} \ge R$, and the number of feedback levels is $K \ge \left\lceil\frac{BR}{M}\right\rceil + 1$. Subject to the power constraint in (7), the optimal rate-diversity-delay tradeoff is given by

$$d_\ell(R) = (1 + BN_tN_r)^{\ell-1}\left(d^\dagger(R) + 1\right) - 1 \qquad (14)$$

for R such that $d^\dagger(R)$ (given in (12)) is continuous.

Proof: A sketch of the proof is given as follows. We first lower-bound the outage diversity by considering a suboptimal ARQ system with $\underline{K} = \left\lceil\frac{BR}{M}\right\rceil + 1$ feedback levels, where the quantization thresholds are placed at $\overline{I}([\boldsymbol{k}_{\ell-1}, k_\ell]) = \frac{k_\ell M}{B}, k_\ell = 0, \ldots, \left\lfloor\frac{BR}{M}\right\rfloor$. Using Proposition 1, we prove by induction that the outage diversity of the suboptimal ARQ system at round ℓ is $d_\ell(R)$.

Conversely, the outage performance of the optimal ARQ system with K level feedback can be improved by adding $\left\lceil\frac{BR}{M}\right\rceil$ extra quantization thresholds (and corresponding feedback levels) at $\frac{tM}{B}, t = 0, \ldots, \left\lfloor\frac{BR}{M}\right\rfloor$. Using Proposition 1, we prove by induction that the outage diversity at round ℓ of the improved systems is also given by $d_\ell(R)$. Therefore, $d_\ell(R)$ is the optimal outage diversity at round ℓ for an ARQ system with $K \ge \left\lceil\frac{BR}{M}\right\rceil + 1$ feedback levels. ■

We now prove that the rate-diversity-delay tradeoff $d_\ell(R)$ is achievable, as given by the following theorem.

Theorem 2: Consider INR-ARQ transmission over the MIMO block-fading channel in (1) using constellation $\mathcal{X}$ of size 2^M and the transmission scheme described in Section III-B with power constraint P given in (7). Further assume that the number of feedback levels is $K \ge \left\lceil\frac{BR}{M}\right\rceil + 1$. With an optimal coding scheme and $J \to \infty$, asymptotic to P, the word error probability $P_e(\ell)$ at round ℓ satisfies $P_e(\ell) \doteq P^{-d_\ell^{(r)}(R)}$, where (with $d^\ddagger(R)$ given in (13))

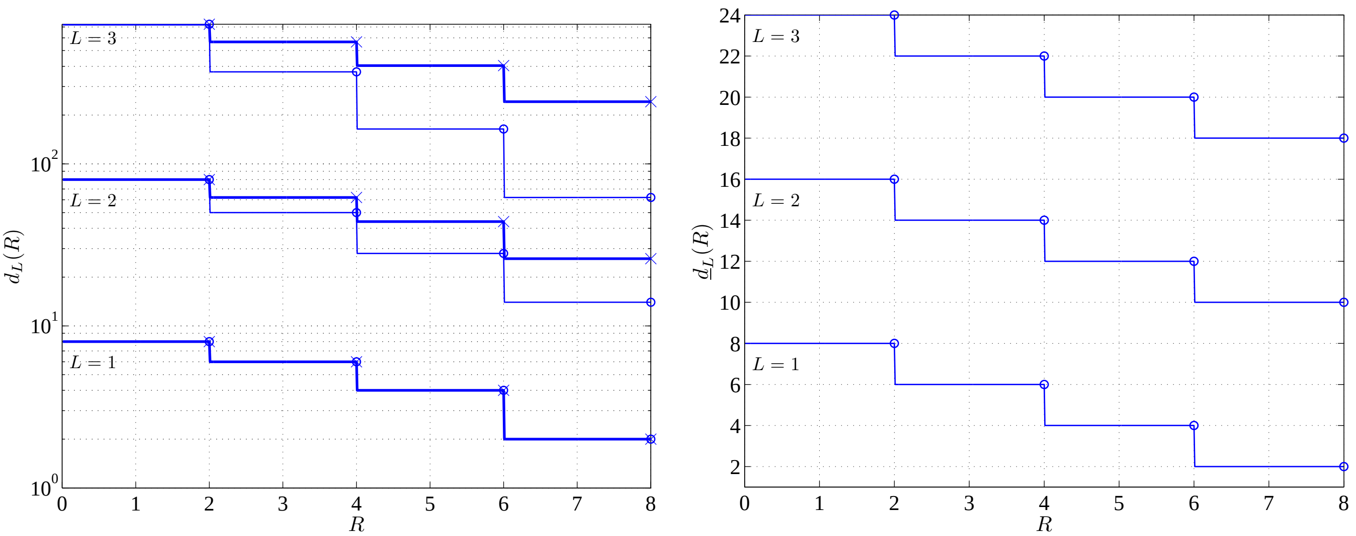
$$d_\ell^{(r)}(R) = (1 + BN_tN_r)^{\ell-1}\left(d^\ddagger(R) + 1\right) - 1$$

