Discount Window Stigma:
An Experimental Investigation

"*In August 2007,… banks were reluctant to rely on discount window credit to address their funding needs. The banks' concern was that their recourse to the discount window, if it became known, might lead market participants to infer weakness -the so-called stigma problem.*" Bernanke (2009)

Abstract

A core function of central banks is to act as a "lender of last resort" to the banking system. In the U.S., the Federal Reserve (the Fed) uses the Discount Window (DW) to fulfill this task. Historically, however, the DW has been little used, even when banks faced acute liquidity shortages. This lack of DW borrowing is commonly attributed to stigma, as illustrated by Chairman Bernanke's quote above. The economic consequences of DW stigma may be severe for the financial system (e.g. fire-sales, excessive self-insurance, failures). Further, DW stigma may prevent central banks from effectively providing emergency liquidity and implementing monetary policy. Despite its relevance, DW stigma is not a well-understood phenomenon, in large part because of the lack of data. In this paper, we use lab experiments to generate data in order to better understand DW stigma and how to eliminate it. To the best of our knowledge, this is the first paper that relies on experimental methods to address DW stigma and more generally the provision of emergency liquidity by central banks.

Keywords: Lender-of-last-Resort, Central Bank, Stigma, Experiment

JEL codes: E58 G01 C92

Introduction

A core function of central banks is to act as a "lender of last resort" to the banking system. In principle, solvent but illiquid banks should be able to obtain funding from banks with excess liquidity in the interbank market (Selgin 1993). When the interbank market becomes dysfunctional, however, even solvent banks may become unable to meet their funding needs. In such cases, most central banks supply emergency credit, in a targeted manner, to illiquid institutions.[1] In the U.S., the Federal Reserve (the Fed) uses the Discount Window (DW) to fulfill this task. Historically, however, the DW has been little used, even when banks faced acute liquidity shortages. For example, despite several measures enacted by the Fed to encourage DW borrowing, banks scarcely accessed the DW at the onset of the 2007 financial crisis. This lack of DW borrowing is commonly attributed to stigma, as illustrated by Chairman Bernanke's quote above.

DW stigma is defined as a reluctance to access the DW out of concerns that, if detected, depositors, creditors, or analysts could interpret DW borrowing as a sign of financial weakness.[2] As argued by e.g. Madigan (2009) and Duke (2010), the economic consequences of DW stigma may be severe. First, a bank may delay access to the DW resorting instead to costly alternatives (e.g. fire-sales of assets), which may further weaken the bank and increase financial instability. Second, DW stigma could lead banks to excessive self-insurance against tail-risks thereby reducing the loans extended to the real economy. Third, DW stigma may prevent central banks from effectively providing emergency liquidity (as illustrated by Chairman Bernanke's quote above).[3] Fourth, DW stigma may prevent the implementation of monetary policy because a hard ceiling on overnight interbank market rates cannot be guaranteed (Keister 2012, Kahn 2010).[4]

Despite its relevance, DW stigma is not a well-understood phenomenon. In fact, empirical evidence of DW stigma has been scarce (see Armantier et al. 2015).[5] The major challenge

[1] Flannery (1996), Freixas et al. (1999), Berger, Davies, and Flannery (2000) and Rochet and Vives (2004) discuss reasons why central banks may be in a better position to provide liquidity than private lenders in the interbank market for funds. For an opposite view see Goodfriend and King (1988).

[2] DW stigma may exist with respect to market participants or with respect to the Fed if banks are concerned that borrowing from the DW sends a negative signal to the Fed which acts both as lender and a regulator (Furfine 2003). In this paper, we focus on DW stigma with respect to the market.

[3] Winters (2012) argues that stigma limited the effectiveness of the Bank of England (BoE) emergency liquidity facility during the 2008 financial crisis and resulted in substantial costs to the financial system.

[4] The fed, along with many central banks, implement monetary policy with a corridor system. The floor of the corridor is the standing deposit facility rate (or the interest on excess reserves in the Fed's case) and the ceiling is the standing borrowing facility (or the DW rate in the Fed's case). In principle, banks have no incentives to borrow above the ceiling or lend below the floor. Thus, the corridor system enables central banks to limit fluctuations in the overnight interbank market rate. As argued by (e.g. Kahn 2010), however, the ceiling becomes "leaky" when there is DW stigma, in which case the central bank loses its ability to moderate the market rate fluctuations around the target rate.

[5] The challenges in formally establishing DW stigma may be best illustrated by the 2009 court's ruling in the Freedom of Information Act lawsuit filed by Bloomberg News seeking to obtain the identity of financial institutions that received loans from the Fed during the 2007-2008 crisis. While the Fed's officials argued that releasing the identities of DW borrowers would stigmatize banks and impede the Fed's ability to respond to

to study DW stigma is the lack of data: If financial institutions do not access the DW for fear of stigma, then analysts have no data to study the phenomenon. Using lab experiment is therefore an effective way to generate data that may help better understand DW stigma and how to eliminate it. To the best of our knowledge, this is the first paper that relies on experimental methods to address DW stigma and more generally the provision of emergency liquidity by central banks.

To be completed

Discuss somewhere necessary conditions for DW stigma: 1) DW borrowing must be observed publically (at least on occasion), 2) learning that a bank borrowed at the DW must be an informative signal that the bank is bad/insolvent. This rules out the assumption that the DW is accessible only to solvent but illiquid banks. If that was the case then DW borrowers should not be stigmatized (non DW borrowers should be). This also rules out the possibility that actions taken by the banks that should access the DW but do not (e.g. fire sale of assets) should not be observed

Discuss related literature.

See Appendix 0 for Institutional Background

The Baseline Model

We propose a simple model, easily implementable in the lab. The model captures the essence of emergency lending and DW stigma. Unlike (e.g.) Ennis and Weinberg (2013), we take a partial equilibrium perspective and do not model the interbank market for funds.

The setting

There are two populations of players: banks and investors. A bank is matched randomly with an investor. The players move sequentially. The bank moves in period 1. The investor moves in period 2. Players are risk neutral and do not discount the future. We focus only on pure strategies.

The investor has a stake in the bank in the form of an asset.[6] The value of the asset to the investor at the end of the game is either V or 0. The bank is said to be insolvent when the

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future crises (Madigan 2009, McLaughlin 2009), Bloomberg's lawyers countered that these assertions were "speculative" and "factually unsupported" (Bloomberg 2009, Glasser et al. 2011). In her ruling in favor of Bloomberg News, the chief federal district judge stated that the Fed had failed to meet its burden of proving that stigma was a real phenomenon (Preska 2009).

[6] The stake may be interpreted as a line of credit or a loan rolled over from period 1 to period 2.

asset is worth 0, and it is said to be solvent when the asset is worth V. The value of the asset is known to the bank at the beginning of the game, but it is revealed to the investor only at the end of the game.

At the beginning of the game, a bank may receive a negative liquidity shock, in which case it is said to be illiquid.[7] An illiquid bank must choose between borrowing 1 unit of liquidity at the DW at a cost dw, or incurring a loss $c > dw$.[8] We say there is DW stigma when an illiquid bank chooses not to borrow at the DW. We also call $c - dw$ the DW premium.

