Intertemporal Optimization: Experimental Evidence on Lifecycle Consumption

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Household Consumption

- The intertemporal problem faced by households in macroeconomic models is to choose, intertemporally, consumption and savings in the face of (possibly uncertain) future interest rates and income.
- Such household consumption choices are a critical component of business cycle acitivity; consumption accounts for more than 70% of GDP in the U.S. and in other developed countries.
- Understanding how households make such choices is therefore critical for stabilization policies.

The Lifecycle Model

- Due to Modigliani and Brunberg (1954) and Friedman (1957).
- The lifecycle model replaced an older behavioral-based theory due to Veblen (1899) and Duesenberry (1949), which emphasized peer effects, and the Keynesian consumption function, Keynes (1936), which posited the relevance of current, disposable income for consumption (and which we still teach to undergraduates today!).
- The lifecycle model has been studied experimentally, in the laboratory, primarily as an individual choice experiment under both certainty and uncertainty.

The Lifecycle Model under Certainty

The basic model is:

$$\max_{\{c_t\}} \sum_{t=1}^{T-1} \frac{u(c_t)}{(1+\delta)^t}$$

subject to:

$$\begin{aligned} a_{t+1} &= (1+r)(a_t + y_t - c_t), \text{for } t = 0, 1, \cdots, T-1 \\ a_0 & \; \textit{given} \\ a_T &= 0 \;\; \text{(no bequest motive)} \end{aligned}$$

Here, c_t is consumption at date t, $u(\cdot)$ is a concave utility function, δ is the subjective rate of time preference, a_t is wealth and y_t is income at time t, and r is the rate of interest on savings.

The Lifecycle Model under Certainty

The lifecycle budget constraint can be written as:

$$\sum_{t=0}^{T-1} \frac{c_t}{(1+r)^t} = a_0 + \sum_{t=0}^{T-1} \frac{y_t}{(1+r)^t}$$

The problem can be solved using the method of Lagrange:

$$\max L = \sum_{t=0}^{T-1} \frac{u(c_t)}{(1+\delta)^t} + \lambda \left(a_0 + \sum_{t=0}^{T-1} \frac{y_t}{(1+r)^t} - \sum_{t=0}^{T-1} \frac{c_t}{(1+r)^t} \right)$$

First order condition:

$$u'(c_t) = \lambda \left(\frac{1+\delta}{1+r} \right)^t$$

Euler equation linking consumption in adjacent periods

$$u'(c_t) = \frac{(1+r)}{(1+\delta)} u'(c_{t+1}) \;\; (\text{if } r = \delta,\; c_t = \bar{c} \;\forall t)$$

$T-1$ Euler equations together with the budget constraint determine $\{c_t\}_{t=0}^{T-1}$

Carbone and Duffy (2014)

- Study the certainty case.
- Two, $T = 25$ period lifecycles (both the same).
- $u(c) = \kappa - \frac{1}{R}e^{-Rc}$, $R = .10$ is coefficient of absolute risk aversion
- $\delta = 0$, $r = .05$, $y_t = y = 10$ for all $t = 1,2,\ldots 25$, $a_0 = 0$