represents the achievable SNR-exponent.

Proof: With an optimal coding scheme and $J \to \infty$, the codeword is correctly decoded with probability one [2], and thus the receiver feeds back an ACK, at round ℓ if $I_{\overline{1,\ell}} > R$ (in contrast to the outage case, where an ACK is fed back if $I_{\overline{1,\ell}} \ge R$). The proof then follows similar arguments as the proof of Theorem 1, noting that[1] $\Pr\{I_\ell \le I\} \dot{\le} P_\ell^{-d^\ddagger(I)}$. ■

From Theorems 1 and 2, we have the following. The optimal outage diversity of an INR-ARQ transmission with rate R and delay constraint L is given by $d_L(R)$. Furthermore, the SNR-exponent $d_L^\ddagger(R)$ of the transmission scheme described

[1] The exponential inequality ($\dot{\le}$) is similarly defined from (10).



(a) Long-term power constraint tradeoff.

(b) Constant transmit power tradeoff.

Fig. 3. Optimal rate-diversity-delay tradeoff of ARQ transmission with long-term power constraint (a) and constant power (b). 16-QAM is used over a MIMO block-fading channel with $N_t = N_r = 2, B = 2, L = 1, 2, 3$. Thick and thin lines in (a) represent the optimal tradeoffs $d_L(R)$ achieved by multi-bit feedback ($K \geq \lceil R/2 \rceil + 1$) and $\hat{d}_L(R)$ achieved by one-bit feedback ($K = 2$), respectively. Crosses and circles correspond to the rate points where the SNR-exponent of random codes does not achieve the optimal diversity.

in Section III-B achieves the optimal outage diversity for all rates such that $d_\ell(R)$ is continuous. The optimal rate-diversity-delay tradeoff for INR-ARQ transmission with $L = 1, 2, 3$ over the MIMO block-fading channel with $N_t = N_r = B = 2$ is illustrated in Figure 3(a). For comparison, we consider the optimal tradeoff $\hat{d}_L(R)$ of the corresponding system with one-bit feedback studied in [8], where $\hat{d}_L(R)$ is recursively obtained from $\hat{d}_1(R) = d_1(R)$ and

$$\hat{d}_\ell(R) = BN_tN_r\left(\ell - 1 + \sum_{l=1}^{\ell-2} \hat{d}_l(R)\right) + (1 + \hat{d}_{\ell-1}(R)). \tag{15}$$

Furthermore, the tradeoff $\underline{d}_L(R)$ of an ARQ system with constant transmit power (short-term power constraint) is also illustrated in Figure 3(b), where [7, 8]

$$\underline{d}_L(R) = N_r\left(1 + \left\lfloor BL\left(N_t - \frac{R}{LM}\right)\right\rfloor\right). \tag{16}$$

The figure shows an order-of-magnitude improvement in outage diversity of INR-ARQ when a long-term power constraint is allowed. Furthermore, significant gain in outage diversity is provided by multi-bit feedback, especially at transmission rates R close to N_tM. Since high R is particularly relevant in ARQ systems, the result suggests that multi-bit feedback will give significant gains in practical implementations.