To simplify, we assume that all insolvent banks are illiquid. Thus, there are only three types of banks: solvent-liquid with probability P_{SL}, solvent-illiquid with probability $P_{S\bar{L}}$ and insolvent-illiquid with probability $P_{\bar{S}\bar{L}} = 1 - P_{S\bar{L}} - P_{SL}$.[9]

In period 1 an illiquid bank decides whether or not to borrow at the DW.[10] In period 2, the investor decides whether or not to keep its stake in the bank until the end of the game.[11] To simplify, we say that the investor either funds or does not fund the bank. When the investor does not fund the bank, the bank effectively ceases to operate. In this case, the investor gets a termination value $v < V$ and the bank gets 0 in period 2. If the investor funds the bank, then the investor gets the value of the asset (i.e. 0 or V) and the bank gets a continuation profit $k > 0$ from operating in period 2.[12]

[7] In practice, the liquidity shock may be caused by a set of depositors who suddenly withdraw their deposits or by a counterparty who fails to send a payment in time. For an example of the latter see "*Barclays admits borrowing hundreds of millions at Bank's emergency rate*," The Guardian, Aug 30, 2007.

[8] The liquidity shock here is net of what the bank can obtain in the interbank market. In other words, the unit of liquidity can be obtained only at the DW. Similar to Ennis and Weinberg (2013) we interpret c as a combination of pecuniary (e.g. overdraft in the bank's account at the central bank) and non-pecuniary (e.g. loss of reputation) elements.

[9] The assumption that a solvent bank may be illiquid reflects some of the rigidities in the interbank market. For instance, Ennis and Weinberg (2013) argue that solvent banks may need to borrow at the DW because they cannot always find a counterparty in the over-the-counter market. Moreover, we do not need to assume that all insolvent banks are illiquid. All we need is the standard assumption that the probability to be illiquid (i.e. to have a need to borrow at the DW) is higher for insolvent banks than for solvent banks.

[10] Following Ennis and Weinberg (2013), we assume that the central bank is not able to distinguish insolvent from illiquid banks, and thus lends to every bank at the DW at the same interest rate. In practice, the Fed is not supposed to lend to insolvent banks, but distinguishing insolvent from illiquid banks is difficult. Here, we assume it is impossible. Adding a probability that a DW loan to an insolvent bank can be declined complicates the model and does not change the equilibria discussed below. Further, the inability for the central bank to distinguish insolvent from illiquid banks is a necessary condition to have DW stigma (as discussed by Ennis and Weinberg 2013).

[11] In practice, pulling its stake out could mean that the investor cut the line of credit extended to the bank or stop rolling over a loan. In this case the investor gets back the loan's principal but looses on the interest it could have collected at the end of the game if the bank turns out to be solvent.

[12] An equivalent interpretation of the model (adopted by e.g. Ennis 2016) is that a firm can embark on a risky project for which it needs funding. The investor decides whether or not to fund the project. If the investor funds the project, then the investor gets either 0 or V, and the firm gets k. If the investor does not fund the project, then the investor gets an outside option $v < V$ and the firm gets 0.

In principle, a bank should be able to access the DW anonymously. In practice however, DW borrowings have been made public on occasions.[13] We denote θ the probability that a bank's DW borrowing is observed. This information is then used by the investor to update its beliefs about the value of the bank's asset. Thus, there is an indirect cost to DW borrowing because, if observed by the investor, the bank appears more likely to be insolvent to, in which case it may not be funded in Period 2. We say that a DW borrower is "stigmatized" when the investor funds only banks not seen borrowing at the DW.

Restrictions on the parameters

We impose the following restrictions on the model's parameters:

- *R1* : $0 < dw < c < k \le v < V$
- *R2* : $\underline{P} < \frac{v}{V} < p$, where $\underline{P} = \frac{P_{S\bar{L}}}{P_{S\bar{L}}+P_{\bar{S}\bar{L}}}$ and $p = 1 - P_{\bar{S}\bar{L}}$

R1 guarantees in particular that i) DW borrowing is not strictly dominated ($dw < c$), ii) it is profitable for a bank to operate in period 2 even after incurring the cost c ($k > c$), and iii) it is socially inefficient to fund an insolvent bank ($k \le v$).[14] Because $\underline{P}$ is the probability that a bank is solvent conditional on being illiquid, the first inequality in *R2* guarantees that ex-ante the investor should not fund a bank that is known to be illiquid ($v > \underline{P}V$). Because p is the unconditional probability that a bank is solvent, the second inequality in *R2* guarantees that it is ex-ante beneficial for the investor to fund a bank.[15]

Equilibrium

Let us define the following two pairs of strategies, the stigma strategy ***St*** and the non stigma strategy ***NoSt***:

St= {No bank borrows at the DW; Investors fund only banks not seen borrowing at the DW}

[13] For instance, during the 2007-2008 financial crisis, the press reported that Deutsche Bank had accessed the DW on the previous business day (see "*Fed fails to calm money markets*," The Financial Times, August 20, 2007). Similarly, Barclays' use of the BoE emergency credit facility was immediately reported in the press (see "*Barclays admits borrowing hundreds of millions at Bank's emergency rate*," The Guardian, August 30, 2007). Finally, a 2009 BoE report concludes that a BBC leak that Northern Rock had received funds from the BoE was instrumental in the bank's demise (see Treasury 2008). Further, as discussed below, (partial) observability is a necessary condition to have DW stigma.

[14] Under *R1*, the social surplus when an insolvent bank is not funded (in which case the bank gets 0 and the investor v) is larger than the social surplus when an insolvent bank is funded (in which case the bank gets k and the investor 0).

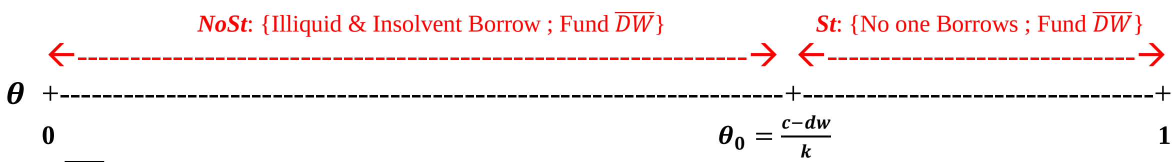
[15] The second inequality in *R2* also implies that in the absence of the DW, the market is not frozen in period 2, as the investor is willing to fund a random bank. This is in contrast with the model of Tirole (2012) and Che et al. (2015) who consider a situation in which a government intervention can jumpstart a market that is otherwise frozen. Unlike other *temporary* government programs implemented during the 2008 financial crisis, however, the DW is a *permanent* facility and its primary role is not to jumpstart a market. Although the presence of the DW does not jumpstart the market under *R2*, we will show that it improves social welfare. Nevertheless we relax *R2* below and show that there cannot be DW stigma in equilibrium in that case.

NoSt = {Illiquid banks borrow at the DW; Investors fund only banks not seen borrowing at the DW}

Proposition 1: When $\theta > \theta_0 = \frac{c-dw}{k}$ the unique equilibrium is ***St***, otherwise the unique equilibrium is ***NoSt***.

Proof: see Appendix 1.

As illustrated in the chart below, Proposition 1 states that illiquid banks use the DW only when the probability of being detected is lower than the threshold θ_0. DW stigma is therefore an equilibrium phenomenon when θ is large enough. Further, DW borrowers are systematically stigmatized in equilibrium.



Note: $\overline{DW}$ means a bank not seen borrowing at the DW

Comparative statics

The comparative statics are straightforward. As θ increases, the equilibrium switches from strategy profile ***NoSt*** (where illiquid banks borrow at the DW) to strategy profile ***St*** (with DW stigma). Thus, consistent with intuition and the positions of many policy makers,[16] making DW borrower easier to identify publically can generate DW stigma.

