Outage Probability of the MIMO Gaussian Free-Space Optical Channel with PPM

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***Abstract*— The main drawback in communicating via the free-space optical channel is the detrimental effect the atmosphere has on a propagating laser beam. Atmospheric turbulence causes random fluctuations in the irradiance of the received laser beam, commonly referred to as *scintillation*. We investigate the mitigation of scintillation through the use of multiple lasers and multiple apertures, thereby creating a multiple-input multiple output (MIMO) channel. We adopt a quasi-static block fading model and study the outage probability of the channel under the assumption of orthogonal pulse-position modulation. Non-ideal photodetection is also assumed such that the combined shot noise and thermal noise are considered as signal-independent additive Gaussian white noise. Assuming perfect receiver channel state information (CSI), we compute the signal-to-noise ratio exponents for the cases when the scintillation is lognormal, exponential and gamma-gamma distributed, which cover a wide range of atmospheric turbulence conditions. Furthermore, we illustrate very large gains when CSI is also available at the transmitter.**

I. Introduction

Free-space optical (FSO) communication offers an attractive alternative to the radio frequency (RF) channel for transmitting data at very high rates. By utilising a high carrier frequency in the optical range, digital communication on the order of gigabits per second is possible. In addition, FSO links are difficult to intercept, immune to interference or jamming from external sources, and are not subject to frequency spectrum regulations. FSO communications has received recent attention in applications such as satellite communications, fiber-backup, RF-wireless back-haul and last-mile connectivity [1].

The main drawback of the FSO channel is the detrimental effect the atmosphere has on a propagating laser beam. The atmosphere is composed of gas molecules, water vapor, pollutants, dust, and other chemical particulates that are trapped by Earth's gravitational field. Since the wavelength of a typical optical carrier is comparable to these molecule and particle sizes, the carrier wave is subject to various propagation effects that are uncommon to RF systems. One such effect is *scintillation*, caused by atmospheric turbulence, refers to random fluctuations in the irradiance of the received optical laser beam (analogous to fading experienced in RF systems) [2–4].

Recent works on the mitigation of scintillation concentrate on the use of multiple-lasers and multiple-apertures to create a multiple-input-multiple-output (MIMO) channel [5–13]. Many of these works consider scintillation as an ergodic fading process, and analyse the channel in terms of its ergodic capacity. However, compared to typical data rates, scintillation is a slow time varying process (with a coherence time on the order of milliseconds), and it is therefore more appropriate to analyse the outage probability of the channel. To some extent, this has been done in the works of [6, 10, 12–14]. In [6, 13] the outage probability of the MIMO FSO channel is analysed under the assumption of ideal photodetection (PD) (i.e. PD is modeled as a Poisson counting process) with no bandwidth constraints. Wilson *et al.* [10] also assume perfect PD, but with the further constraint of pulse-position modulation (PPM). Lee and Chan [12], study the outage probability under the assumption of on-off keying (OOK) transmission and non-ideal PD, i.e. the combined shot noise and thermal noise process is modeled as zero mean signal independent additive white Gaussian noise (AWGN). Farid and Hranilovic [14] extend this analysis to include the effects of pointing errors. Recently, the outage probability was analysed in [15] for the single-laser single aperture channel with PPM, subject to lognormal and exponential scintillation corresponding to weak and strong turbulence conditions, respectively.

In this paper, we extend the analysis of [15] to the MIMO case, under the assumption of equal gain combining (EGC) at the receiver. In addition to the lognormal and exponential cases, we also analyse the gamma-gamma case, which was recently proposed to model the scintillation for a wide range of turbulence conditions [16]. When perfect CSI is only known at the receiver (CSIR case), we show that the SNR exponent is proportional to the number of lasers times the number of apertures, times a channel related parameter, times the Singleton bound [17–19]. When perfect CSI is also known at the transmitter (CSIT case), we show that with the use of MIMO one need only to code over a single fading realisation to ensure positive *delay-limited capacity* [20]. These results highlight the benefits of MIMO and block diversity in reducing the outage probability in FSO systems.