V. Power Adaptation and Feedback Design

The design of optimal feedback and transmission rules for an ARQ system with multi-bit feedback includes joint optimization of the overall set of quantization thresholds $\{\bar{I}(\boldsymbol{k}_{L-1}), \boldsymbol{k}_{L-1} \in \{0, \ldots, K-2\}^{L-1}\}$ and the corresponding power adaptive rule $P_\ell(\boldsymbol{k}_{\ell-1})$. The optimal feedback and power adaptation rule is obtained by minimizing

$$\sum_{\boldsymbol{k}_{L-1}} q(\boldsymbol{k}_{L-1})p(L|\boldsymbol{k}_{L-1}) \tag{17}$$

subject to the power constraint in (7). To the best of our knowledge, the optimization problem is not analytically tractable. We propose to separate the design problem into two steps.

Step 1: At round ℓ, determine a set of feedback thresholds $\bar{I}([\boldsymbol{k}_{\ell-1}, k])$ for every feedback vector $\boldsymbol{k}_{\ell-1} \in \{1, \ldots, K-2\}^{\ell-1}$.

Step 2: Given the set of feedback thresholds in Step 1, determine the corresponding transmit power rule, aiming at minimizing the outage probability.

The procedure suboptimally separates the joint optimization problem into two problems. Moreover, as shown in the next sections, each individual problem is sub-optimally solved. Still, the procedure guarantees that the optimal diversity shown in the previous section is achieved.

A. Selecting the Set of Feedback Thresholds

Assuming that the number of feedback levels K satisfies $K \geq \left\lceil \frac{BR}{M} \right\rceil + 1$, the proof of Theorem 1– not included due to space limitation– suggests the following procedure for setting the quantization thresholds [11]. Consider the feedback levels at round ℓ for a given feedback vector $\boldsymbol{k}_{\ell-1}$. Let $\tau \triangleq \left\lfloor \frac{BR}{M} \right\rfloor$ and $\hat{I}_t = \frac{Bt}{M}$. Let $t' \triangleq \left\lfloor \frac{M\bar{I}(\boldsymbol{k}_{\ell-1})}{B} \right\rfloor$, the feedback thresholds in round ℓ, given $\boldsymbol{k}_{\ell-1}$ is determined as follows.

1) Place a threshold at $\bar{I}(\boldsymbol{k}_{\ell-1})$, and a threshold at R;
2) Place $\tau - t'$ thresholds at $\hat{I}_t, t = t' + 1, \ldots, \tau$;
3) Place the remaining $K-2-\tau+t'$ thresholds sequentially within

$$\left(\bar{I}(\boldsymbol{k}_{\ell-1}), \hat{I}_{t'+1}\right), \left(\hat{I}_\tau, R\right), \left(\hat{I}_{t'+1}, \hat{I}_{t'+2}\right), \left(\hat{I}_{\tau-1}, \hat{I}_\tau\right), \ldots$$

until no more thresholds are left to place, and such that the thresholds uniformly partition each region.

The feedback thresholds for INR-ARQ transmission over the block-fading channel with $N_t = N_r = 1$, $B = 2$, $L = 2$, and $R = 3.5$ using 16-QAM constellations are illustrated in Figure 2, where $\overline{I}(\boldsymbol{k}_{\ell-1}) = \overline{I}([\boldsymbol{k}_{\ell-1}, 0])$, and the values of $\overline{I}(\boldsymbol{k}_2)$ are reported in the table below.

	$k_2 = 0$	$k_2 = 1$	$k_2 = 2$
$k_1 = 0$	0	1	2
$k_1 = 1$	1	2	2.75
$k_1 = 2$	2	2.75	3

B. Power Adaptation

The suboptimal power adaptation rule is obtained from the following simplifications.