All else equal, the range of θ over which DW stigma is an equilibrium phenomenom is wider when the direct and indirect costs of borrowing at the DW are larger. That is, when i) the DW rate dw is higher, ii) the outside option cost c is lower, and iii) the profit k from operating in period 2 is higher.

As long as conditions *R1* and *R2* are satisfied, changes in the other parameters (i.e. v, V, θ, P_{SL}, $P_{S\bar{L}}$ and $P_{\bar{S}\bar{L}}$) do not affect the equilibrium.

Social Welfare

I guess I assume that lending at the DW to insolvent has no cost (because terms of loan already take counterparty risk into consideration: i.e. loan is adequately collaterized, and rate is high). May be add slight social cost to lending to insolvent.

[16] In the 2009 Freedom of Information Act lawsuit filed by Bloomberg News, Fed's officials argued that revealing the identity of DW borrowers would stigmatize banks and impede the Fed's ability to respond to future crises (Madigan 2009, McLaughlin 2009). Recognizing the possible adverse consequences of real-time disclosure, the Dodd-Frank Act of 2010 requires the Fed to reveal the identity of DW borrowers only after a lag of two years. Similarly, to mitigate DW stigma, the BoE has recently extended its DW disclosure lag to five quarters (see Bank of England 2013).

The social optimum is obtained when all illiquid banks borrow at the DW and the investor funds solvent banks only.[17] Social welfare in that case is:

$$W^* = (P_{SL} + P_{S\bar{L}})(k+V) + P_{S\bar{L}}(-dw+k) + P_{\bar{S}\bar{L}}(-dw+v)$$

In the absence of a DW, the investor lends to every bank and illiquid banks incur a loss $-c$. Social welfare in that case is:

$$W_{\overline{DW}} = k + V(P_{SL} + P_{S\bar{L}}) - c(P_{\bar{S}\bar{L}} + P_{S\bar{L}}) \ ,$$

and we have

$$W^* - W_{\overline{DW}} = P_{\bar{S}\bar{L}}(v-k) + (P_{\bar{S}\bar{L}} + P_{S\bar{L}})(c-dw) > 0 \ ,$$

where the last inequality comes from *R1*. W^* and $W_{\overline{DW}}$ can therefore serve as an upper and a lower benchmark to evaluate social welfare when DW borrowing is allowed.

Compared to the case without DW, social welfare remains unchanged when the probability to detect a DW borrower is high and the strategy profile ***St*** is the equilibrium: $W_A = W_{\overline{DW}}$ when $\theta > \theta_0$. When $\theta < \theta_0$, under the equilibrium strategy profile ***NoSt***, social welfare is

$$W_{NoSt} = (P_{\bar{S}\bar{L}} + P_{S\bar{L}})(-dw + (1-\theta)k + \theta v) + P_{S\bar{L}}(1-\theta)V + P_{SL}(k+V) \ ,$$

and we have

$$W_{NoSt} - W_{\overline{DW}} = (P_{SL} + P_{S\bar{L}})(c - dw - \theta k) + \theta[(P_{\bar{S}\bar{L}} + P_{S\bar{L}})v - VP_{S\bar{L}}].$$

Observe that under *R1*, $c - dw - \theta k > 0$ when $0 < \theta < \theta_0 = \frac{c-dw}{k}$. Further, under *R2* $(P_{\bar{S}\bar{L}} + P_{S\bar{L}})v - VP_{S\bar{L}} > 0$. Thus $W_{NoSt} - W_{\overline{DW}} > 0$. In other words, social welfare improves with a well functioning DW that is used by illiquid banks in equilibrium.

How does W_B compares to W^*?

Model Extensions

Model with random requirement to borrow at the DW ("arm twisting")

We now modify the baseline model by imposing that a bank may be required to borrow at the DW. Specifically, each bank at the beginning of period 1 may be selected at random with a probability α, where we will assume that α is small. The selected banks, regardless of whether they are liquid or illiquid, are told they must borrow at the DW. Thus, in period 1, the illiquid non-selected banks have to decide whether or not to borrow at the DW. In contrast, liquid banks and the randomly selected banks have no decisions to make in period 1. Below, we differentiate involuntary DW borrowing by the selected banks, from voluntary borrowing by the illiquid non-selected banks.

The two pairs of strategies:

[17] It can be argued that the social optimum is obtained when insolvent banks are not allowed to borrow at the DW. This would not change the nature of the welfare comparisons conducted below.

Proposition 2: When $\theta < \theta_0 = \frac{c-dw}{k}$ the unique equilibrium is $\boldsymbol{NoSt}$, otherwise there is no equilibrium.

Proposition 2 shows that imposing that even a small fraction α>0 of banks must borrow at the DW eliminates the stimgma strategy $\boldsymbol{St}$ as an equilibrium even when the probability of detection $\theta < 1$ is high. The only possible equilibrium, is the no stigma equilibrium $\boldsymbol{NoSt}$ (in which every non-selected illiquid bank borrow at the DW) when the probability of detection is small enough (i.e. $\theta < \theta_0$).

Model with endogenous probability of detection

Practitioners often argue that the probability to identify DW borrower at the DW decreases with the number of banks that borrow at the DW. In other words, θ is endogenous and $\theta'(n) < 0$, where n is the number of banks that borrow at the DW. We now have a coordination problem with asymmetric information. Define the threshold n_0 such that $\theta(n_0) = \frac{c-dw}{k}$. It is easy to verify that if there is a critical mass $n > n_0$ of DW borrowers, then borrowing at the DW is a dominant strategy for an illiquid bank. Thus $\boldsymbol{NoSt}$ is an equilibrium. Conversely, if there is not a critical mass (i.e. $n < n_0$) of DW borrowers, then borrowing at the DW is dominated for an illiquid bank. Thus, $\boldsymbol{St}$ is also an equilibrium.

To sum up, when the probability of being detected is endogenous, there are two pure strategy equilibria, one bad equilibrium (i.e. with lower social welfare) with DW stigma in which no bank borrows at the DW, and one good equilibrium without DW stigma in which every illiquid bank borrows at the DW.

If we combine an endogenous probability of detection and a (small) random requirement to borrow at the DW, then there is only a unique equilibrium without stigma, $\boldsymbol{NoSt}$, in which every non-selected illiquid bank borrow at the DW.

Note that this might be a particularly interesting model to implement in the lab because it has two equilibria (one good and one bad) and it will be interesting to know which one subjects are drawn to. Then, it will be interesting to study which lever needs to be pulled to get subjects to switch from one to the other. In particular, changing the critical mass of DW borrowers n_0 by modifying dw, c or k, may help steer subjects toward the good equilibrium. Note also that one of the policy enacted by the Fed in 2007 was a form of "arm twisting" in which it "forced" a number of liquid-solvent banks to borrow at the DW, which is equivalent to trying to create a critical mass.

Heterogeneous illiquidity cost

We now relax the assumption that the cost of avoiding the DW is the same across illiquid banks. Instead, we now assume that the cost of not borrowing at the DW is c_l for the some banks and $c_h > c_l$ for others. To simplify, we assume that insolvent banks have a cost c_h while solvent-illiquid banks may have a cost c_l or c_h. So there are now four types of banks:

solvent-liquid (with probability P_{SL}), solvent-illiquid with a low illiquidity cost (with probability $P_{S\bar{L}c_l}$), solvent-illiquid with a high illiquidity cost (with probability $P_{S\bar{L}c_h}$) and insolvent-illiquid (with probability $P_{\bar{S}\bar{L}} = 1 - P_{S\bar{L}c_l} - P_{S\bar{L}c_h} - P_{SL}$).