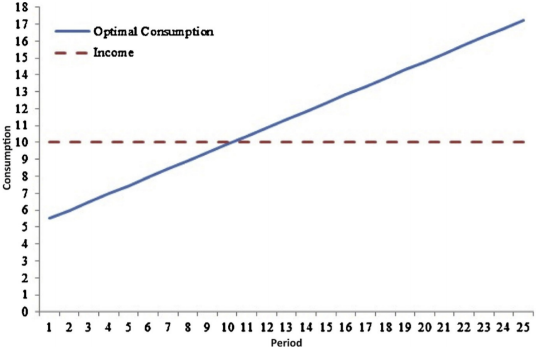


Figure: Optimal Consumption in Carbone and Duffy 2014

Overconsumption followed by underconsumption

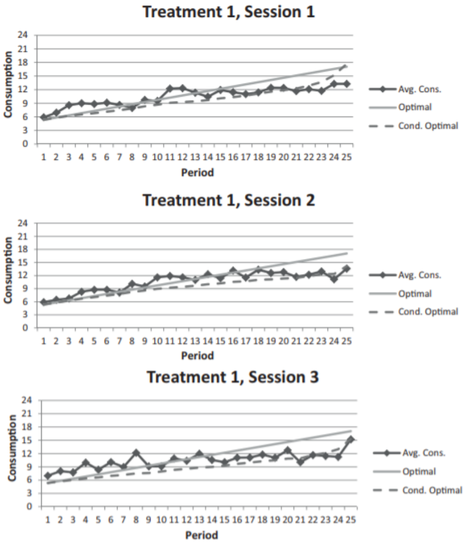


Figure: 3 Sessions, 10 Subjects Per Session, 2nd 25 Period Lifecycle

Consumption is significantly different:

- From the *un*conditionally optimal path.
- From the *conditionally* optimal path.
  - Subject i enters period t with cash on hand (COH) that consists of $y + (1+r)a^i_{t-1}$.
  - We thus treat each individuals COH for period t as though it were the initial wealth level that a subject brought to solving a reduced, $T - t + 1$ period consumption planning problem, and we calculate the optimal consumption and savings plan for subject i conditional on subject i's COH^i_t, as of the start of period t.
  - In the final period T, it is optimal for all subjects to consume all of their COH_T.
  - In essence, we use only the current period optimal consumption amount, c^{i*}_t, given the current period COH^i_t as our measure of the conditionally optimal consumption amount for subject i and using this value we calculate the MSD, $(c^i_t - c^{i*}_t)^2$ to evaluate the fit.

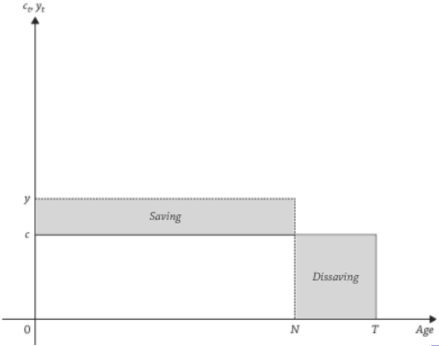
Saving for Retirement

Suppose:

$$y_t = \begin{cases} y & \text{for } t < N \\ 0 & \text{for } N \leq t \leq T \end{cases}$$

(This is the Modigliani-Brunberg variant). Suppose that $r = \delta = 0$ so $c_t = c \; \forall \; t$. Then,

$$c = \frac{a_0 + Ny}{T} = \frac{N}{T}y + \frac{1}{T}a_0$$

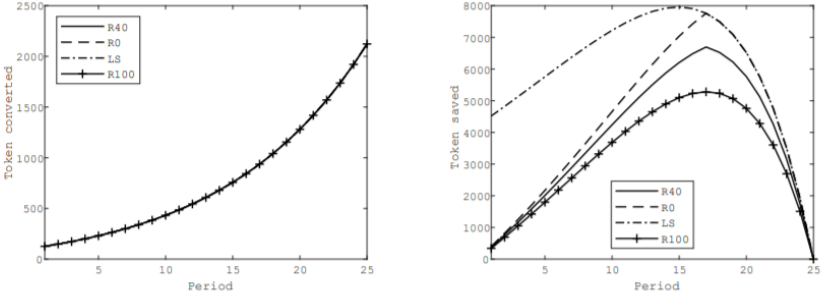


Duffy and Li (2019)

Explore lifecycle consumption/savings under different income profiles.

- Each period represents 2.3 years and $T = 25$: begin life at age 23, and exit at age 79; first 17 periods=ages 23-60=worker, last 8 periods=ages 61-79=retirees
- $u(c) = 0.2\ln(0.01c + 1)$
- $r = 0.10$: annual real return of 4.5 percent
- The present value of endowments is the **same** for all four treatments
  - R40 (benchmark, 40% replacement rate): $y = 500$ for $t = 1, 2, \cdots, 17$, and 200 for $t = 18, 19, \cdots, 25$
  - R0 (0% replacement rate): $y = 526$ for $t = 1, 2, \cdots, 17$, and 0 for $t = 18, 19, \cdots, 25$
  - LS (lump-sum): $y = 4,644$ for $t = 1$, 0 for $t = 2, 3, \cdots, 25$
  - R100 (100% replacement rate): $y = 465$ for $t = 1, 2, \cdots, 25$

