Optimal Rate-Diversity-Delay Tradeoff in ARQ Block-Fading Channels

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Abstract — **We consider coded modulation schemes for the automatic-repeat-request (ARQ) block-fading channel. We show that the optimal SNR exponents of these schemes are given by a modified form of the Singleton bound that includes the effects of code rate and maximum ARQ delay. The role of maximum distance separable (MDS) codes over the ARQ block-fading channel is investigated and simulation results are presented, demonstrating that practical MDS codes achieve the optimal SNR exponent dictated by the Singleton bound.**

I. Introduction

The block-fading channel is a relevant channel model for data transmission over slowly-varying fading channels [1]. The duration of a block-fading period is determined by the channel coherence time. Within such a block-fading period, the fading channel coefficient remains constant, while between periods the channel coefficient change randomly according to a fading distribution. In this setting, transmission typically extends over multiple block-fading periods. This is a reasonable model for slow frequency hopping systems such as GSM, EDGE and orthogonal frequency division multiplexing (OFDM) modulation. Despite its simplicity, the model captures important aspects of slow fading channels and proves useful to develop coding design criteria.

In this paper, we consider an automatic-repeat-request (ARQ) system signaling over a block fading channel with L maximum allowable ARQ rounds and N fading blocks per round. In contrast to the work [2], we constrain the transmitter to *fixed rate*[2] codes constructed over complex signal constellations. The receiver is able to generate a finite number of one-bit repeat-requests, subject to a latency constraint, whenever an error is detected in the decoded message. The main focus of our work is to characterize the optimal diversity gain (or signal-to-noise ratio (SNR) exponent), defined as [3, 4]

$$d \triangleq -\lim_{\rho\to\infty} \frac{P_e(\rho)}{\log \rho}, \quad (1)$$

where ρ denotes the SNR and $P_e(\rho)$ denotes the probability that the transmitted message is decoded incorrectly, namely, the frame error rate (FER). We also investigate the system in terms of throughput and delay. We show that the optimal SNR exponent of the system can be upper bounded by a modified form of the Singleton bound, and that maximum distance separable (MDS) codes can achieve the Singleton bound over the ARQ block-fading channel. Finally, we demonstrate that while the optimal SNR exponent of the system is an increasing function of the maximum number of allowed ARQ rounds L, the throughput of the system becomes independent of L for sufficiently high SNR. This result provides strong incentive to use ARQ as a way to increase reliability without suffering code rate penalties.

The effect of introducing modulation constraints on ARQ systems has been recently investigated in [5]. The authors present the Singleton bound as an upper bound to the SNR exponent. The fundamental difference in our work is that we prove the optimality of the Singleton bound applied to ARQ systems. Further, we also demonstrate that asymptotically optimal throughput can be achieved by a class of codes that attains the optimal SNR exponent, namely, the MDS codes.

The following notation is used in the paper. Vectors and matrices are denoted by bold lower case and bold upper case letters, respectively. Sets are denoted by calligraphic fonts with the complement denoted by superscript c. The exponential equality $f(z) \doteq z^d$ indicates that $\lim_{z\to\infty} \frac{\log f(z)}{\log z} = d$. The exponential inequality $\dot{\leq}, \dot{\geq}$ are similarly defined. $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. Vector/matrix transpose is denoted by $'$ (e.g. $\mathbf{v}'$) and $\|\cdot\|_F$ is the Frobenius norm.

II. System model

Consider a single-input single-output (SISO) ARQ system. The transmission medium is modeled as a block-fading channel with coherence time denoted by T in terms of channel uses. We investigate the use of a simple stop-and-wait ARQ protocol where the maximum number of ARQ rounds is denoted by L. Each ARQ round consists of N independent block-fading periods and thus each ARQ round spans NT channel uses.

The information sequence to be transmitted is passed through an encoder with codebook $\mathcal{C}$ and code rate R_0, where $R_0 \triangleq \frac{R_1}{L}$ and $R_1 \triangleq \frac{1}{NT}\log_2|\mathcal{C}|$ is the code rate of the first ARQ round. The rate R_0 codeword is partitioned into a sequence of LN coded vectors, denoted $\mathbf{x}_{\ell,n} \in \mathbb{C}^T$, where $n = 1\dots N$ and $\ell = 1\dots L$. The transmitted codewords are normalized in energy such that $\forall \mathbf{x} \in \mathcal{C}$, $\frac{1}{LNT}\mathbb{E}[\|\mathbf{x}\|_F^2] = 1$.