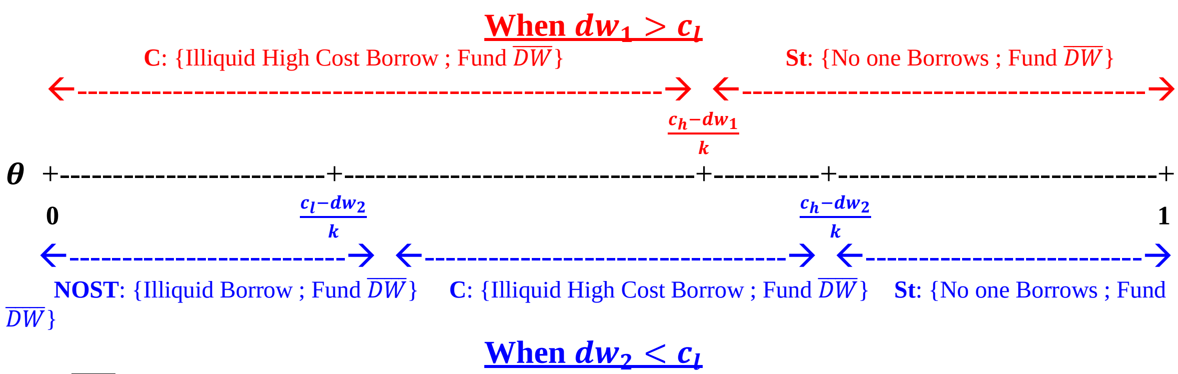
When $c_l < dw < c_h$ the low cost illiquid banks do not borrow at the DW. The model is then formally equivalent to the baseline model under $c = c_h$. Hence, as illustrated in the chart below, we have the same equilibria strategy profiles **St** and **NOST** as in the baseline model. We now consider the case $dw < c_l$ in which case the probability that a bank is solvent conditional on being illiquid in *R2* becomes $\underline{P} = \frac{P_{S\bar{L}c_l}+P_{S\bar{L}c_h}}{P_{S\bar{L}c_l}+P_{S\bar{L}c_h}+P_{\bar{S}\bar{L}}}$. Consider the additional strategy profile

C = {Only high cost illiquid banks borrow at the DW; Investors fund only banks not seen borrowing at the DW}

<u>Proposition 2</u>: **St** is an equilibrium when $\theta > \frac{c_h - dw}{k}$, **NOST** is an equilibrium when $\theta < \frac{c_l - dw}{k}$, and **C** is an equilibrium when $\frac{c_l - dw}{k} < \theta < \frac{c_h - dw}{k}$.

Proof: see Appendix 2.

The different equilibria are illustrated in the chart below.



Note: $\overline{DW}$ means a bank not seen borrowing at the DW

Thus, decreasing the cost of the DW from $dw_1 > c_l$ to $dw_2 < c_l$ has now two effects, it widen the range of θ values for which DW borrowing by high cost illiquid banks is an equilibrium, and it creates an additional equilibrium under which all illiquid banks borrow.[18]

Note: This model will be particularly interesting/necessary when we look at the impact of a liquidity auction like TAF (in which you can entice low cost banks to bid at the auction

[18] Other forms of heterogeneity between illiquid banks produce similar types of equilibria. For instance, consider the baseline model and assume that the continuation profit is equal to $k + \alpha$ (with $\alpha > 0$) for a solvent bank and k for an insolvent bank, so that, when funded, solvent banks earn more in period 2 than insolvent banks. In that case **St** is an equilibrium when $\theta > \frac{c-dw}{k}$, **NOST** is an equilibrium when $\theta < \frac{c-dw}{k+\alpha}$, and **D** is an equilibrium when $\frac{c-dw}{k+\alpha} < \theta < \frac{c-dw}{k}$, where the strategy profile **D** is {Only insolvent banks borrow at the DW; Investors fund only banks not seen borrowing at the DW}.

even when they have no incentive to go to the DW). This model may also be interesting to look more precisely at the impact of changing the DW rate dw.

Model with frozen markets

Consider the baseline model and denote $\bar{P}$ the probability that a bank is solvent conditional on <u>not</u> being seen borrowing at the DW when every illiquid bank borrow at the DW:

$$\bar{P} = \frac{P_{SL}+(1-\theta)P_{S\bar{L}}}{P_{SL}+(1-\theta)\left(P_{S\bar{L}}+P_{\bar{S}\bar{L}}\right)} > p \ ,$$

where $p = 1 - P_{\bar{S}\bar{L}}$ is the unconditional probability of being solvent. We now replace restriction *R2*, in the baseline model by:

- *R2'*: $p < \frac{v}{V} < \bar{P}$,

Because $v > pV$, the investor does not fund a random bank in period 2. Thus, in the absence of the DW, the market is frozen in period 2.

<u>Proposition 3</u>: When $\theta < \theta_0 = \frac{c-dw}{k}$ the unique equilibrium is ***NoSt***, otherwise there is no equilibrium in pure strategy.

Proof: See Appendix 3.

NOST: {Illiquid Borrow ; Fund $\overline{DW}$} (Jumpstart Market)

←--→

$\boldsymbol{\theta}$ +--+---------------------------------+

0 $\boldsymbol{\theta_0 = \frac{c-dw}{k}}$ **1**

Note: $\overline{DW}$ means a bank not seen borrowing at the DW

As illustrated in the chart above, there are no equilibria in which illiquid banks do not borrow at the DW. This means that DW stigma cannot be an equilibrium phenomenon in this case. Further, when $\theta < \theta_0$, the investors funds the banks not seen borrowing at the DW in equilibrium. Thus, the DW serves as a screening device which enables to jumpstart the funding markets in period 2.

Note: while the fact that the DW can jumpstart the funding market in period 2 is interesting, I am not sure it is worth implementing this model in the lab, in part because it cannot generate DW stigma in equilibrium

Parameters for experiment

Possible values for the parameters are: $k = 50$, $c = 40$, and $dw = 20$. The first part of restriction *R1* $(0 < dw < c < k)$ is satisfied. These parameters values imply that the threshold for the probability of being detected is $\theta_0 = \frac{c-dw}{k} = 40\%$. This set of parameters enables easy changes in the cost of the DW (a treatment variable). In particular, subtracting (respectively adding) 5 to $dw = 20$ increases (respectively decreases) the threshold θ_0 by 10%. For instance, $dw = 10$ yields $\theta_0 = 60\%$.

For the case in which the probability of being seen borrowing at the DW is endogenous we could have groups of 5 banks with the following probabilities: $\theta(1) = 2/3$, $\theta(2) = 1/2$, $\theta(3) = 1/3$, $\theta(4) = 1/6$, and $\theta(5) = 0$. So in this case, it would require 3 DW borrowers to cross the threshold of $\theta_0 = 40\%$. These probabilities, as well as the number of banks required to cross the threshold θ_0, would then be adjusted depending on the treatment.