Optimal Decisions Implied by Rational Choice Theory

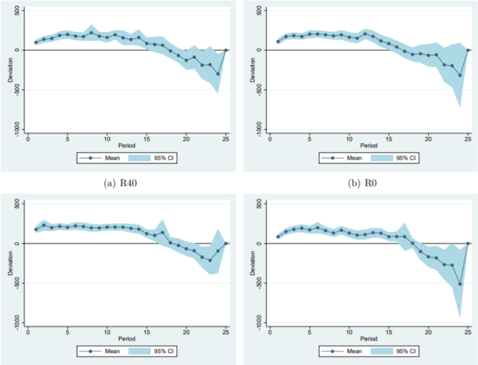


(a) Consumption

(b) Assets

Figure 1: Optimal decisions implied by standard rational choice theory

Duffy and Li findings



(a) R40 (b) R0

(c) LS (d) R100

Figure: Average consumption deviation from the conditionally optimal path by treatment

A rational inattention model, where subjects consider whether to deviate from the hand-to-mouth consumption heuristic or adopt the conditionally optimal strategy provides a good fit to the data.

Lifecycle model with uncertain income

Carbone and Hey (2004), Carbone (2006), Ballinger et al. (2003, 2011), Brown et al. (2008), Meissner (2016)
More complicated, but realistic setting where income is unknown.

Effect of Unemployment on Consumption: Carbone and Hey (2004)

- $T = 25$ periods.
- Savings earn interesr rate r.
- Two income levels, High $y =$ employed $>$ Low z $=$unemployed.
- Markov transition process for remaining (becoming) employed was p(q), and these probabilities were made known to subjects,

t \ $t+1$	y	z
y	p	$1-p$
z	q	$1-q$

- Induced CARA utility function.
- Solution via guess and verify numerical procedure!

$$
\begin{aligned}
c &= 0 \text{ if } a + bW \leq 0 \\
c &= a + bW \text{ if } 0 < a + bW < W \\
c &= W \text{ if } W < a + bW
\end{aligned}
$$

Carbone and Hey (2004) Findings

- Subjects adopt a shorter horizon than is optimal.
- Regression analysis of how consumption responds to changes in p, q, r, and y/z (z is held constant):

Change (Δ) in treatment variable	Unemployed		Employed	
(from low value to high value)	Optimal	Actual	Optimal	Actual
Δp (Pr. remaining employed)	5.03	23.64	14.57	39.89
Δq (Pr. becoming employed)	14.73	-1.08	5.68	0.15
Δ ratio high-low income	0.25	0.24	0.43	0.76

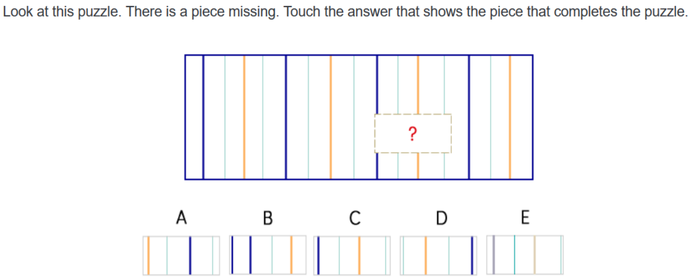
Table: Average Change in Consumption in Response to Parameter Changes and Conditional on Employment Status, taken from Carbone and Hey (2004,Table 5).

Precautionary Savings / Intergenerational Learning, Ballinger et al. 2003

- Precautionary motives for savings arise from either a convex marginal utility of consumption (third derivative of the utility function is positive, "prudence") or from strict borrowing constraints.
- Ballinger et al. use an induced CARA function to get the precautionary savings motive.
- Main treatment variable concerns the variance of the stochastic income process (high or low), which affects the extent of precautionary savings.
- In the high case they also explore the role of allowing communication/mentoring or not (while maintaining observability of actions by overlapping cohorts at all times).
- Overconsumption and undersaving. But savings are greater in the high as compared with the low variance case which is consistent with the precautionary savings rational choice prediction.
- Most interestingly, the consumption behavior of generation 3 is significantly closer to the optimal consumption program than in the consumption behavior of generation 1 suggesting that social learning by observation plays an important role, and may be a more reasonable characterization of the representative agent.