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[2]Fixed rate codes operate at zero multiplexing gain (as defined in [3]).

The received signal at the nth block and ℓth ARQ round is written

$$\mathbf{y}_{\ell,n} = \sqrt{\rho} h_{\ell,n} \mathbf{x}_{\ell,n} + \mathbf{w}_{\ell,n}, \quad (2)$$

where $\mathbf{y}_{\ell,n}, \mathbf{w}_{\ell,n} \in \mathbb{C}^T$ and $h_{\ell,n} \in \mathbb{C}$ denote the received vector, the noise vector and the channel fading gain, respectively, and ρ denotes the average receive SNR. Both the channel fading gain $h_{\ell,n}$ and the elements of the noise vector $\mathbf{w}_{\ell,n}$ are assumed i.i.d. zero mean complex circularly symmetric complex Gaussian with normalized variance $\sigma^2 = 0.5$ per dimension. The channel coefficients are assumed to be perfectly known to the receiver.

Decoding begins following the reception of an ARQ round. If the received codeword can be decoded, the receiver sends back a one-bit acknowledgement signal to the transmitter via a zero-delay and error-free feedback link. The transmission of the current codeword ends immediately following the acknowledgment signal and the transmission of the next message in the queue starts. If an error is detected in the received codeword before the Lth ARQ round, then the receiver requests another ARQ round by issuing a one-bit negative acknowledgment along the perfect feedback path. A decision is always made at the end of the Lth ARQ round, regardless of whether errors are detected.

In general, the optimal ARQ decoder makes use of all available coded blocks and corresponding channel state information up to the current ARQ round in the decoding process. This leads to the concept of information accumulation, where individually incomplete data blocks are combined, along with any other side information. We hence introduce the ARQ channel model, completely analagous to (2), but allow for a more concise notation. The received signal up to the ℓth ARQ round is written

$$\widetilde{\mathbf{y}}_\ell = \sqrt{\rho} \widetilde{\mathbf{H}}_\ell \widetilde{\mathbf{x}}_\ell + \widetilde{\mathbf{w}}_\ell, \quad (3)$$

where

$$\begin{aligned}
\widetilde{\mathbf{y}}_\ell &= [\mathbf{y}'_{1,1}, \ldots, \mathbf{y}'_{1,N}, \ldots, \mathbf{y}'_{\ell,1}, \ldots, \mathbf{y}'_{\ell,N}]', \\
\widetilde{\mathbf{x}}_\ell &= [\mathbf{x}'_{1,1}, \ldots, \mathbf{x}'_{1,N}, \ldots, \mathbf{x}'_{\ell,1}, \ldots, \mathbf{x}'_{\ell,N}]', \\
\widetilde{\mathbf{w}}_\ell &= [\mathbf{w}'_{1,1}, \ldots, \mathbf{w}'_{1,N}, \ldots, \mathbf{w}'_{\ell,1}, \ldots, \mathbf{w}'_{\ell,N}]', \\
\widetilde{\mathbf{H}}_\ell &= \operatorname{diag}(h_{1,1}\mathbf{I}_T, \ldots, h_{1,N}\mathbf{I}_T, \ldots, h_{\ell,1}\mathbf{I}_T, \ldots, h_{\ell,N}\mathbf{I}_T).
\end{aligned}$$

That is, $\widetilde{\mathbf{y}}_\ell, \widetilde{\mathbf{w}}_\ell \in \mathbb{C}^{\ell NT}$ and $\widetilde{\mathbf{x}}_\ell \in \mathbb{C}^{\ell NT}$ are simply collections of the received vectors, the code vectors and the noise vectors, respectively, available at the end of the ℓth ARQ round, concatenated into block column vectors. The new channel matrix $\widetilde{\mathbf{H}}_\ell \in \mathbb{C}^{\ell NT \times \ell NT}$ is a diagonal matrix with the diagonal entries composed of the respective channel state during each block-fading period.