With respect to the types' probabilities, I would suggest P_{SL}=1/2 , $P_{S\bar{L}}$ =1/6 and $P_{\bar{S}\bar{L}}$ =1/3. So the unconditional probability that a bank is solvent $p = 1 - P_{\bar{S}\bar{L}}$=2/3 and the probability that a bank is solvent conditional on being illiquid $\underline{P} = \frac{P_{S\bar{L}}}{P_{S\bar{L}}+P_{\bar{S}\bar{L}}}$ =1/3. The constraint *R2* is then $\underline{P} = \frac{1}{3} < \frac{v}{V} < p = \frac{2}{3}$. Setting $V = 100$ and $v = 50$ would be such that $\frac{v}{V} = 1/2$ is half way between $\underline{P}$ and p. As a result, an investor has as much incentive to not fund a bank know to be illiquid ($\underline{P}V = 33.3 < v = 50$), than to fund a random bank ($v = 50 < pV = 66.6$).

In each session, the subjects are divided in an equal number of banks and investors. In each round, a bank is matched randomly to an investor. I suggest using the strategy method: In period 1 of a round, the subject playing the bank is asked to decide i) whether he will borrow at the DW if he turns out to be solvent-illiquid and ii) whether he will borrow at the DW if he turns out to be insolvent-illiquid. Similarly, we ask the subject playing the bank to decide i) whether he will fund a bank seen borrowing at the DW and ii) whether he will fund a bank not seen borrowing at the DW. After decisions are made, uncertainty is resolved (i.e. the type of the bank is drawn and the bank is seen or not seen borrowing at the DW). One of the main advantages of using the strategy method in this experiment is that we do not have to recruit subjects to play the solvent-liquid banks (which have no decision to make).

I also suggest providing plenty of feedback between rounds. In particular, both the bank and the investor should be told what their own type and payoff were at the end of each round. Further, summary measures about the rest of the subjects should also be provided. In particular, we should report the proportion of banks that ended up being seen borrowing at the DW, the proportion of banks that were funded conditional on being seen at the DW, and the proportion of banks that were funded conditional on not being seen at the DW.

Finally, I do not see an issue framing the experiment using terms like "bank," "investor," "insolvent," and "illiquid." However, I think we could use something like "emergency lending facility" instead "discount window" because the latter is unlikely to mean anything to subjects.

Questions and Experimental Treatments

I think question 1.1 and 1.2 could be a first paper. Question 2 and 3 could each be a different paper.

Question 1.1: Is DW stigma an equilibrium or a behavioral phenomenon?

To address this question we could take the baseline model and set the probability of detection θ above and below the threshold $\theta_0 = \frac{c-dw}{k}$. When θ is below the threshold there should not be DW stigma in equilibrium. When θ is above the threshold there should be DW stigma in equilibrium. With the parameters in the previous section in which $\theta_0 = 0.4$, we could start with $\theta = 0.3$ and $\theta = 0.5$. If there is DW stigma when $\theta = 0.3$ we should try a lower value such as $\theta = 0.1$. Further, it might be interesting to test some of the comparative statics. In particular, for a given θ, how do subjects react when we change the DW rate dw, or the loss c vary? Are subjects able to switch from a bad equilibrium with DW stigma to a good equilibrium without DW stigma? In all of those cases, it would also be interesting to verify how social welfare compares to the theoretic benchmarks. Finally, it will interesting to look at this question with the model where the probability of detection is endogenous because there are two equilibria, one good (without DW stigma) and one bad. In fact, this model with endogenous probability of detection may be the most interesting.

Question 1.2: If DW stigma is a behavioral phenomenon, then how can it be eliminated?

In August 2007, the Fed tried at least three strategies to alleviate DW stigma that could be tested in the lab: i) lower the cost of the DW (dw), ii) "cheap talk," i.e. tell banks it would be better for them to go to the DW, iii) "arm twisting," force a number of solvent-liquid banks to borrow at the DW. I am sure there other policies I have not thought about that should be tested. Again, it might be interesting to look at this question with the model where the probability of detection is endogenous to test whether subjects switch from the bad to the good equilibrium.

Question 2: Does a liquidity auction (like TAF) solve DW stigma?

Consider the model in which illiquid banks have different illiquidity costs. Say there are N illiquid banks. Set the parameters such that the low cost illiquid banks should not borrow at the DW in equilibrium. Run a first treatment in which illiquid banks can get liquidity only at the DW. Run a second treatment in which there is an additional period, period 0. In period 0 there is a liquidity auction in which we (i.e. the central bank) allocate $n<N$ units of liquidity with a sufficiently low reserve price so that low cost illiquid bank should be bidding. Similar to the TAF we could use a uniform price auction. Those who did not get liquidity at the auction can then borrow at the DW is they wish to do so. At the beginning of period 2 the investor observes with probability θ whether the bank received liquidity from the central bank (i.e. at the auction or at the DW). Again it might be interesting to have θ endogenous. In 2008, such an auction (the TAF) seem to have solved the DW problem. Is it the same in the experiment? If so, why? Does it depend on the number of units n allocated at the auction? Does it depend on the low reserve price at the auction? Does it depend on the fact that we are attracting at the auction banks with low liquidity cost that would not go to the DW otherwise?

Question 3: How best to provide emergency lending without stigma? (To be Completed)

This issue is particularly relevant, as several central banks, including the Fed and the Bank of England, have been considering several possible modifications to their emergency lending facility in the wake of the recent financial crisis. The mechanisms we could compare would include in particular a standard DW, a TAF like auction facility, an insurance mechanism (in which financial institutions could purchase ex-ante an insurance against negative liquidity shock), and an option mechanism (in which financial institutions could pre-purchase options to withdraw a liquidity from the central bank at a certain rate). I have not thought too much at this point about how to implement these different mechanisms in the lab.

Appendix 0: Institutional Background[19]

In this section, we provide a brief historical perspective on the DW and the provision of emergency liquidity by the Fed emphasizing the issue of stigma.

The Discount Window

Lending from the DW is in the form of "advances," which are (typically) overnight loans evidenced by promissory notes of the borrowing bank and secured by adequate collateral. All depository institutions that maintain reservable transaction accounts or non-personal time deposits are entitled to borrow at the DW. These include any FDIC-insured bank, savings or mutual bank, insured credit union, and US branch and agency of a foreign bank.[20]

The question of stigma has been a lingering issue throughout the history of the DW and led to fundamental reforms in 2003. Prior to 2003, banks in distress could borrow from the DW at a rate below the Fed target rate. Because of the subsidized rate, the Fed was concerned about "opportunistic overborrowing" by banks. Accordingly, before accessing the DW, a bank had to satisfy the Fed that it had exhausted private sources of funding and that it had a genuine business need for the funds. Hence, if market participants learned that a bank had accessed the DW, then they could conclude that the bank had limited sources of funding. The *old* DW regime therefore created a perception of stigma since DW borrowers revealed financial weakness to the Fed and possibly to competitors. These concerns may have deterred banks from accessing the DW even if they had an urgent need for funds.

To address concerns about DW stigma, the Fed fundamentally changed its DW policy on January 9, 2003. In Regulation A, the Fed classified DW loans into primary credit, secondary credit and seasonal credit. Financially strong and well-capitalized banks can borrow under the primary credit program at a penalty rate above the target fed funds rate (rather than a subsidized rate under the *old* DW regime). Other banks use the secondary credit program and pay a rate higher than the primary credit rate. Finally, seasonal credit is for relatively small banks with seasonal fluctuations in reserves. By far, the most common form of DW borrowing (roughly 99% of the volume over recent years) is through the primary credit program. Therefore, our focus in this paper is exclusively on the primary credit facility. Further, whenever we refer to the DW, we mean the DW primary credit facility.