The paper is organised as follows. In Section II, we define the channel model and assumptions. In Section III we review the lognormal, exponential and gamma-gamma scintillation models. Then in Sections IV and V we present the main results of our asymptotic outage probability analysis. Concluding remarks are then given in Section VI.

II. System Model

We consider an FSO system with M transmit lasers an N aperture receiver. Information data is first encoded by a binary code of rate R_c. The encoded stream is modulated according to a Q-ary PPM scheme, resulting in rate $R = R_c \log_2 Q$ (bits/channel use). Repetition transmission is employed such that the same PPM signal is transmitted in perfect synchronism by each of the M lasers through an atmospheric turbulent channel and collected by N receive apertures. We assume the distance between the individual lasers and apertures is sufficient so that spatial correlation is negligible. At each aperture, the received optical signal is converted to an electrical signal via PD. Non-ideal PD is assumed such that the combined shot noise and thermal noise processes can be modeled as zero mean, signal independent AWGN (an assumption commonly used in the literature, see e.g. [3–5, 12, 14, 21–26]).

In FSO communications, channel variations are typically much slower than the signaling period. As such, we model the channel as a non-ergodic block-fading channel, for which a given codeword of length BL *sees* only a finite number B of scintillation realisations [27, 28]. The received signal at aperture $1 \le n \le N$ can be written as

$$\boldsymbol{y}_b^n[\ell] = \left(\sum_{m=1}^{M} \tilde{h}_b^{m,n}\right)\sqrt{\tilde{p}_b}\,\boldsymbol{x}_b[\ell] + \tilde{\boldsymbol{z}}_b^n[\ell], \tag{1}$$

for $b = 1, \dots, B, \ell = 1, \dots, L$, where $\boldsymbol{y}_b^n[\ell], \tilde{\boldsymbol{z}}_b^n[\ell] \in \mathbb{R}^Q$ are the received and noise signals at block b, time instant ℓ and aperture n, $\boldsymbol{x}_b[\ell], \in \mathbb{R}^Q$ is the transmitted signal at block b and time instant ℓ, and $\tilde{h}_b^{m,n}$ denotes the scintillation fading coefficient between laser m and aperture n. Each transmitted symbol is drawn from a PPM alphabet, $\boldsymbol{x}_b[\ell] \in \mathcal{X}^{\text{ppm}} \triangleq \{\boldsymbol{e}_1, \dots, \boldsymbol{e}_Q\}$, where $\boldsymbol{e}_q$ is the canonical basis vector, i.e., it has all zeros except for a one in position q, the time slot where the pulse is transmitted. The noise samples of $\tilde{\boldsymbol{z}}_b^n[\ell]$ are independent realisations of a random variable $Z \sim \mathcal{N}(0, 1)$, and $\tilde{p}_b$ denotes the received electrical power of block b at each aperture in the absence of scintillation. The fading coefficients $\tilde{h}_b^{m,n}$ are independent realisations of a random variable $\tilde{H}$ with probability density function (pdf) $f_{\tilde{H}}(h)$. At the receiver, we assume equal gain combining (EGC) is employed, such that the entire system is equivalent to a single-input single-output (SISO) channel, i.e.

$$\boldsymbol{y}_b[\ell] = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} \boldsymbol{y}_b^n[\ell] = \sqrt{p_b}h_b\boldsymbol{x}_b[\ell] + \boldsymbol{z}_b[\ell], \tag{2}$$

where $\boldsymbol{z}_b[\ell] = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} \tilde{\boldsymbol{z}}_b^n[\ell] \sim \mathcal{N}(0, 1)$, and h_b, a realisation of the random variable H, is defined as the normalised combined fading coefficient, i.e. $h_b = \frac{c}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} \tilde{h}_b^{m,n}$, where $c = 1/(\mathbb{E}[\tilde{H}]\sqrt{1 + \sigma_I^2/(MN)})$ is a constant to ensure $\mathbb{E}[H^2] = 1$.[1] Thus, the total instantaneous received electrical power at block b is $p_b = M^2N\tilde{p}_b/c$, and the total average received SNR is $\mathsf{snr} \triangleq \mathbb{E}[h_b^2 p_b] = \mathbb{E}[p_b]$.