We will make use of the ARQ decoder proposed in [2], which behaves as a typical set decoder for the first $L-1$ ARQ rounds and finally performs ML decoding at the last ARQ round. Let $\mathcal{M} = \{1, 2, \ldots, 2^{R_0 LNT}\}$ denote the set of possible information sequences and let $\mathbf{x}(m) \in \mathbb{C}^{LNT}$ denote the rate R_0 codeword associated with message m. The decoding function at ARQ round ℓ, denoted $\psi_\ell(\widetilde{\mathbf{y}}_\ell, \widetilde{\mathbf{H}}_\ell)$, outputs the message index $\hat{m} \in \mathcal{M}$ whenever the received vector can be decoded and $\psi_\ell(\widetilde{\mathbf{y}}_\ell, \widetilde{\mathbf{H}}_\ell) = 0$ whenever errors are detected.

III. ARQ Throughput and Latency

To determine the average latency of the system, first let

$$\mathcal{A}_\ell \triangleq \left\{ \bigcup_{\hat{m} \neq 0} \psi_\ell(\widetilde{\mathbf{y}}_\ell, \widetilde{\mathbf{H}}_\ell) = \hat{m} \right\}, \quad (4)$$

denote the event of decoding a valid message at ARQ round ℓ. Further, let $q(\ell) \triangleq \Pr(\mathcal{A}_1^c, \ldots, \mathcal{A}_{\ell-1}^c, \mathcal{A}_\ell)$ and $p(\ell) \triangleq \Pr(\mathcal{A}_1^c, \ldots, \mathcal{A}_\ell^c)$ denote the probability of a frame being accepted at the ℓth ARQ round and the probability of a frame being rejected at the ℓth ARQ round, respectively. Then the expected latency of the system κ, expressed as the average transmitted ARQ rounds is given by

$$\kappa = 1 + \sum_{\ell=1}^{L-1} p(\ell). \quad (5)$$

Note that κ is derived based on the assumption of a code and decoder which is capable of outputting the correct message whenever the channel is not in deep fade. Therefore κ behaves as a lower bound for practical codes.

Next, we apply the renewal-reward theorem [6] to obtain an expression for the *transmit* throughput $\eta(R_1, L)$, where the transmit throughput is defined to be the average number of information bits transmitted per channel use (note this implies $R_0 \le \eta(R_1, L) \le R_1$). We recognize $\{\mathcal{A}_1^c, \ldots, \mathcal{A}_{\ell-1}^c, \mathcal{A}_\ell\}$, $\ell = 1, \ldots, L$ as the recurrent events and associate a reward of R_1 with every recurrent event. Note that the probability distribution of the recurrent event is given by $q(\ell)$. Finally, let the random time between two consecutive recurrent events be denoted by S (inter-renewal time), where the probability distribution of S is

$$\Pr(S = s) = \begin{cases} q(s) & 1 \le s \le L-1 \\ p(L-1) & s = L \\ 0 & \text{otherwise} \end{cases} . \quad (6)$$

The transmit throughput of the system is obtained by applying the renewal-reward theorem to get [2]

$$\eta(R_1, L) = \frac{R_1}{1 + \sum_{\ell=1}^{L-1} p(\ell)}, \quad (7)$$

where $\eta(R_1, L)$ is expressed in bits per channel use.

IV. Rate-diversity-delay tradeoff

At ARQ round ℓ, let

$$I_{\widetilde{\mathbf{G}}_\ell}(\widetilde{\mathbf{x}}_\ell; \widetilde{\mathbf{y}}_\ell) \triangleq \frac{1}{NT} I(\widetilde{\mathbf{x}}_\ell; \widetilde{\mathbf{y}}_\ell | \widetilde{\mathbf{H}}_\ell = \widetilde{\mathbf{G}}_\ell) \quad (8)$$

denote the normalized mutual information between the accumulated received vector $\widetilde{\mathbf{y}}_\ell$ and the coded blocks $\widetilde{\mathbf{x}}_\ell$, given the instantaneous channel state matrix $\widetilde{\mathbf{G}}_\ell$. $\widetilde{\mathbf{G}}_\ell$ is a random matrix and $I_{\widetilde{\mathbf{G}}_\ell}(\widetilde{\mathbf{x}}_\ell; \widetilde{\mathbf{y}}_\ell)$ is a non-negative random variable representing the instantaneous mutual information at ARQ round ℓ.