The new DW is a "no questions asked" facility for primary credit. Namely, the Fed no longer establishes a bank's sources and needs for funding. Instead, primary credit for overnight maturity is allocated with minimal administrative burden. Hence, borrowing from the new DW need not be motivated by pressing funding needs or signal financial weakness.

[19] This section borrows from Armantier et al. (2015)

[20] For exact DW eligibility criteria see http://www.frbdiscountwindow.org/discountwindowbook pf.pdf.

Despite these changes, DW borrowing remained sparse after 2003 and perceptions of stigma resurfaced at the onset of the recent financial crisis. By the end of the summer of 2007, financial institutions were perceived to face serious liquidity shortages for term funding (Hilton and McAndrews 2011). To encourage borrowing, the Fed reduced the DW penalty (i.e. the spread over the target rate) from 100 basis points to 50 basis points on August 17, 2007 and increased the term of DW financing from overnight to as long as 30 days. In addition, the Fed issued statements that DW borrowing would be viewed as a sign of strength and not a sign of weakness (Hilton and McAndrews 2011). These policy changes, however, generated little DW borrowing in the second half of 2007. Stigma was believed to be the main contributing factor to the persistent reluctance in accessing the DW, as illustrated by the earlier quote from then Chairman Bernanke.

The Term Auction Facility

In response to persistently adverse liquidity conditions in the interbank markets, the Fed announced the creation of the Term auction facility (TAF) on December 12, 2007. The TAF was designed as an alternative to the DW to provide term funding. One of the primary objectives of the TAF was to eliminate any perception of stigma attached to borrowing from the DW. A total of 60 TAF auctions were conducted every two weeks between December 17, 2007 and March 8, 2010 when the TAF program was terminated. The amount of credit allocated by the Fed at each auction varied from $20 billion initially to $150 billion at the peak of the crisis. With a few exceptions, the terms of the funds allocated were 28 days, and after August 11, 2008, 84 days.

Since the TAF was introduced as an alternative to the DW, the two facilities shared a number of important features. Funding was offered under the two facilities against the same collateral, using identical haircut calculations. In addition, the same institutions, namely those deemed in sound financial condition by their Federal Reserve District Bank, had access to both facilities. Finally, at the time, the identities of borrowers were not disclosed at either facility.

The TAF and the DW facilities are also different in several respects. First, the DW offers a posted rate determined by the Fed, while the borrowing rate at the TAF was set competitively at an auction. More precisely, TAF bids were accepted in descending order of rates until the funds supplied at the auction were exhausted. The borrowing rate for all winning bidders was then set to the lowest accepted bid rate (the “stop-out rate”).[21] Second, the amount of credit allocated at a TAF was fixed by the Fed before each auction, while the amount of credit available to the banking system at the DW is limited by the amount of collateral posted by banks at the Fed. At the bank level, borrowing at both facilities was limited by the amount of collateral the bank had posted at the Fed, but a bank’s TAF borrowing was further limited to 10% of the total amount supplied by the Fed at the

[21] For undersubscribed auctions, the TAF borrowing rate was automatically set to the overnight index swap (OIS) rate until January 12, 2009 and the Fed’s rate of interest on excess reserve balances afterward.

auction. Third, whereas DW loans are credited on the same day, TAF awards were only credited three days after the auction. Fourth, while most TAF auctions allocated funds for 28 days during our sample period, DW loans could be obtained for any term up to 30 days after August 17, 2007 and up to 90 days after March 16, 2008. Fifth, the DW facility is available every business day, whereas the TAF was operated at two-week intervals. Finally, TAF loans could not be prepaid, while DW loans can be repaid at any time.

Some features of the TAF facility were purposely introduced by the Fed to remove the concerns of stigma that were attached to the DW. In particular, having banks approach the Fed collectively, rather than individually, and obtaining funds at a competitive rate after a three day delay, rather than immediately at a premium set by the Fed, were expected to mitigate any perception that TAF participation was primarily motivated by a pressing need for funding.[22] In addition, a fully subscribed TAF auction would have at least 10 winners (given the 10% cap on bid size) which further reduced the likelihood of an individual institution being publicly singled out. In contrast to the DW, the TAF was an immediate success in terms of amounts bid and allocated (see Figure 1), which provides prima facie evidence that less, if any, stigma was attached to TAF borrowing.

Appendix 1 : Proof of Proposition 1

The restrictions on the model's parameters are:

- *R1* : $0 < dw < c < k \leq v < V$
- *R2* : $\underline{P} < \frac{v}{V} < p$, where $\underline{P} = \frac{P_{S\bar{L}}}{P_{S\bar{L}} + P_{\bar{S}\bar{L}}}$ and $p = 1 - P_{\bar{S}\bar{L}}$

The two pairs of strategies:

St = {No bank borrows at the DW; Investors fund only banks not seen borrowing at the DW}

NOST = {Illiquid banks borrow at the DW; Investors fund only banks not seen borrowing at the DW}

Proposition 1: When $\theta > \theta_0 = \frac{c-dw}{k}$ the unique equilibrium is **St**, otherwise the unique equilibrium is **NOST**.

Proof of Proposition 1: First, observe that there cannot be a separating equilibrium in which one type of illiquid bank borrows at the DW while the other type does not. Indeed, the two types of illiquid banks have the same payoff function. Thus, if it is strictly better for a solvent-illiquid bank to borrow at the DW, then an insolvent-illiquid bank has an incentive to mimic this strategy and behave as if solvent-illiquid.

[22] For further details on how the TAF was designed to remove stigma, see Armantier et al. (2008). For a transcript of the discussions surrounding the creation of the TAF see http://www.federal reserve.gov/monetarypolicy /files/FOMC20071206 confcall.pdf.

Consider now the case where <u>no bank borrows at the DW</u>. In that case, the investor cannot update its beliefs. An investor expected profit at the beginning of period 2 is pV (where $p = 1 - P_{\overline{SL}}$ is the unconditional probability that a bank is solvent) if he funds the bank and v otherwise. Because $v < pV$ under *R2*, the investor funds the bank. The profit of an illiquid bank (whether solvent or insolvent) is then $-c + k$ in that case. Does a bank have an incentive to deviate and borrow at the DW? It depends on the strategy of the investor if it were to observe a bank borrowing at the DW. If the investor funds a bank that is seen borrowing at the DW, then an illiquid bank has a strict incentive to deviate, as this would yield a higher profit $(-dw + k)$ even when the bank is seen borrowing at the DW. If instead the investor does not fund a bank seen borrowing at the DW, then a bank expected profit when it deviates and borrows at the DW becomes $-dw + (1 - \theta)k$. The bank has no incentive to deviate when $-c + k > -dw + (1 - \theta)k$, or equivalently when $\theta > \frac{c-dw}{k}$. *Thus, the strategy profile* ***St*** *is an equilibrium only when θ is large enough, i.e. $\theta > \frac{c-dw}{k}$.*