[1]Since we consider only the asymptotic behaviour of the outage probability, the specific normalisation is irrelevant and does not affect our results.

III. Scintillation Distributions

The scintillation pdf, $f_{\tilde{H}}(h)$, is parameterised by the *scintillation index* (SI), $\sigma_I^2 \triangleq \frac{\text{Var}(\tilde{H})}{(\mathbb{E}[\tilde{H}])^2}$. Under weak atmospheric turbulence conditions (defined as those regimes for which $\sigma_I^2 < 1$), the SI is proportional to the so called *Rytov variance* which represents the SI of an unbounded plane wave in weak turbulence conditions, and is also considered as a measure of the strength of the optical turbulence under strong-fluctuation regimes [4]. The distribution of the irradiance fluctuations is dependent on the strength of the optical turbulence. For weak turbulence, fluctuations are generally considered to be lognormal distributed, and for strong turbulence, exponential distributed [2, 29]. For moderate turbulence, the distribution of the fluctuations is not well understood, and a number of distributions have been proposed, such as the lognormal-Rice distribution, K-distribution and gamma-gamma distribution (see [4] and references therein). In this paper we focus on the lognormal, exponential, and gamma-gamma distributed scintillation. However, our analysis can be extended to the other aforementioned distributions.

For *lognormal distributed scintillation*,

$$f_{\tilde{H}}(h) = \frac{1}{h\sigma\sqrt{2\pi}} \exp\left(-(\log h - \mu)^2/(2\sigma^2)\right), \tag{3}$$

where μ and σ are related to the SI via $\mu = -\log(1 + \sigma_I^2)$ and $\sigma^2 = \log(1 + \sigma_I^2)$.

For *exponential distributed scintillation*,

$$f_{\tilde{H}}(h) = \lambda \exp(-\lambda h). \tag{4}$$

Note that this corresponds to the super-saturated turbulence regime, for which $\sigma_I^2 = 1$.

The *gamma-gamma distribution* arises from the product of two independent Gamma distributed random variables and [16],

$$f_{\tilde{H}}(h) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h^{\frac{\alpha+\beta}{2}-1}\,\mathrm{K}_{\alpha-\beta}(2\sqrt{\alpha\beta h}), \tag{5}$$

where $\mathrm{K}_\nu(x)$ denotes the modified Bessel function of the second kind. The parameters α and β are related with the scintillation index via $\sigma_I^2 = \alpha^{-1} + \beta^{-1} + (\alpha\beta)^{-1}$.

IV. Outage Probability Analysis with CSIR

The channel described by (2) under the quasi-static assumption is not information stable [30] and therefore, the channel capacity in the strict Shannon sense is zero. The codeword error probability of any coding scheme can be lower bounded by the information outage probability [27, 28],

$$P_{\text{out}}(\mathsf{snr}, R) = \Pr(I(\boldsymbol{p}, \boldsymbol{h}) < R), \tag{6}$$

where R is the transmission rate and [31],

$$I(\boldsymbol{p}, \boldsymbol{h}) = \frac{1}{B}\sum_{b=1}^{B} I^{\text{awgn}}(p_b h_b^2), \tag{7}$$

is the instantaneous mutual information for a given power allocation $\boldsymbol{p} = (p_1, \ldots, p_b)$ and vector channel realisation $\boldsymbol{h} \triangleq (h_1, \ldots, h_B)$. For PPM over an AWGN channel [21],

$$I^{\mathrm{awgn}}(\rho) = \log_2 Q - \mathbb{E}\left[\log_2\left(1 + e^{-\rho}\sum_{q=2}^{Q} e^{\left(\sqrt{\rho}(Z_q - Z_1)\right)}\right)\right],$$

where ρ is the SNR and $Z_q \sim \mathcal{N}(0,1)$ for $q = 1, \ldots, Q$.