Following [6, Lemma 1], we get that for $|\mathcal{M}| = 2^{R_1 NT}$, there exists a codebook $\mathcal{C}$ such that the conditional probability of error $P_e(\rho|\widetilde{\mathbf{H}}_\ell) < \epsilon$ for any $\epsilon > 0$ whenever the instantaneous mutual information satisfies $I_{\widetilde{\mathbf{G}}_\ell}(\widetilde{\mathbf{x}}_\ell;\widetilde{\mathbf{y}}_\ell) \geq R_1$ (for any $\ell = 1,\dots,L$), provided that the equivalent block length NT is sufficiently large. We hence define information outage as the event when the instantaneous mutual information drops below R_1 (i.e. $I_{\widetilde{\mathbf{G}}_\ell}(\widetilde{\mathbf{x}}_\ell;\widetilde{\mathbf{y}}_\ell) < R_1$). The corresponding outage probability is then defined as

$$P_{\text{out}}(\rho,\ell,R_1) \triangleq \Pr\left(I_{\widetilde{\mathbf{G}}_\ell}(\widetilde{\mathbf{x}}_\ell;\widetilde{\mathbf{y}}_\ell) < R_1\right). \quad (9)$$

Further, we will refer to the channel as being in outage whenever the instantaneous channel state belongs to the outage region $\mathcal{O}_\ell \triangleq \left\{\widetilde{\mathbf{G}}_\ell \in \mathbb{C}^{\ell NT\times \ell NT} : I_{\widetilde{\mathbf{G}}_\ell}(\widetilde{\mathbf{x}}_\ell;\widetilde{\mathbf{y}}_\ell) < R_1\right\}$.

We now present the main results of this paper concerning the optimal SNR exponents of ARQ systems.

Theorem 1. *Consider the channel model* (3) *with input constellation satisfying the average power constraint* $\frac{1}{LNT}\mathbb{E}[\|\mathbf{x}(m)\|_F^2] \leq 1$. *The optimal SNR exponent* $d_G(N,L)$ *is given by*

$$d_G(N,L) = LN. \quad (10)$$

Further, this is achieved at all positive code rates by Gaussian random codes.

Proof. Theorem 1 follows immediately as a corollary of [3, Theorem 2] after taking into account the introduction of N in the system. □

Theorem 1 states that Gaussian codes achieve maximal diversity gain for any positive rate. As we show in the following, this is not the case with discrete signal constellations (PSK, QAM). In particular, due to the discrete nature of these signal sets, a tradeoff between rate and diversity arises.

Theorem 2. *Consider the channel model* (3) *with discrete input constellation* $\mathcal{X}$ *of cardinality* 2^Q *satisfying the average power constraint* $\frac{1}{LNT}\mathbb{E}[\|\mathbf{x}(m)\|_F^2] \leq 1$. *The optimal SNR exponent* $d_D(R_1,N,L,\mathcal{X})$ *is upper bounded by the modified Singleton bound*

$$d_D(R_1,N,L,\mathcal{X}) = 1 + \left\lfloor LN\left(1 - \frac{R_1}{LQ}\right)\right\rfloor. \quad (11)$$

Further, (11) *is achieved with random codes wherever* (11) *is continuous, provided that the block length grows sufficiently fast with SNR.*

Proof (Sketch). We first prove the converse and show that the diversity gain $d \leq d_D(R_1,N,L,\mathcal{X})$. We can use Fano's inequality to show that the outage probability P_{out} lower-bounds the error probability P_e for a sufficiently large block length. Then we bound the maximum SNR exponent by considering the diversity gain of the outage probability. For large SNR, the instantaneous mutual information is either zero or Q, corresponding to when the channel is in deep fade and when the channel is not in deep fade, respectively [4]. Achievability is proved by considering random codes coupled with the previously described ARQ decoder of [2], as well as the use of the union Bhattacharyya bound. For finite T, we obtain similar conditions to those in [4]. Finally, as $T\to\infty$, we show that the SNR exponent of random codes is given by the Singleton bound. □

The bound (11) is also applicable to any systems using block codes over LN independent block-fading periods. The significance of the ARQ framework is that it provides a way of achieving the optimal SNR exponent attained by a block code with LN coded blocks, without always having to transmit all LN code blocks. Indeed, following [2], observe that

$$\begin{aligned} p(\ell) &\triangleq \Pr(\mathcal{A}_1^c,\dots,\mathcal{A}_\ell^c) \\ &\leq \Pr(\mathcal{A}_\ell^c) \\ &= \Pr(\psi_\ell(\widetilde{\mathbf{y}}_\ell\widetilde{\mathbf{H}}_\ell) = 0) \\ &\leq P_{\text{out}}(\rho,\ell,R_1) + \epsilon \\ &\doteq \rho^{-d_D(R_1,N,\ell,\mathcal{X})}. \end{aligned} \quad (12)$$