Consider now the case where <u>every illiquid bank borrows at the DW</u>. In that case, an investor can update its beliefs when it faces a bank that has been seen borrowing at the DW. In particular, because all illiquid banks borrow at the DW, the probability that a bank is solvent conditional on being seen borrowing at the DW is the same as the probability that a bank is solvent conditional on being illiquid: $\underline{P} = \frac{P_{S\bar{L}}}{P_{S\bar{L}} + P_{\overline{SL}}}$. Under *R2*, $v > \underline{P}V$, so an investor does not fund a bank seen borrowing at the DW. In contrast an investor funds a bank not seen borrowing at the DW. Indeed, because all insolvent banks are illiquid and borrow at the DW, the probability $\bar{P}$ that a bank is solvent conditional on <u>not</u> being seen borrowing at the DW is higher than the unconditional probability p. Under *R2*, $v < pV < \bar{P}V$, so an investor funds a bank that has not been observed borrowing at the DW. The expected profit of illiquid banks in that case is $-dw + (1 - \theta)k$. However, if an illiquid bank deviates and does not go to the DW, then it gets a profit of $-c + k$. An illiquid bank has no incentive to deviate when $-dw + (1 - \theta)k > -c + k$, or equivalently when $\theta < \frac{c-dw}{k}$. *Thus, the strategy profile* ***NOST*** *is an equilibrium only when θ is small enough, i.e. $\theta < \frac{c-dw}{k}$.* ■

Appendix 2 : Proof of Proposition 2

Consider the case $dw < c_l$, replace $\underline{P}$, the probability that a bank is solvent conditional on being illiquid, in *R2* by $\underline{P} = \frac{P_{S\bar{L}c_l} + P_{S\bar{L}c_h}}{P_{S\bar{L}c_l} + P_{S\bar{L}c_h} + P_{\overline{SL}}}$, and consider the additional strategy profile

C = {Only high cost illiquid banks borrow at the DW; Investors fund only banks not seen borrowing at the DW}

Proposition 2: **St** is an equilibrium when $\theta > \frac{c_h - dw}{k}$, **NOST** is an equilibrium when $\theta < \frac{c_l - dw}{k}$, **C** is an equilibrium when of θ. $\frac{c_l - dw}{k} < \theta < \frac{c_h - dw}{k}$

Proof of Proposition 2: First, observe that because the two types banks with high illiquidity cost have the same payoff function, there cannot be a separating equilibrium in which one type borrows at the DW while the other type does not. Further, there cannot be an equilibrium in which only illiquid-solvent banks with low illiquidity cost borrow at the DW. Indeed, the investor would fund DW borrowers in that case and a high cost bank would have an incentive to deviate and mimic the strategy of the low cost.

Consider now the case where every illiquid bank borrows at the DW. In that case, an investor can update its beliefs when it faces a bank that has been seen borrowing at the DW. In particular, because all illiquid banks borrow at the DW, the probability that a bank is solvent conditional on being seen borrowing at the DW is the same as the probability that a bank is solvent conditional on being illiquid: $\underline{P} = \frac{P_{S\bar{L}c_l} + P_{S\bar{L}c_h}}{P_{S\bar{L}c_l} + P_{S\bar{L}c_h} + P_{\bar{S}\bar{L}}}$. Under *R2*, $v > \underline{P}V$, so an investor does not fund a bank seen borrowing at the DW. In contrast, an investor funds a bank not seen borrowing at the DW. Indeed, because all insolvent banks are illiquid and borrow at the DW, the probability $\bar{P}$ that a bank is solvent conditional on not being seen borrowing at the DW is higher than the unconditional probability p. Under *R2*, $v < pV < \bar{P}V$, so an investor funds a bank that has not been observed borrowing at the DW. The expected profit of illiquid banks in that case is $-dw + (1-\theta)k$. However, if a low cost illiquid bank deviates and does not go to the DW, then it gets a profit of $-c_l + k$. A low cost illiquid bank has no incentive to deviate when $-dw + (1-\theta)k > -c_l + k$, or equivalently when $\theta < \frac{c_l - dw}{k}$. Similarly, it is easy to see that a high cost illiquid bank has no incentive to deviate when $\theta < \frac{c_h - dw}{k}$. Because $c_l < c_h$ only the first inequality must be binding. *Thus, the strategy profile **NOST** is an equilibrium only when θ is small enough, i.e. $\theta < \frac{c_l - dw}{k}$.*

Consider now the case where every high cost illiquid bank borrows at the DW but the low cost illiquid do not. In that case, an investor can update its beliefs when it faces a bank that has been seen borrowing at the DW. In particular, because all high cost illiquid banks borrow at the DW, the probability that a bank is solvent conditional on being seen borrowing at the DW is the same as the probability that a bank is solvent conditional on being high cost: $\underline{p} = \frac{P_{S\bar{L}c_h}}{P_{S\bar{L}c_h} + P_{\bar{S}\bar{L}}}$. Observes that $\underline{p} < \underline{P} = \frac{P_{S\bar{L}c_l} + P_{S\bar{L}c_h}}{P_{S\bar{L}c_l} + P_{S\bar{L}c_h} + P_{\bar{S}\bar{L}}}$, the probability that a bank is solvent conditional on being illiquid. Thus, under *R2*, we have $v > \underline{P}V > \underline{p}V$, and an investor does not fund a bank seen borrowing at the DW. In contrast, an investor funds a bank not seen borrowing at the DW. Indeed, because all insolvent banks are illiquid and borrow at the DW, the probability $\bar{P}$ that a bank is solvent conditional on not being seen borrowing at the DW is higher than the unconditional probability p. Under *R2*, $v < pV < \bar{P}V$, so the investor funds a bank not seen borrowing at the DW. In that case, the expected profit a high cost illiquid bank is $-dw + (1-\theta)k$ and the expected profit of a low cost illiquid bank is $-c_l + k$. If a high cost illiquid bank deviates and does not go to the DW, then it gets a profit of $-c_h + k$. A high cost illiquid bank has no incentive to deviate when $-dw + (1-\theta)k > -c_h + k$, or equivalently when $\theta < \frac{c_h - \mathrm{dw}}{\mathrm{k}}$. If a low cost illiquid bank deviates and goes to the DW, then it gets a profit of $-dw + (1-\theta)k$. A low cost illiquid bank has no incentive to deviate when $-c_l + k > -dw + (1-\theta)k$, or equivalently when $\theta > \frac{c_l - dw}{k}$. *The strategy profile* ***C*** *is therefore an equilibrium when* $\frac{c_l - dw}{k} < \theta < \frac{c_h - dw}{k}$.

Consider now the case where <u>no bank borrows at the DW</u>. In that case, an investor cannot update its beliefs. An investor expected profit at the beginning of period 2 is pV if he funds the bank and v otherwise. Because $v < pV$ under *R2*, the investor funds the bank. The profit of a low cost illiquid bank is $-c_l + k$ while the profit of a high cost illiquid bank is $-c_h + k$. Does a bank have an incentive to deviate and borrow at the DW? It depends on the strategy of the investor if it were to observe a bank borrowing at the DW. If the investor funds a bank that is seen borrowing at the DW, then every illiquid bank has a strict incentive to deviate as it would get a higher profit ($-dw + k$) even when seen borrowing at the DW. If instead the investor does not fund a bank seen borrowing at the DW, then an illiquid bank expected profit when it deviates and borrows at the DW becomes $-dw + (1-\theta)k$. The high cost illiquid bank has no incentive to deviate when $-c_h + k > -dw + (1-\theta)k$, or equivalently when $\theta > \frac{c_h - dw}{k}$. The low cost illiquid bank has no incentive to deviate when $-c_l + k > -dw + (1-\theta)k$, or equivalently when $\theta > \frac{c_l - dw}{k}$. Because $c_l < c_h$ only the first inequality must be binding. *Thus, the strategy profile* ***St*** *is an equilibrium only when* θ *is large enough, i.e.* $\theta > \frac{c_h - dw}{k}$. ■

Appendix 3: Proof of Proposition 3

Consider the baseline model and denote $\bar{P}$ the probability that a bank is solvent conditional on <u>not</u> being seen borrowing at the DW when every illiquid bank borrow at the DW (while liquid banks do not borrow at the DW). We have

$$\bar{P} = \frac{P_{SL} + (1-\theta)P_{S\bar{L}}}{P_{SL} + (1-\theta)(P_{S\bar{L}} + P_{\bar{S}\bar{L}})} > p$$

where $p = 1 - P_{\bar{S}\bar{L}}$ is the unconditional probability of being solvent. Replace restriction *R2*, in the baseline model by:

- *R2'*: $p < \frac{v}{V} < \bar{P}$,

Proposition 3: When $\theta < \theta_0 = \frac{c-dw}{k}$ the unique equilibrium is **NOST**, otherwise there is no equilibrium in pure strategy.