Cognitive ability and intertemporal optimization Ballinger et al. 2011

- Focus on whether cognitive and/or personality measures might account for the observed heterogeneity in subject's savings behavior, in particular, their use of shorter-than optimal planning horizons.
- Use various cognitive and personality tests, e.g. Raven's progressive matrix test:



Ballinger et al. 2011 findings

- Using a careful multivariate regression analysis that accounts for potentially confounding demographic variables, they report that cognitive measures and NOT personality measures are good predictors of heterogeneity in savings behavior.
- In particular, they report that variations in subjects' cognitive abilities as assessed using visually oriented "pattern completion" tests and "working memory" tests that assess a subject's ability to control both attention and thought, can explain variations in subject lifecycle savings behavior, and that the median subject is thinking just three periods ahead.

Habit Formation/ Private or Social Learning: Brown et al. 2008

- Objective is to maximize

$$E\sum_{s=t}^{T} u(c_t, h_{t-1})$$

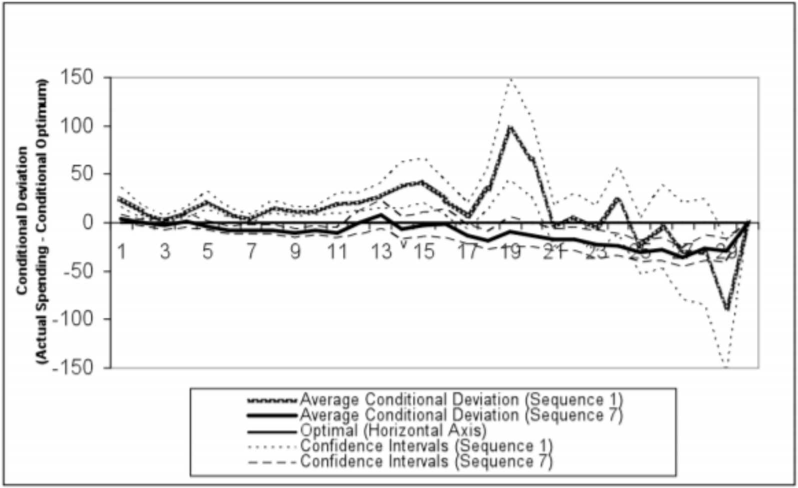
subject to

$$\begin{aligned}
c_t &\leq s_t + y_t \\
s_t &= s_{t-1} + y_{t-1} - c_{t-1} \\
y_t &= P_t\eta_t, \log\eta_t \sim N(-1/2,\ 1) \\
P_t &= (1+\mu)P_{t-1} \\
h_t &= \lambda h_{t-1} + c_t, \quad \lambda < 1
\end{aligned}$$

- where $u(c_t, h_{t-1}) = \frac{\theta}{1-\rho}\left(\frac{c_t}{h_{t-1}^{\gamma}}\right)^{1-\rho}$
- $r = \delta = 0$, and $T = 30$