For the CSIR case, we employ uniform power allocation, i.e. $p_1 = \ldots = p_B = \mathsf{snr}$. For codewords transmitted over B blocks, obtaining a closed form analytic expression for the outage probability is intractable. Even for $B = 1$, in some cases, for example the lognormal or gamma-gamma distributions, determining the exact distribution of H can be a difficult task. Instead, as we shall see, obtaining the asymptotic behaviour of the outage probability is substantially simpler. Towards this end, and following the footsteps of [19, 32], we derive the *SNR exponent*.

Theorem 4.1: The outage SNR exponent for a MIMO FSO communications system modeled by (2) is given as follows:

$$d^{\mathrm{ln}}_{(\log \mathsf{snr})^2} = \frac{MN}{8\log(1+\sigma_I^2)}\left(1 + \lfloor B\left(1 - R_c\right)\rfloor\right) \quad (8)$$

$$d^{\mathrm{exp}}_{(\log \mathsf{snr})} = \frac{MN}{2}\left(1 + \lfloor B\left(1 - R_c\right)\rfloor\right), \quad (9)$$

$$d^{\mathrm{gg}}_{(\log \mathsf{snr})} = \frac{MN}{2}\min(\alpha,\beta)\left(1 + \lfloor B\left(1 - R_c\right)\rfloor\right), \quad (10)$$

for lognormal, exponential and gamma-gamma respectively, where $R_c = R/\log_2(Q)$ is the rate of the binary code and

$$d_{(\log \mathsf{snr})^k} \triangleq -\lim_{\mathsf{snr}\to\infty}\frac{\log P_{\mathrm{out}}(\mathsf{snr}, R)}{(\log \mathsf{snr})^k} \quad k = 1, 2. \quad (11)$$

From (8)-(10) we immediately see the benefits of spatial and block diversity on the system. In particular, each exponent is proportional to: the number of lasers times the number of apertures, reflecting the spatial diversity; a channel related parameter that is dependent on the scintillation distribution; and the Singleton bound, which is the optimal rate-diversity tradeoff for Rayleigh-faded block fading channels [17–19].

Comparing the channel related parameters in (8)-(10) the effects of the scintillation distribution on the outage probability are directly visible. For the lognormal case, the channel related parameter is $8\log(1+\sigma_I^2)$ and hence is directly linked to the SI. Moreover, for small $\sigma_I^2 < 1$, $8\log(1+\sigma_I^2) \approx 8\sigma_I^2$ and the SNR exponent is inversely proportional to the SI. For the exponential case, the channel related parameter is a constant $1/2$ as expected, since the SI is constant. For the gamma-gamma case the channel related parameter is $\min(\alpha,\beta)/2$, which highlights an interesting connection between the outage probability and recent results in the theory of optical scintillation. For gamma-gamma distributed scintillation, the fading coefficient results from the product of two independent random variables, i.e. $\tilde{H} = XY$, where X and Y model fluctuations due to large scale and small scale cells. Large scale cells cause refractive effects that mainly distort the wave front of the propagating beam, and tend to steer the beam in a slightly different direction (i.e. beam wander). Small scale cells cause scattering by diffraction and therefore distort the amplitude of the wave through beam spreading and irradiance fluctuations [4, p. 160]. The parameters α, β are related to the large and small scale fluctuation variances via $\alpha = \sigma_X^{-2}$ and $\beta = \sigma_Y^{-2}$. For a plane wave (neglecting inner/outer scale effects) $\sigma_Y^2 > \sigma_X^2$, and as the strength of the optical turbulence increases, the small scale fluctuations dominate and $\sigma_Y^2 \to 1$ [4, p. 336]. This implies that the SNR exponent is exclusively dependent on the small scale fluctuations. Moreover, in the strong fluctuation regime, $\sigma_Y^2 \to 1$, the gamma-gamma distribution reduces to a K-distribution [4, p. 368], and the system has the same SNR exponent as the exponential case typically used to model very strong fluctuation regimes.