- We consider a power constraint more stringent than (7),
$$\sum_{\boldsymbol{k}_\ell \in \{0,\ldots,K-1\}^\ell} q(\boldsymbol{k}_\ell) P_{\ell+1}(\boldsymbol{k}_\ell) \leq \frac{P}{L}, \qquad (18)$$
for $\boldsymbol{k}_\ell \in \{0, \ldots, K-1\}^\ell, \ell = 0, \ldots, L-1$, where $q(\boldsymbol{k}_0) = 1$ by definition.
- When feedback $\boldsymbol{k}_{\ell-1}$ is received, we have that $I_{\overline{1,\ell-1}} \geq \overline{I}(\boldsymbol{k}_{\ell-1})$. Then, the feedback probability is approximated from (6) by replacing $I_{\overline{1,\ell-1}}$ with $\overline{I}(\boldsymbol{k}_{\ell-1})$; and the outage probability can be upper bounded as
$$\hat{p}(\ell|\boldsymbol{k}_{\ell-1}) \triangleq \Pr\left\{I_\ell + \overline{I}(\boldsymbol{k}_{\ell-1}) < R\right\}, \qquad (19)$$
where I_ℓ is given by (4) with $P_\ell = P_\ell(\boldsymbol{k}_{\ell-1})$.
- To further simplify the problem, we consider minimizing $\hat{p}(\ell), \ell = 1, \ldots, L$ sequentially.

Based on the simplifications, the corresponding power adaptation rule $P_\ell(\boldsymbol{k}_{\ell-1})$ is obtained by solving

$$\begin{cases} \text{Minimize} & \sum_{\boldsymbol{k}_{\ell-1}} q(\boldsymbol{k}_{\ell-1}) \hat{p}(\ell|\boldsymbol{k}_{\ell-1}) \\ \text{Subject to} & \sum_{\boldsymbol{k}_{\ell-1}} q(\boldsymbol{k}_{\ell-1}) P_\ell(\boldsymbol{k}_{\ell-1}) \leq \frac{P}{L}. \end{cases} \qquad (20)$$

The optimization problem is separable, and thus can be solved via a branch-and-bound simplex algorithm using piece-wise linear approximation [12].

C. Numerical Results

The outage performance of ARQ systems with multi-bit feedback is illustrated in Figure 4 for ARQ transmissions ($L = 2$) at rate $R = 3.5$ over the block-fading channel in (1) with $N_t = N_r = 1, B = 2$ using 16-QAM input constellation. We consider systems with $K = 2, 3, 8, 16$, where the power adaptive rule for $K = 2$ is as that proposed in [11]. We observe that the outage diversity achieved by constant transmit power and by power adaptation for $K = 2$ is given by $3, 4$ as given in (16) and (15) respectively. For $K \geq 3$, the outage diversity is 5 as predicted from (14). This leads to significant improvement in outage performance for power adaptive ARQ transmission with multi-bit feedback. Furthermore, the simulation results suggest that increasing K beyond 8 does not substantially improve the outage performance; and thus, even for $K = 3$, the suboptimal choice of feedback thresholds in Section V-A performs within 1dB of systems with large K.

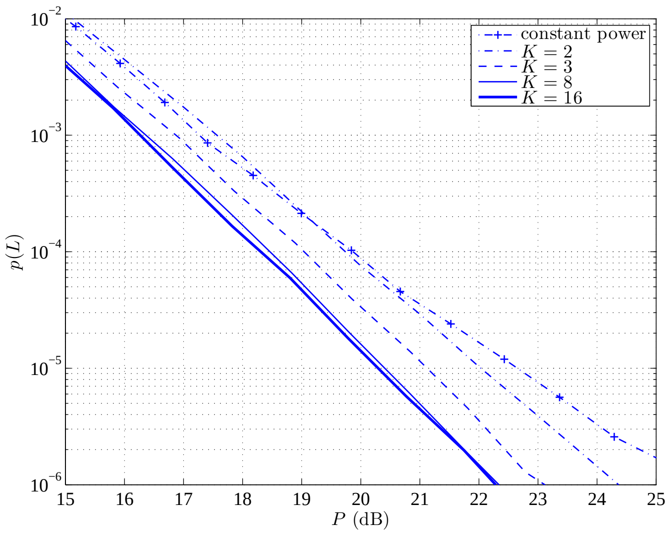


Fig. 4. Outage performance of ARQ transmission schemes for a 16-QAM input block-fading channel with $L = 2, N_t = N_r = 1, B = 2, R = 3.5$.

VI. Conclusions

We have studied the outage performance of power adaptation for multi-bit feedback INR-ARQ transmission over the MIMO block-fading channel. We have shown the large gain in outage diversity provided by multi-bit feedback in INR-ARQ systems with a long-term power constraint. A suboptimal feedback and power adaptation rule is proposed, showing the significant gain in outage probability.

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