On substitution of (12) into (7), we find

$$\eta(R_1,L) \dot{\geq} \frac{R_1}{1 + \sum_{\ell=1}^{L-1}\rho^{-d_D(R_1,N,\ell,\mathcal{X})}} \doteq R_1, \quad (13)$$

which shows that the transmit throughput is asymptotically equal to R_1 (since $R_1 \geq \eta(R_1,L)$), the rate of a single ARQ round. In other words, provided the SNR is sufficiently high, ARQ systems which send *on average* N coded blocks can achieve the same diversity gain as that achieved by a block code system which sends LN coded blocks *every time*. This is because in the high SNR regime, most frames can be decoded correctly with high probability based only on the first transmitted code block. ARQ retransmissions are used to correct the rare errors which occur almost exclusively whenever the channel is in outage. While the throughput $\eta(R_1,L)$ is a function of L at mid to low SNR, it converges towards R_1 independent of L at sufficiently high SNR. Since the optimal diversity gain is an increasing function of L, this behavior can be exploited to increase reliability without suffering code rate losses. However, as noted in [2], this behavior is exhibited only by decoders capable of near perfect error detection (PED). Therefore, the performance of practical error detection schemes can be expected to significantly influence the throughput of ARQ systems.

Examining the rate-diversity-delay tradeoff (11) in more detail, first note that $R_1/LQ = R_0/Q$ is the code rate of a binary code. i.e. $0 \leq R_0/Q \leq 1$. The expression (11) implies that the higher we set the target rate R_1 (equivalently, R_0), the lower the achievable diversity order. In particular, *uncoded* sequences (i.e. $R_1 = Q$) achieve an optimal diversity gain of $1 + \lfloor N(L-1)\rfloor$, while any code with non-zero $R_1 \leq Q$ (equivalently $R_0 \leq Q/L$) achieve optimal diversity less than or equal to LN. This is an intuitively satisfying result as LN is precisely the number of independent fading periods.

Figure 1-3 are graphs of the SISO tradeoff function with varying Q, N and L plotted against the rate of a single ARQ round R_1. First we examine the effect of the constellation size Q on the optimal diversity tradeoff function. Figure 1 shows the tradeoff curve for three different values of Q. We can see from the plot that the tradeoff curves for higher Q are strictly better than lower Q in terms of achievable diversity gain. This implies that a high-order modulation scheme always outperforms lower-order modulation schemes in the limit of high SNR in terms of error rate performance, for any code rate. Alternatively, a system with high Q can choose to operate at higher code rates than a low Q system and still maintain the same diversity gain.

Figure 2 shows the diversity tradeoff curve for different values of N. Similar to the previous tradeoff curve with constellation size Q, we observe that systems with high values of N are strictly better than systems with low N (in terms of diversity gain). In addition, we notice that N corresponds to the number of "steps" in the tradeoff function of (11). Systems with low values of N maintain the same diversity gain over wider intervals of rates than systems with high N. Relatively, the penalty for using codes with high spectral efficiency is much higher for systems with large N (although these systems will still achieve higher diversity gains than systems with low N).

Figure 3 illustrates the effect of the maximum number of allowed ARQ rounds L on the diversity of the system. It is clear from the plot that the effect of L is to simply shift tradeoff curves upwards. This is intuitively satisfying, since each additional ARQ round represents incremental redundancy, which can be considered as a form of advanced repetition coding. Each additional ARQ round contains N additional independent fading blocks and hence the diversity gain is simply $d_D(R_1, N, L+1, \mathcal{X}) = d_D(R_1, N, L, \mathcal{X}) + N$.

Having established the main effects of each parameter in (11), we now consider the practical coding aspects of Theorem 2. The SISO diversity function (11) can be viewed as a modified version of the Singleton bound [7] with the diversity gain corresponding to the Hamming distance. The number of independent fading blocks corresponds to the new block length while the constellation of the code can be thought of as containing $|\mathcal{X}|^T$ elements. The code rate is given by R_0/Q. This is a useful interpretation and naturally leads us to investigate the role of Singleton-bound-achieving MDS codes. The role of MDS codes as block codes in block-fading channel has been examined extensively in [4, 8]. In the following section, we make use of the MDS convolutional codes presented in [8] to illustrate the meaning of the diversity tradeoff curve in a practical sense.