In comparing (8) to (9) and (10) we observe a striking difference. For the lognormal case (8) implies the outage probability is dominated by a $(\log(\mathsf{snr}))^2$ term, whereas for the other cases it is dominated by a $\log(\mathsf{snr})$ term. Thus the outage probability decays much more rapidly with SNR for the lognormal case than it does for the exponential or gamma-gamma cases. Furthermore, for the lognormal case, the slope of the outage probability curve, when plotted on a log-log scale, will not converge to a constant value. In fact, a constant slope curve will only be observed when plotting the outage probability on a log-$(\log)^2$ scale.

For the special case of single block transmission, $B = 1$, it is straightforward to express the outage probability in terms of the cumulative distribution function (cdf) of the scintillation random variable, i.e. $P_{\mathrm{out}}(\mathsf{snr}, R) = F_H(\sqrt{\mathsf{snr}_R^{\mathrm{awgn}}/\mathsf{snr}})$ where $F_H(h)$ denotes the cdf of H, and $\mathsf{snr}_R^{\mathrm{awgn}} \triangleq I^{\mathrm{awgn},-1}(R)$ denotes the SNR value at which the mutual information is equal to R. Therefore, for $B = 1$, we can compute the outage probability analytically when the distribution of H is available, i.e., in the exponential case for $M, N \geq 1$ or in the lognormal and gamma-gamma cases for $M, N = 1$. It is however possible to evaluate the distribution numerically using the fast-Fourier transform (FFT). This approach involves performing the FFT of the truncated distribution of $\tilde{H}$, raising it to the MN-th power and then computing the inverse FFT (IFFT).The accuracy of this method depends on the truncation, the sampling of the distribution as well as the number of FFT points. In any case, very accurate computations of the outage probability for $B = 1$ and $M, N \geq 1$ can be done in only a few seconds.

Outage probability curves for the $B = 1$ case are shown in Fig. 1 (left). For the lognormal case, we see that the curves do not have constant slope for large SNR, while, for the exponential and gamma-gamma cases, a constant slope is clearly visible. We also see the benefits of MIMO, particularly in the exponential and gamma-gamma cases, where the SNR exponent has increased from $1/2$ and 1 to 2 and 4 respectively.

V. Outage Probability Analysis with CSIT

In this section we consider the case where the transmitter and receiver both have perfect CSI knowledge. In this case, the transmitter determines the optimal power allocation that minimises the outage probability for a fixed rate, subject to a power constraint [33]. For the SISO case with CSIT, long-term and short-term power constraints were considered in [15], the results of which were based on the application of results from [34]. Since the MIMO channel with EGC can be considered as a SISO channel (as evidenced by (2)) the same optimal power allocation algorithms as described in [15] apply.

For a short-term power constraint P, such that $\frac{1}{B}\sum_{b=1}^{B} p_b \le P$, the optimal power allocation is given by mercury-waterfilling at each channel realisation [34, 35],

$$p_b = \frac{1}{h_b^2}\mathsf{mmse}^{-1}\left(\min\left\{1, \frac{\eta}{h_b^2}\right\}\right), \qquad (12)$$

for $b = 1, \ldots, B$ where $\mathsf{mmse}^{-1}(u)$ is the inverse-MMSE function and η is chosen to satisfy the power constraint. The MMSE for PPM was derived in [15] and can be computed via Monte Carlo simulations. From [34, Prop. 1] it is apparent that the SNR exponent for the CSIT case under short-term power constraints is the same as the CSIR case.