Proof of Proposition 3: Because the two types of illiquid banks have the same payoff function, there cannot be a separating equilibrium in which one type of illiquid bank borrows at the DW while the other type does not.

Consider first the case where no bank borrows at the DW. In that case, an investor cannot update its beliefs. The investor expected profit at the beginning of period 2 is pV if he funds the bank and v otherwise. Because $v > pV$ under *R2'*, an investor does not fund a bank not seen borrowing at the DW. The profit of an illiquid bank (whether solvent or insolvent) is then $-c$ in that case. Regardless of the strategy of the investor, a bank always has an incentive to deviate and borrow at the DW as it would yield at least $-dw > -c$. *Thus, the strategy profile St is NOT an equilibrium.*

Consider now the case where every illiquid bank borrows at the DW. In that case, an investor can update its beliefs when it faces a bank that has been seen borrowing at the DW. In particular, because all illiquid banks borrow at the DW, the probability that a bank is solvent conditional on being seen borrowing at the DW is the same as the probability that a bank is solvent conditional on being illiquid: $\underline{P} = \frac{P_{S\bar{L}}}{P_{S\bar{L}}+P_{\bar{S}\bar{L}}}$. Under *R2'*, $v > pV > \underline{P}V$, so an investor does not fund a bank seen borrowing at the DW. In contrast, the investor funds a bank that has not been observed borrowing at the DW since $v < \bar{P}V$ under *R2'*, where $\bar{P}$ is the probability that a bank is solvent conditional on not being seen borrowing at the DW. The expected profit of illiquid banks in that case is $-dw + (1-\theta)k$. However, if an illiquid bank deviates and does not go to the DW, then it gets a profit of $-c + k$. An illiquid bank has no incentive to deviate when $-dw + (1-\theta)k > -c + k$, or equivalently when $\theta < \frac{c-dw}{k}$. *Thus, the strategy profile **NOST** is an equilibrium only when θ is small enough, i.e. $\theta < \frac{c-dw}{k}$.* ■

The restrictions on the model's parameters are:

- *R1* : $0 < dw < c < k \leq v < V$
- *R2* : $\underline{P} < \frac{v}{V} < p$, where $\underline{P} = \frac{P_{S\bar{L}}}{P_{S\bar{L}}+P_{\bar{S}\bar{L}}}$ and $p = 1 - P_{\bar{S}\bar{L}}$

The two pairs of strategies:

ST = {No bank borrows at the DW; Investors fund only banks not seen borrowing at the DW}

$\overline{ST}$ = {Illiquid banks borrow at the DW; Investors fund only banks not seen borrowing at the DW}

Proposition 1: When $\theta > \theta_0 = \frac{c-dw}{k}$ the unique equilibrium is **St**, otherwise the unique equilibrium is **NOST**.

Proof of Proposition 1:

At the beginning of period 1, each bank may be selected at random with a probability α. The selected banks, regardless of whether they are liquid or illiquid, are told they must borrow at the DW. Thus, in period 1, the illiquid non-selected banks have to decide whether or not to borrow at the DW. In contrast, liquid banks and the randomly selected banks have no decisions to make in period 1. Below, we differentiate involuntary DW borrowing by the selected banks, from voluntary borrowing by the illiquid non-selected banks.

First, observe that there cannot be a separating equilibrium in which one type of illiquid bank voluntarily borrows at the DW while the other type does not. Indeed, the two types of illiquid banks have the same payoff function. Thus, if it is strictly better for a solvent-illiquid bank to borrow voluntarily at the DW, then an insolvent-illiquid bank has an incentive to mimic this strategy and behave as if solvent-illiquid.

Consider now the case where no bank voluntarily borrows at the DW. In that case, the investor cannot update its beliefs. Indeed, if a bank is observed borrowing at the DW, then the investor knows that this borrowing was involuntary and therefore the investor does not receive any information about the bank's type. An investor expected profit at the beginning of period 2 is pV (where $p = 1 - P_{\bar{S}L}$ is the unconditional probability that a bank is solvent) when the investor funds a bank, whether the bank was observed or not observed borrowing at the DW. When the investor does not fund a bank its profit is v. Because $v < pV$ under *R2*, the investor funds every bank, that is, whether or not a bank is observed borrowing at the DW. The profit of a non-selected illiquid bank (whether solvent or insolvent) is then $-c + k$ when it does not borrow at the DW. Because the investor also funds banks observed borrowing at the DW, a non-selected illiquid bank has an incentive to deviate and borrow at the DW. Indeed, doing so would yield a higher profit ($-dw + k$) even when the bank is seen borrowing at the DW. *Thus, the strategy profile* ***ST*** *can no longer be an equilibrium.*

Consider now the case where every non-selected illiquid bank voluntarily borrows at the DW. In that case, an investor can update its beliefs when it faces a bank that has been seen borrowing at the DW. In particular, because all illiquid banks borrow at the DW, the probability that a bank is solvent conditional on being seen borrowing at the DW is

$\underline{\mathcal{P}} = \alpha p + (1-\alpha)\underline{P}$ where p is the unconditional probability that a bank is solvent and $\underline{P} = \frac{P_{S\bar{L}}}{P_{S\bar{L}}+P_{\bar{S}\bar{L}}}$ is the probability that a bank is solvent conditional on being illiquid. Observe that $\underline{P} < \underline{\mathcal{P}} < p$. <u>We now assume that α is small enough so that $\underline{\mathcal{P}}$V $< v$</u>. In that case, an investor has no incentives to fund a bank seen borrowing at the DW. In contrast, an investor funds a bank not seen borrowing at the DW. Indeed, because all insolvent banks are illiquid and borrow at the DW, the probability $\bar{\mathcal{P}}$ that a bank is solvent conditional on <u>not</u> being seen borrowing at the DW is higher than the unconditional probability p. Under *R2*, $v < pV < \bar{\mathcal{P}}V$, so an investor funds a bank that has not been observed borrowing at the DW. The expected profit of a non-selected illiquid bank is $-dw + (1-\theta)k$ in that case. However, if an illiquid bank deviates and does not go to the DW, then it gets a profit of $-c + k$. An illiquid bank has no incentive to deviate when $-dw + (1-\theta)k > -c + k$, or equivalently when $\theta < \frac{c-dw}{k}$. *Thus, the strategy profile $\overline{ST}$ is an equilibrium only when θ is small enough, i.e. $\theta < \frac{c-dw}{k}$.* ■