V. Numerical results

Figure 4 illustrates the performance of two different ARQ systems. The first system has maximum number of ARQ rounds $L = 2, N = 1$ and code rate $R_0 = 1/2$ with BPSK signaling. The second system has maximum ARQ rounds $L = 4$, $N = 1$, code rate $R_0 = 1/4$ and also BPSK signaling. The 4-state $[5, 7]_8$ convolutional code is used for the first ARQ system and the 4-state $[5, 5, 7, 7]_8$ convolutional code is used for the second system. Both systems have $T = 100$. We apply the list Viterbi decoder proposed in [9] to perform joint error detection and decoding.

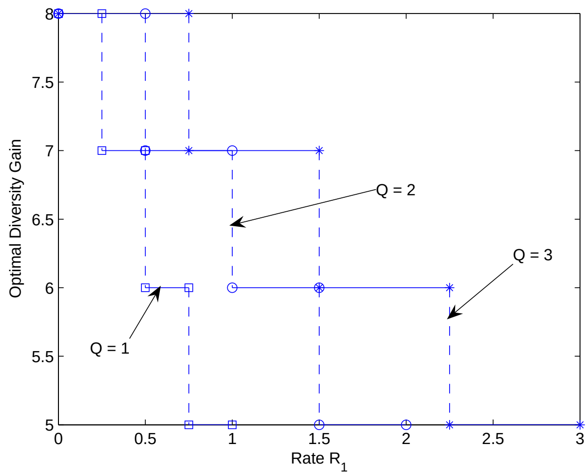

Figure 1: Optimal diversity tradeoff curve corresponding to $L = 2, N = 4$ for a SISO channel.

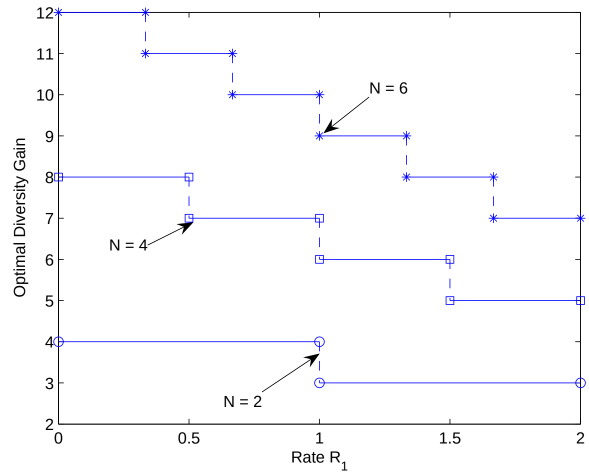

Figure 2: Optimal diversity tradeoff curve corresponding to $L = 2, Q = 2$ for a SISO channel.

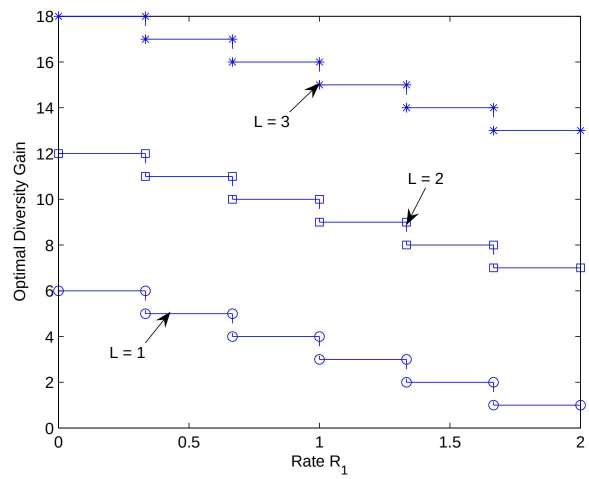

Figure 3: Optimal diversity tradeoff curve corresponding to $N = 6, Q = 2$ for a SISO channel.

Consider the first system, the top three curves in figure 4 show the corresponding outage probability, FER with list decoding and FER with PED. The FER curves are parallel to the outage curve at high SNR, which show that the convolutional MDS codes indeed achieve the optimal diversity gain. The second system corresponds to the bottom three curves of figure 4. Again, we see that the optimal diversity gain is achieved by the MDS convolutional code.