For a long-term power constraint P, such that $\mathbb{E}\left[\frac{1}{B}\sum_{b=1}^{B} p_b\right] \le P$ the optimal power allocation is [34]

$$\boldsymbol{p} = \begin{cases} \boldsymbol{\wp}, & \sum_{b=1}^{B} \wp_b \le s \\ \boldsymbol{0}, & \text{otherwise,} \end{cases} \qquad (13)$$

where

$$\wp_b = \frac{1}{h_b^2}\mathsf{mmse}^{-1}\left(\min\left\{1, \frac{1}{\eta h_b^2}\right\}\right),\ b = 1, \ldots, B \qquad (14)$$

and s is a threshold such that $s = \infty$ if $\lim_{s\to\infty} \mathbb{E}_{\mathcal{R}(s)}\left[\frac{1}{B}\sum_{b=1}^{B} \wp_b\right] \le P$, and

$$\mathcal{R}(s) \triangleq \left\{\boldsymbol{h} \in \mathbb{R}_+^B : \frac{1}{B}\sum_{b=1}^{B} \wp_b \le s\right\}, \qquad (15)$$

otherwise, s is chosen such that $P = \mathbb{E}_{\mathcal{R}(s)}\left[\frac{1}{B}\sum_{b=1}^{B} \wp_b\right]$. In (14), η is now chosen to satisfy the rate constraint

$$\frac{1}{B}\sum_{b=1}^{B} I^{\text{awgn}}\left(\mathsf{mmse}^{-1}\left(\min\left\{1, \frac{1}{\eta h_b^2}\right\}\right)\right) = R \qquad (16)$$

From [34], the long-term SNR exponent is given by

$$d^{\text{lt}}_{(\log \mathsf{snr})} = \begin{cases} \frac{d^{\text{st}}_{(\log \mathsf{snr})}}{1 - d^{\text{st}}_{(\log \mathsf{snr})}} & d^{\text{st}}_{(\log \mathsf{snr})} < 1 \\ \infty & d^{\text{st}}_{(\log \mathsf{snr})} > 1 \end{cases}, \qquad (17)$$

where $d^{\text{st}}_{(\log \mathsf{snr})}$ is the short-term SNR exponent. Note that $d^{\text{lt}}_{(\log \mathsf{snr})} = \infty$ implies the outage probability curve is vertical, i.e. delay-limited capacity [20] is positive. From (8) we see that $d^{\text{st}}_{(\log \mathsf{snr})} = \infty$ for the lognormal case, i.e. delay-limited capacity is always positive. Whereas for the exponential and gamma-gamma cases, from (9) and (10), we require $MN(1 + \lfloor B(1 - R_c)\rfloor) > 2$ and $MN\min(\alpha, \beta)(1 + \lfloor B(1 - R_c)\rfloor) > 2$, respectively. Otherwise delay-limited capacity is zero for these cases. Thus, for these cases, M, N, B and R_c need to be chosen carefully to ensure positive delay-limited capacity.

Single block transmission ($B = 1$) is most relevant in FSO communications since the coherence time is on the order of milliseconds which is large compared to typical data rates. In this case the solution (14) can be determined explicitly since $\eta = \left(h^2\mathsf{mmse}(I^{\text{awgn},-1}(R))\right)^{-1} = \left(h^2\mathsf{mmse}(\mathsf{snr}_R^{\text{awgn}})\right)^{-1}$. Therefore,

$$\wp^{\text{opt}} = \frac{\mathsf{snr}_R^{\text{awgn}}}{h^2}. \qquad (18)$$

Intuitively, (18) implies that for single block transmission, whenever $\mathsf{snr}_R^{\text{awgn}}/h^2 \le s$, one simply transmits at the minimum power necessary so that the received instantaneous SNR is equal to the SNR threshold ($\mathsf{snr}_R^{\text{awgn}}$) of the code. Otherwise, one does not transmit. Thus an outage occurs whenever $h < \sqrt{\mathsf{snr}_R^{\text{awgn}}/s}$ and hence $P_{\text{out}}(\mathsf{snr}, R) = F_H\left(\sqrt{\frac{\mathsf{snr}_R^{\text{awgn}}}{\gamma^{-1}(\mathsf{snr})}}\right)$ where $\gamma^{-1}(\mathsf{snr})$ is the solution to the equation $\gamma(s) = \mathsf{snr}$, $\gamma(s) = \mathsf{snr}_R^{\text{awgn}}\int_\nu^\infty \frac{f_H(h)}{h^2}\,dh$, where $\nu = \sqrt{\frac{\mathsf{snr}_R^{\text{awgn}}}{s}}$ [15].