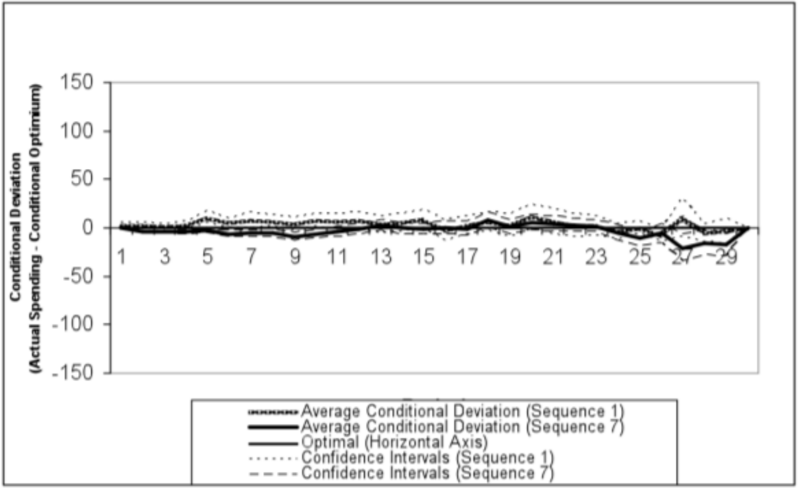
Private Learning Deviations

Figure III: Deviations from conditional optima, lifecycle 1 and 7, private learning



Learning from Others (Social Learning) Deviations

Figure IV: Deviations from conditional optima, lifecycle 1 and 7, social learning



Debt aversion, Meissner (2016)

Meissner (2016) modifies the finite horizon, lifecycle planning environment to allow subjects to borrow and not just to save.

- Two treatment orders
- Order 1: 3 20-period lifetimes with an upward sloping stochastic income profile followed by 3 20-period lifetimes with a downward sloping stochastic income process.
- Order 2: 3 20-period lifetimes with a downward sloping stochastic income profile followed by 3 20-period lifetimes with an upward sloping stochastic income process.
- Optimal policy is to borrow when young in the upward sloping case (borrow first) and save when young in the downward sloping profile (save first).
- Subjects seem adverse to borrowing but not to saving. Note interest rate on borrowing/saving is 0 in the experiment.

Meissner 2016 Results

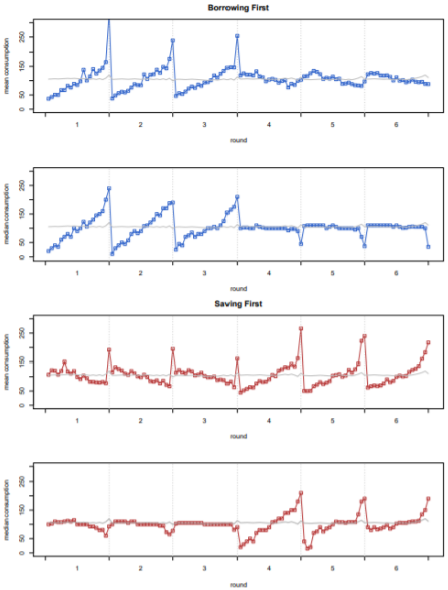


Figure 1: Mean and median consumption over all subjects in the Borrowing First and Saving First conditions.

Exponential Growth Bias: Levy and Tasoff (2016)

- Specifically, agent's perception of an asset is divided into two accounts: a fraction $0 < \alpha < 1$ grows with compounding interest and a fraction $1 - \alpha$ grows with simple interest.
- The perception of the future period-T value of one dollar invested at time $t \leq T$ is given by:

$$p(i, t, \alpha) = \prod_{s=t}^{T-1} (1 + \alpha i_s) + \sum_{s=t}^{T-1} (1 - \alpha) i_s$$

- Lifecycle problem is distorted as a result:

$$\max \sum_{t=0}^{T} \delta^t u(c_t)$$

subject to:

$$\sum_{t=0}^{T} c_t \cdot p(i, t, \alpha) \leq \sum_{t=0}^{T} y_t \cdot p(i, t, \alpha)$$

- Rational type has $\alpha = 1$.

Exponential Growth Bias: Levy and Tasoff (2016)

- Consequently, the consumer misperceives the value of his income over time. With positive interest rates, the consumer overestimates the value of future income.
- Also, agents misperceive the relative prices of consumption over time.
- The extent of the bias, α, is measured by incentivized questions giving subjects a choice between two assets "Asset A that has an initial value of \$100 and grows at an interest rate of 10% each period" and "Asset B that has an initial value of \$ X and does not grow." What value of X which would make the two assets equal value after 20 periods?
- Payment is based on accuracy of answer to X.
- They find that 85% of the population has an α between 0 and 1 The median is 0.53, and the mean is 0.60
- Underestimation of exponential growth can possibly account for under-saving.