Comparing the two ARQ systems, it is clear that significant performance gains can be obtained at the expense of higher delays. At FER of 10^{-2}, the performance gain of the $L = 4$ system over the $L = 2$ system is already 5 dB. The performance gap increases even more dramatically at higher SNR.

Figure 5 shows the average ARQ rounds of the two ARQ systems considered above. For each system, we plot the average ARQ rounds with PED, and with the list decoder and the lower bound (LB) given by (5), respectively. It is clear from the plot that at medium to low SNR, significant loss in throughput is incurred by codes that do not approach the outage probability limit, like convolutional code. Even more loss in throughput is observed when list decoding is used as the error detection mechanism.

Finally, note that the average ARQ round curves converge towards one at high SNR. This agrees with (13) and shows that regardless of the maximum number of allowed ARQ rounds L, no spectral efficiency penalties are incurred at sufficiently high SNR. In the limit of high SNR, the transmit throughput $\eta(R_1, L) = R_1$.

VI. Conclusion

In this paper, we derived an expression for the optimal ARQ SNR reliability function over the block-fading channel. The discrete reliability function (11) characterizes the tradeoff between diversity gain, code rate, signal set and delay. We showed that ARQ transmissions can significantly increase the level of diversity in the system. Further, the additional diversity gain due to ARQ comes with no throughput or delay penalty at high SNR. We recognize the optimal SNR reliability function as the Singleton bound in a modified form which lead us to investigate the class of MDS codes. Finally, we showed via simulation that practical MDS codes can achieve the optimal SNR reliability function with low-complexity decoders on the ARQ block-fading channel.

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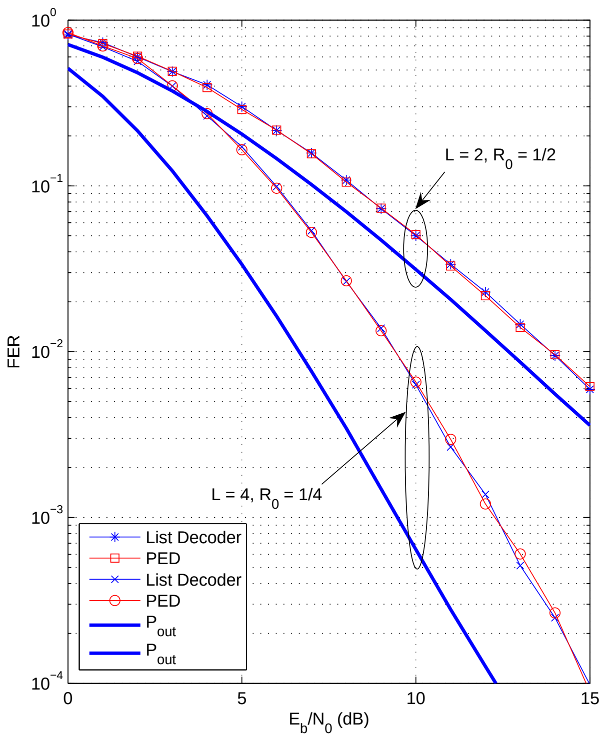

Figure 4: FER with MDS convolutional code over a SISO channel corresponding to $N = 1$, $Q = 1$ and $T = 100$.

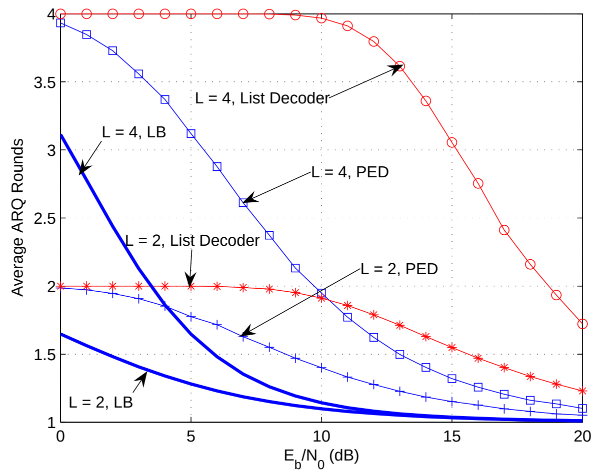

Figure 5: Average ARQ rounds of MDS convolutional code over a SISO channel.