Fig. 1 (right) compares the outage probability for the $B = 1$ CSIT case (with long-term power constraints) for each of the scintillation distributions. For the $MN = 1$ case we see that delay-limited capacity is positive only for the lognormal case, since for the other two distributions $d^{\text{st}}_{(\log \mathsf{snr})} < 1$. For the other cases, one must code over more blocks to ensure positive delay-limited capacity. When $MN = 4$, delay-limited capacity is positive in all three distribution cases since $d^{\text{st}}_{(\log \mathsf{snr})} > 1$. Note that the SNR threshold at which $P_{\text{out}} \to 0$ can be determined by computing the expectation $\mathsf{snr}_R^{\text{awgn}}\mathbb{E}\left[H^{-2}\right]$, and can also be determined explicitly for some cases [15]. Comparing the CSIR and CSIT cases (left and right curves) we can see that very large gains are possible when CSI is known at the transmitter.

VI. Conclusions

In this paper we have analysed the outage probability of the MIMO Gaussian FSO channel under the assumption of PPM and non-ideal PD. When CSI is known only at the receiver, we have shown that the SNR exponent is proportional to the number lasers and apertures, times a channel related parameter (dependent on the scintillation distribution), times the Singleton bound. When the scintillation is lognormal distributed, we have shown that the outage probability is dominated by a $(\log(\mathsf{snr}))^2$ term, whereas for the exponential and gamma-gamma cases it is dominated by a $\log(\mathsf{snr})$ term. When CSI is also known at the transmitter, we applied the techniques of [34] to show very significant power savings.

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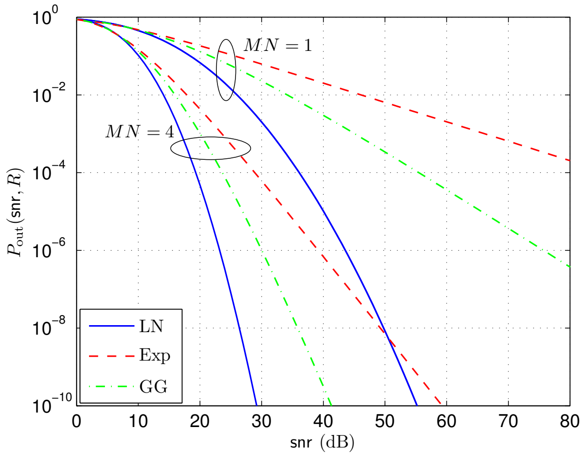

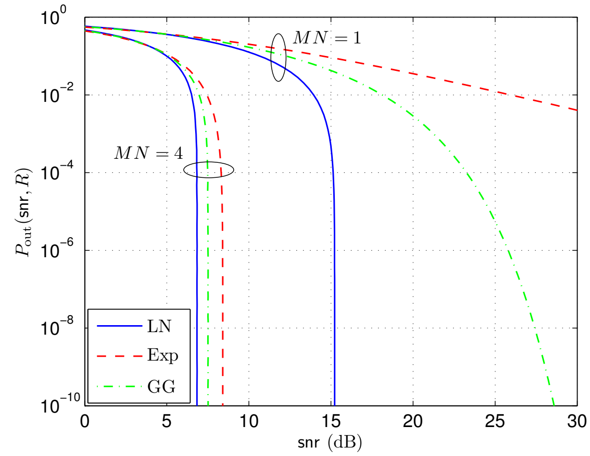

Fig. 1. Outage probability for lognormal (solid), exponential distributed (dashed) and gamma-gamma distributed scintillation (dot-dashed) with $\sigma_I^2 = 1$, $\alpha = 2$, $\beta = 3$, $B = 1$, $Q = 2$, $R_c = 1/2$, $\mathsf{snr}_{1/2}^{\rm awgn} = 3.18$ dB, and CSIR only (left) and CSIT (right).

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